WJEC – AS Unit 1 Pure Mathematics

17th May 2023 2 hours 30 minutes

PAPER WITH TEACHER'S SOLUTIONS

Total marks available 120

Reminder: Sufficient working must be shown to demonstrate the mathematical method employed.

1. **a)** Using the binomial theorem, write down and simplify the first three terms in the expansion of $(1 - 3x)^9$ in ascending powers of x. [3]

b) Hence, by writing $x = 0{\cdot}001$ in your expansion in part (a), find an approximate value for $(0{\cdot}997)^9$. Show all your working and give your answer correct to three decimal places. [3]

a) Using formula booklet :-

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots$ (only asked for 3 terms)

$(1-3x)^9 = 1 + 9(-3x) + \frac{9(8)}{2}(-3x)^2$

$= 1 - 27x + 324x^2$

b) $(1-3(0.001))^9 = 1 - 27(0.001) + 324(0.001)^2$

$= 1 - 0.027 + 0.000324$

$= 0.973324$

$= 0.973$ (3dp)

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2. Solve the following equation for values of θ between 0° and 360°.

$$3\sin^2\theta - 5\cos^2\theta = 2\cos\theta$$ [7]

Using $\sin^2\theta + \cos^2\theta \equiv 1$

$\sin^2\theta \equiv 1 - \cos^2\theta$

equation becomes

$3(1-\cos^2\theta) - 5\cos^2\theta - 2\cos\theta = 0$

$3 - 3\cos^2\theta - 5\cos^2\theta - 2\cos\theta = 0$

$-8\cos^2\theta - 2\cos\theta + 3 = 0$

$8\cos^2\theta + 2\cos\theta - 3 = 0$

x ac	+b
-24	2

+6; -4

$8\cos^2\theta - 4\cos\theta + 6\cos\theta - 3 = 0$

$4\cos\theta(2\cos\theta - 1) + 3(2\cos\theta - 1) = 0$

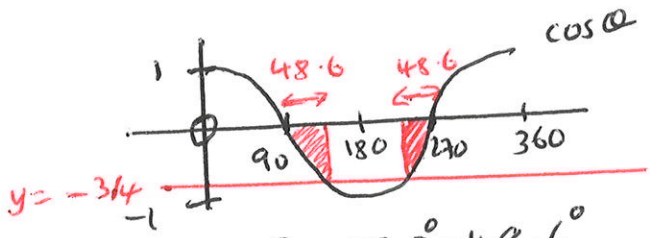
$(4\cos\theta + 3)(2\cos\theta - 1) = 0$

$4\cos\theta + 3 = 0$

$4\cos\theta = -3$

$\cos\theta = -\frac{3}{4}$

$\theta = 138.6°$



$\theta = 270° - 48.6°$

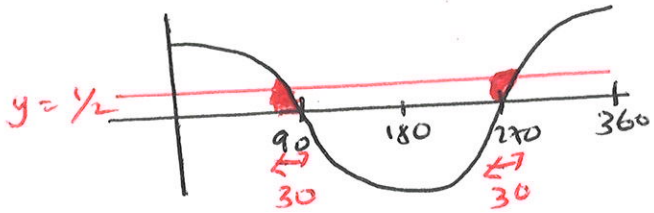
$\theta = 221.4°$

$2\cos\theta - 1 = 0$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\theta = 60°$



$\theta = 270° + 30°$

$\theta = 300°$

$\theta = 60°, 138.6°, 221.4°, 300°$

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3. The point A has coordinates $(-2, 5)$ and the point B has coordinates $(3, 8)$. The point C lies on the x-axis such that AC is perpendicular to AB.

a) Find the equation of AB. [3]

b) Show that C has coordinates $(1, 0)$. [3]

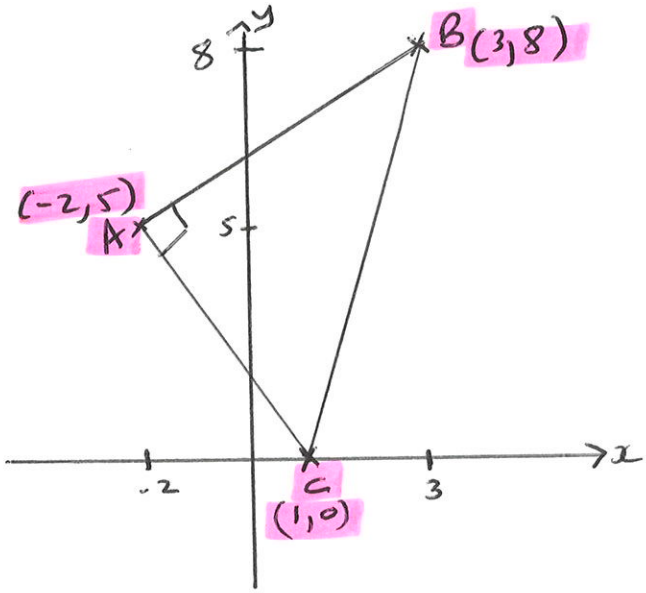
c) Calculate the area of triangle ABC. [4]

d) Find the equation of the circle which passes through the points A, B and C. [5]

a) gradient $AB = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{8-5}{3--2}$

$= \frac{3}{5}$



$(y - y_1) = m(x - x_1)$

$y - 5 = \frac{3}{5}(x - -2)$

$5y - 25 = 3x + 6$

$5y = 3x + 31$

b) gradient $AC = -\frac{5}{3}$

$y - y_1 = m(x - x_1)$ using $A = (-2, 5)$ as (x_1, y_1)

$y - 5 = -\frac{5}{3}(x - -2)$

$3y - 15 = -5x - 10$

$3y = -5x + 5$

at C, given $y = 0$, lies on x axis

$0 = -5x + 5$

$5x = 5$

$x = 1$, $C = (1, 0)$ as required

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c) Area $\Delta ABC = \frac{1}{2} \times$ base $\times$ perp. height

Base = AC

length AC $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-2-1)^2+(5-0)^2}$

$= \sqrt{9+25}$

$= \sqrt{34}$

Perp. height = AB

length AB $= \sqrt{(3-(-2))^2+(8-5)^2}$

$= \sqrt{25+9}$

$= \sqrt{34}$

Area $\Delta ABC = \frac{1}{2} \times \sqrt{34} \times \sqrt{34} = 17$

d) BC must be the diameter of the required circle

Centre $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{1+3}{2}, \frac{0+8}{2}\right) = (2, 4)$

Radius $= \frac{1}{2}$ length of BC

$= \frac{1}{2}\sqrt{(3-1)^2+(8-0)^2}$

$= \frac{1}{2}\sqrt{4+64}$

$= \frac{1}{2}\sqrt{68} = \sqrt{17}$

Equation of circle

$(x-a)^2+(y-b)^2 = r^2$

$(x-2)^2+(y-4)^2 = 17$

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4. **a)** Find the remainder when the polynomial $3x^3+2x^2+x-1$ is divided by $(x-3)$. [3]

b) The polynomial $f(x)=2x^3-3x^2+ax+6$ is divisible by $(x+2)$, where a is a real constant.

i) Find the value of a. [3]

ii) Showing all your working, solve the equation $f(x)=0$. [4]

a) $f(3) = 3(3)^3+2(3)^2+(3)-1$

$= 81+18+3-1 = 101$

b) i) If $f(x)$ is divisible by $(x+2)$

$f(-2)=0$

$2(-2)^3-3(-2)^2+a(-2)+6=0$

$-16-12-2a+6=0$

$-2a = 22$

$a=-11$

ii) $2x^3-3x^2-11x+6 = (x+2)(2x^2+bx+c)$

comparing coefficients

constant :- $6=2c$

$c=3$

x :- $-11 = 2b+c$

$-11 = 2b+3$

$-14 = 2b$

$b=-7$

quadratic factor :- $(2x^2-7x+3)$

$= (2x-1)(x-3)$

$f(x) = (x+2)(2x-1)(x-3)$

$x=-2,\ x=\frac{1}{2},\ x=3$

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5. Simplify the expression $\sqrt[3]{512a^2} - \dfrac{a^{\frac{7}{6}} \times a^{-\frac{1}{3}}}{a^{\frac{1}{6}}}$. [4]

$\sqrt[3]{512a^2} = 8a^{2/3}$

$\dfrac{a^{7/6} \times a^{-1/3}}{a^{1/6}} = \dfrac{a^{7/6 - 1/3}}{a^{1/6}}$

$= a^{5/6 - 1/6}$

$= a^{4/6}$

$= a^{2/3}$

$8a^{2/3} - a^{2/3} = 7a^{2/3}$

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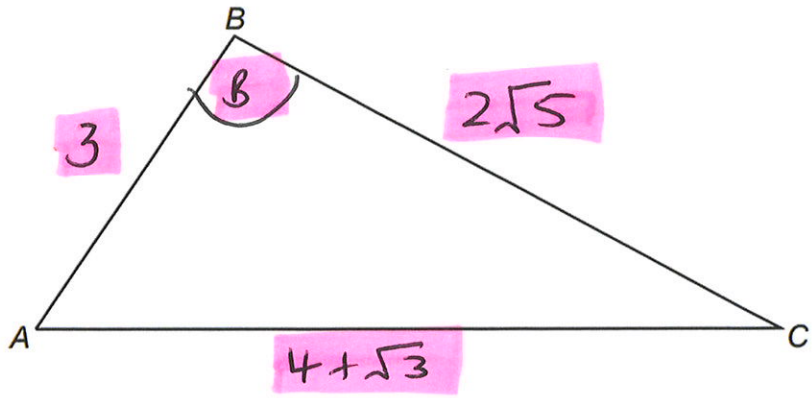
6. The diagram below shows a triangle ABC.



Given that $AB = 3, BC = 2\sqrt{5}, AC = 4+\sqrt{3}$, find the value of $\cos A\hat{B}C$. Show all your working and give your answer in the form $\frac{(a-b\sqrt{3})}{6\sqrt{5}}$, where a, b are integers. [7]

Using cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos\hat{A}$$
$$b^2 = a^2 + c^2 - 2ac\cos\hat{B}$$

$$\cos\hat{B} = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos\hat{B} = \frac{(2\sqrt{5})^2 + (3)^2 - (4+\sqrt{3})^2}{2\times 2\sqrt{5}\times 3}$$

$$\cos\hat{B} = \frac{20 + 9 - (16 + 8\sqrt{3} + 3)}{12\sqrt{5}}$$

$$\cos\hat{B} = \frac{10 - 8\sqrt{3}}{12\sqrt{5}}$$

$$\cos\hat{B} = \frac{2(5-4\sqrt{3})}{2(6\sqrt{5})}$$

$$\cos\hat{B} = \frac{5-4\sqrt{3}}{6\sqrt{5}}$$

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7. The curve C has equation $y = 2x^2 + 5x - 12$ and the line L has equation $y = mx - 14$, where m is a real constant.

a) Given that L is a tangent to C,

i) show that m satisfies the equation

$$m^2 - 10m + 9 = 0,$$ [5]

ii) find the coordinates of the two possible points of contact of C and L. [6]

b) Given instead that L intersects C at two distinct points, find the range of values of m. [2]

ai) Curve C and line L intersect when

$2x^2 + 5x - 12 = mx - 14$

$2x^2 + (5-m)x + 2 = 0$

Now, if L is a tangent, $b^2 - 4ac = 0$, because there is exactly one point of contact

$b^2 - 4ac = 0$

$(5-m)^2 - 4(2)(2) = 0$

$25 - 10m + m^2 - 16 = 0$

$m^2 - 10m + 9 = 0$ as required

ii) $(m-1)(m-9) = 0$

$m = 1$ and $m = 9$

If $m = 1$,

$2x^2 + 4x + 2 = 0$

$x^2 + 2x + 1 = 0$

$(x+1)^2 = 0$

$x = -1$

$y = -1 - 14$

$y = -15$

$(-1, -15)$

If $m = 9$

$2x^2 - 4x + 2 = 0$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$

$y = 9 - 14$

$y = -5$

$(1, -5)$

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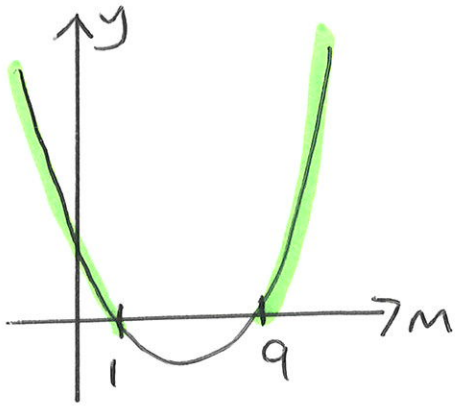
b) If L and C intersect at 2 distinct points

$b^2 - 4ac > 0$

$(m-1)(m-9) > 0$

Consider critical values

$m < 1$ or $m > 9$



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8. Show, by counter example, that the following statement is false.

"For all positive integer values of n, n^2+1 is a prime number." [3]

Consider $n=3$

$n^2=9$

$n^2+1=10$

10 is not a prime number

(factors of 10 are 1, 2, 5, 10)

Hence, the statement is false.

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9. **a)** Given that $y = x^2 - 3x$, find $\frac{dy}{dx}$ from first principles. [5]

b) The function f is defined by $f(x) = 4x^{\frac{3}{2}} + \frac{6}{\sqrt{x}}$ for $x > 0$.

i) Find $f'(x)$. [2]

ii) When $x > k$, $f(x)$ is an increasing function. Determine the least possible value of k. Give your answer correct to two decimal places. [4]

a) $f(x) = x^2 - 3x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2xh + h^2 - 3h}{h}$$

$$f'(x) = \lim_{h \to 0} (2x + h - 3)$$

$$f'(x) = 2x - 3 \quad , \quad \frac{dy}{dx} = 2x - 3$$

OR

$$y = x^2 - 3x$$

$$(y + \delta y) = (x + \delta x)^2 - 3(x + \delta x)$$

$$y + \delta y = x^2 + 2x\delta x + (\delta x)^2 - 3x - 3\delta x$$

subtracting y from both sides

$$\delta y = 2x\delta x + (\delta x)^2 - 3\delta x$$

dividing by δx

$$\frac{\delta y}{\delta x} = 2x + \delta x - 3$$

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 2x - 3$$

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b i) $f(x) = 4x^{3/2} + 6x^{-1/2}$

$f'(x) = \frac{3}{2} \cdot 4x^{1/2} + \left(-\frac{1}{2}\right) \cdot 6x^{-3/2}$

$f'(x) = 6x^{1/2} - 3x^{-3/2}$

ii) Increasing function, $\therefore f'(x) > 0$

$6x^{1/2} - 3x^{-3/2} > 0$

$6x^{1/2} > 3x^{-3/2}$ $(\div 3)$

$2x^{1/2} > x^{-3/2}$ $(\div x^{-3/2})$

$\frac{2x^{1/2}}{x^{-3/2}} > 1$

$2x^{1/2 - -3/2} > 1$

$2x^2 > 1$

$x^2 > \frac{1}{2}$

$x > \sqrt{\frac{1}{2}}$

$x > 0.7071\ldots$

$x > 0.71$ (2dp)

$\therefore k = 0.71$

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10. Solve the following equations for values of x.

a) $\ln(2x+5)=3$ [2]

b) $5^{2x+1}=14$ [3]

c) $3\log_7(2x) - \log_7(8x^2) + \log_7 x = \log_3 81$ [6]

a) $\ln(2x+5)=3$

rewrite as a power

$e^3 = 2x+5$

$2x = e^3 - 5$

$x = \frac{e^3-5}{2}$, $x = 7.542768\ldots$

$x = 7.54$ (2dp)

b) $5^{2x+1} = 14$ (rewrite as a logarithm)

$\ln 5^{2x+1} = \ln 14$

$(2x+1)\ln 5 = \ln 14$ (power law)

$2x+1 = \frac{\ln 14}{\ln 5}$

$2x = \frac{\ln 14}{\ln 5} - 1$

$x = \frac{1}{2}\left(\frac{\ln 14}{\ln 5} - 1\right)$

$x = 0.319869\ldots$

$x = 0.32$ (2dp)

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c) $3\log_7(2x) - \log_7(8x^2) + \log_7(x) = \log_3(81)$

$\log_7(2x)^3 + \log_7(x) - \log_7(8x^2) = \log_3(81)$ (power law)

$\log_7\left(\frac{8x^3 \times x}{8x^2}\right) = 4$ (addition & subtraction laws and $3^4 = 81$ so $\log_3 81 = 4$)

$\log_7(x^4) = 4$

rewriting as a power

$7^4 = x^2$

$49^2 = x^2$

$x = 49$

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11. The function f is defined by $f(x) = \frac{8}{x^2}$.

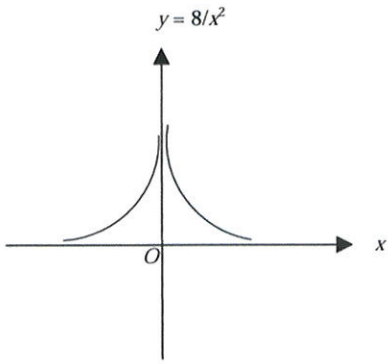
a) Sketch the graph of $y = f(x)$. [2]

b) On a separate set of axes, sketch the graph of $y = f(x-2)$. Indicate the vertical asymptote and the point where the curve crosses the y-axis. [3]

c) Sketch the graphs of $y = \frac{8}{x}$ and $y = \frac{8}{(x-2)^2}$ on the same set of axes.

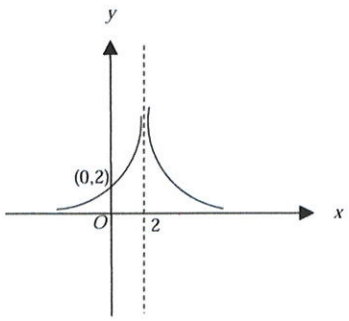
Hence state the number of roots of the equation $\frac{8}{(x-2)^2} = \frac{8}{x}$. [2]

(a)



B2 B1 each branch

(b)



M1 ft shift entire graph to the right

B1 (0, 2) cao

A1 $x = 2$ as **asymptote**

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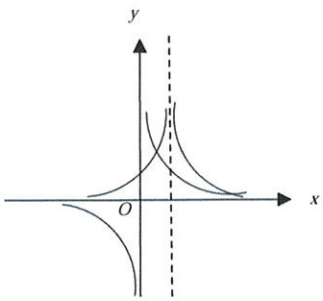
(c)



B1 correct curve $y = \frac{8}{x}$, both branches.

May be seen in (b).

Equation has two solutions

B1 award only if both graphs correct in first quadrant.

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12. The position vectors of the points A and B, relative to a fixed origin O, are given by

$$\mathbf{a} = -3\mathbf{i} + 4\mathbf{j}, \quad \mathbf{b} = 5\mathbf{i} + 8\mathbf{j},$$

respectively.

a) Find the vector **AB**. [2]

b) **i)** Find a unit vector in the direction of **a**. [2]

ii) The point C is such that the vector **OC** is in the direction of **a**. Given that the length of **OC** is 7 units, write down the position vector of C. [1]

c) Calculate the angle AOB. [3]

a) $\vec{OA} = -3\underline{i} + 4\underline{j}$

$\vec{OB} = 5\underline{i} + 8\underline{j}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$\vec{AB} = (5\underline{i} + 8\underline{j}) - (-3\underline{i} + 4\underline{j})$

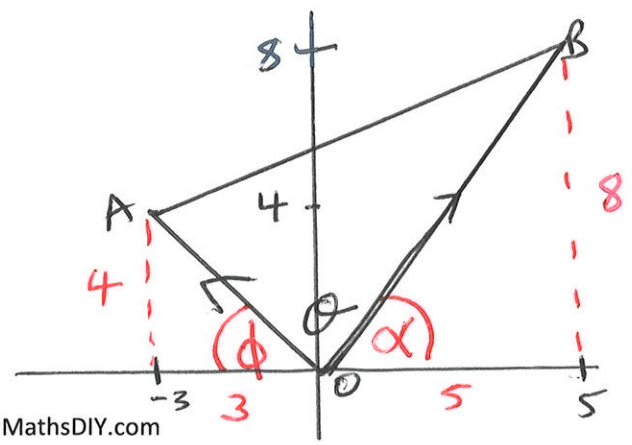
$\vec{AB} = 8\underline{i} + 4\underline{j}$

b)i) $|\vec{OA}| = \sqrt{(-3)^2 + (4)^2} = 5$

$\therefore$ unit vector $= \frac{1}{5}(-3\underline{i} + 4\underline{j})$

ii) $\vec{OC} = 7 \times \frac{1}{5}(-3\underline{i} + 4\underline{j}) = -\frac{21}{5}\underline{i} + \frac{28}{5}\underline{j}$

c)



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$\alpha = \tan^{-1}\left(\frac{8}{5}\right)$

$\alpha = 57.99°$

$\phi = \tan^{-1}\left(\frac{4}{3}\right)$

$\phi = 53.130°$

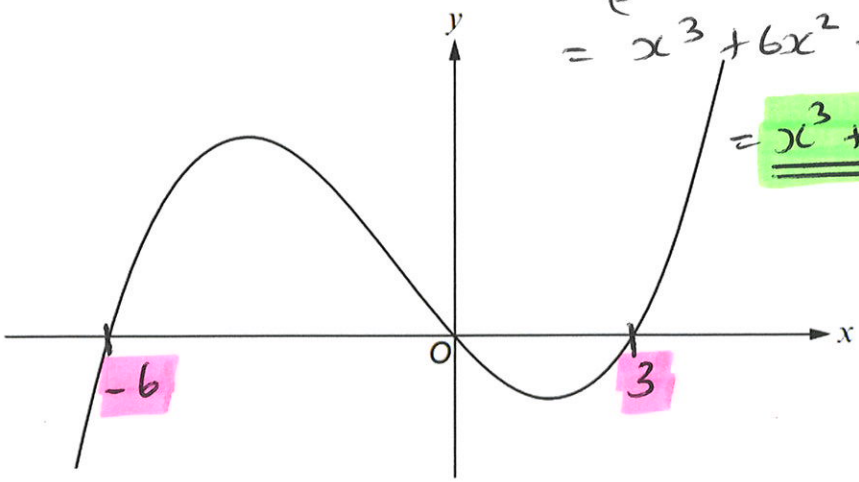
$\theta = 180 - 53.13 - 57.99$

$\theta = 68.88°$

13.

a) Find $\int\left(4x^{-\frac{2}{3}}+5x^3+7\right)\mathrm{d}x$. [3]

b) The diagram below shows the graph of $y=x(x+6)(x-3)$.



Calculate the total area of the regions enclosed by the graph and the x-axis. [9]

$= (x^2+6x)(x-3)$

$= x^3+6x^2-3x^2-18x$

$= x^3+3x^2-18x$

a) $\int(4x^{-2/3}+5x^3+7)\,dx$

$= \frac{4x^{1/3}}{1/3} + \frac{5x^4}{4} + 7x + C$

b) Required area $= \int_{-6}^{0}(x^3+3x^2-18x)\,dx - \int_{0}^{3}(x^3+3x^2-18x)\,dx$

$= \left[\frac{x^4}{4}+\frac{3x^3}{3}-\frac{18x^2}{2}\right]_{-6}^{0} - \left[\frac{x^4}{4}+\frac{3x^3}{3}-\frac{18x^2}{2}\right]_{0}^{3}$

$= \left[0-(324-216-324)\right] - \left[\frac{81}{4}+27-81-0\right]$

$= 216+\frac{135}{4} = \frac{999}{4}$, 249.75

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