WJEC – AS unit 2 Applied Mathematics A

25th May 2023 1 hour 45 minutes

PAPER WITH TEACHER'S SOLUTIONS

Section A Statistics 40 marks

Section B Mechanics 35 marks **Total marks available: 75**

Reminder: *Sufficient working must be shown to demonstrate the **mathematical** method employed.*

Section A: Statistics

0 1 The events A and B are such that $P(A) = 0{\cdot}3$, $P(B) = 0{\cdot}2$ and $P(A \cap B) = 0{\cdot}1$.

a) Find $P(A \cup B)$. [2]

The event C is such that $P(C) = 0{\cdot}45$. The events B and C are independent.

b) Find $P(B \cap C)$. [2]

The events A and C are mutually exclusive.

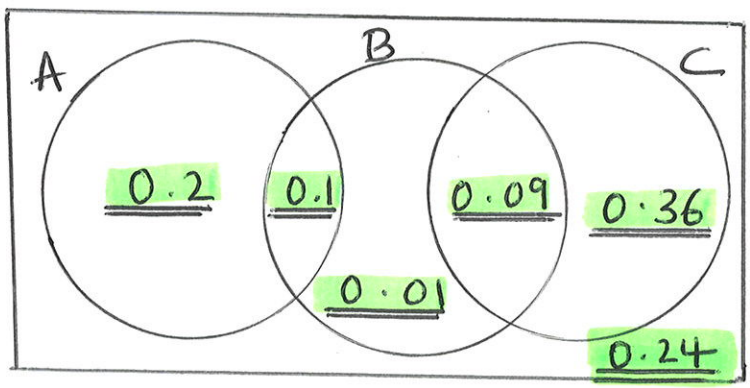
c) Draw a Venn diagram to show the events A, B and C. Include the probability for each distinct region on your diagram. [4]

d) Find $P((B \cup C)')$. [2]

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0{\cdot}3 + 0{\cdot}2 - 0{\cdot}1 = 0{\cdot}4$

b) If independent, $P(B \cap C) = P(B) \times P(C)$
$= 0{\cdot}2 \times 0{\cdot}45 = 0{\cdot}09$

c)



(mutually exclusive events A and C means $P(A \cap C) = 0$)

d) From venn diagram
$P(B \cup C)' = 0{\cdot}24 + 0{\cdot}2$
$= 0{\cdot}44$

©MathsDIY.com

0	2	A card is selected at random from a pack of 20 cards numbered 1 to 20. What distribution models the number on the card? Give a reason for your answer. [2]

A discrete uniform distribution.

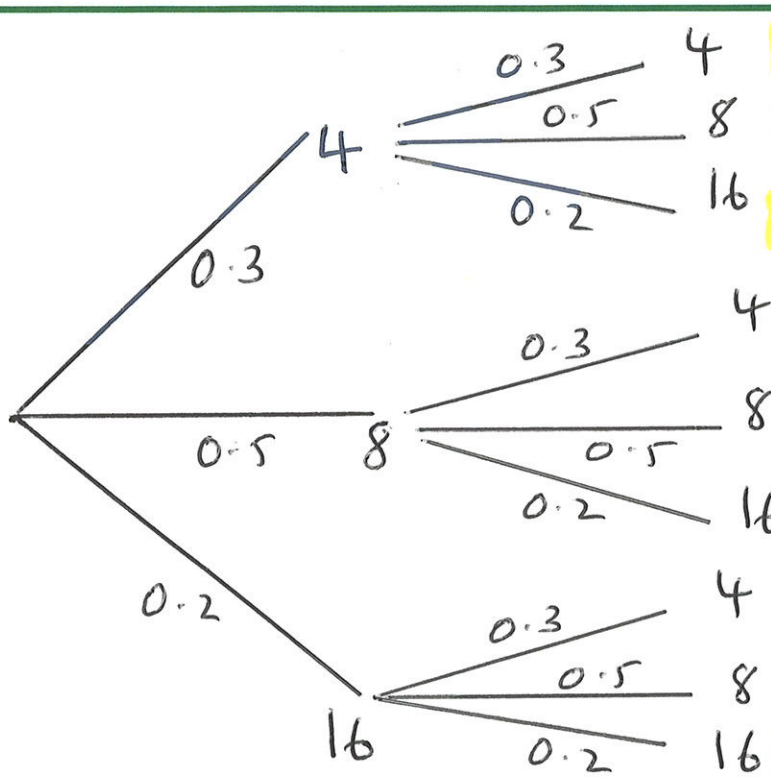
Because each card has the same probability $\frac{1}{20}$ of being selected.

©MathsDIY.com

0 3 The probability distribution of a discrete random variable X is given below.

x	4	8	16
$P(X = x)$	0·3	0·5	0·2

Two independent values of X are generated. Find the probability that the first value of X is less than or equal to the second value of X. [4]



$P(4,4) = 0.3^2$

$P(4,8) = 0.3 \times 0.5$

$P(4,16) = 0.3 \times 0.2$

4 ✗

$P(8,8) = 0.5^2$

$P(8,16) = 0.5 \times 0.2$

4 ✗

8 ✗

$P(16,16) = 0.2^2$

P(first is less than or equal to second)

$= 0.3^2 + 0.3 \times 0.5 + 0.3 \times 0.2 + 0.5^2 + 0.5 \times 0.2 + 0.2^2$

$= 0.09 + 0.15 + 0.06 + 0.25 + 0.1 + 0.04$

$= 0.69$

©MathsDIY.com

0 4 Sources of the radioactive element caesium each emit radioactive particles at an average rate of 4 per second. The numbers of emissions are modelled by a Poisson distribution.

a) Find the probability that, in a 3-second interval, one such source will emit at least 10 radioactive particles, giving your answer correct to 4 decimal places. [3]

b) A random sample of 9 of these sources is taken. Calculate the probability that at most 5 of them will emit at least 10 radioactive particles in a 3-second interval. [3]

a) $X \sim Po(12)$ (3 second interval, $3 \times 4 = 12$ per 3 seconds)

$$P(X \geq 10) = 1 - P(X \leq 9)$$
$$= 1 - 0.2424$$
$$= 0.7576$$

b) This can be modelled as a binomial distribution

$Y \sim B(n, p)$
where $n = 9$ (random sample of 9)
$p = 0.7576$ (probability source emits at least 10 radioactive particles)
↖ from a

$Y \sim B(9, 0.7576)$

$P(Y \leq 5) = 0.1518$

©MathsDIY.com

0 5 A doctor knows that 34% of his patients currently take too little exercise. He believes he can reduce this proportion by putting patients through a behaviour change programme.

To test his belief, he takes a random sample of 156 of his patients and puts each of them through the programme.

It is found that 40 of these patients take too little exercise following the behaviour change programme.

a) The doctor initially considers using a two-tailed test. Explain why this is not appropriate. [1]

b) The doctor then decides to use a one-tailed test.

i) Explain what is meant by a *p*-value in the context of this question. [1]

ii) Test the doctor's belief at the 1% level of significance, stating clearly the conclusion that the doctor should reach. [6]

a) The doctor is only interested in a reduction in the proportion of patients who take too little exercise.

bi) the p value is the probability of 40 or fewer patients taking too little exercise after the programme

given { that there has been no change; $p = 0.34$; H_0 is true } (one of these)

ii) $H_0: p = 0.34$ $H_1: p < 0.34$

Let the random variable X represent the number of patients who take too little exercise.

Under H_0, $X \sim B(156, 0.34)$

Method 1 (p value)

$P(X \leq 40) = 0.015488$

$0.015488 > 0.01 \Rightarrow$ insufficient evidence to reject H_0.

©MathsDIY.com

Method 2 (critical region)

Critical region is $X \leq k$ where $P(X \leq k) \leq 0.01$

$P(X \leq 39) = 0.009768$

$P(X \leq 40) = 0.015488$

Hence, $k = 39$. The critical region is $X \leq 39$

Since 40 is not in the critical region, there is insufficient evidence to reject H_0.

Conclusion

There is insufficient evidence at the 1% level of significance to suggest the behaviour change programme has had the desired effect.

©MathsDIY.com

0 6 Poppi and Clare are investigating Welsh language skills in Wales. Poppi randomly selects a sample of 20 different students from her school and asks them to state their Welsh language skills according to the categories in the 2011 census. She numbers the students in her school from 001 to 578. She uses a random number generator to select her sample. The first 6 numbers generated are shown below.

195 752 023 195 432 271

a) Write down the numbers of the first 4 students in Poppi's sample. [1]

Clare selects a random sample of 20 students from a different school.

b) Give two reasons why Clare's conclusion about Welsh language skills in Wales may be different from Poppi's conclusion. [2]

The data collected from the 2011 census for two regions in Wales are shown below.

		No skills in Welsh	Understand spoken Welsh only	Speak Welsh	Other skills in Welsh
Region A	Number of people	111 232	5517	13 103	4858
	Percentage	82·6	4·1	9·7	3·6
Region B	Number of people	102 153	4083	13 189	2744
	Percentage	83·7	3·3	10·8	2·2

On looking at these data, Poppi concludes that people in Region B have better Welsh language skills than people in Region A.

c) Without carrying out any calculations, use data from the table

i) to argue for Poppi's conclusion,

ii) to argue against Poppi's conclusion. [2]

d) Seven 4-year-old children who "Speak Welsh" were asked to concentrate on a task. The times, in minutes, until they lost concentration are given below.

7·2 5·4 7·4 4·6 13·2 8·4 7·7

i) Calculate the mean and standard deviation for these data. [3]

ii) For a similar group of 4-year-olds with "No skills in Welsh", the mean concentration time is 6·5 minutes and the standard deviation is 3·5 minutes. Compare the concentration times for these two groups of 4-year-olds. [2]

a) 195, 023, 432, 271

©MathsDIY.com

b) Valid reasons could be :-

- One school is English medium, one is Welsh medium
- Different samples can lead to different conclusions
- The schools might be in different areas of Wales

c) i) In Region B, 10.8% can speak Welsh compared to 9.79% in Region A.

ii) In Region B, 83.7% have no skills in Welsh compared to 82.6% in Region A.

d) i) $\bar{x} = \frac{\Sigma x}{n} = \frac{(7\cdot2+5\cdot4+7\cdot4+4\cdot6+13\cdot2+8\cdot4+7\cdot7)}{7}$

$= \frac{53.9}{7}$

$= 7.7$

$\sigma = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$

$= \sqrt{\left(\frac{7\cdot2^2+5\cdot4^2+7\cdot4^2+4\cdot6^2+13\cdot2^2+8\cdot4^2+7\cdot7^2}{7}\right) - (7.7)^2}$

$= \sqrt{\frac{4\,6101}{700} - \frac{5929}{100}} = 2.56 \text{ minutes}$

ii) The children who can speak Welsh can concentrate for longer on average than those who have no skills in Welsh. (Must include on average)

©MathsDIY.com

The length of time children who can speak Welsh can concentrate for is more consistent than those who have no skills in Welsh.

©MathsDIY.com

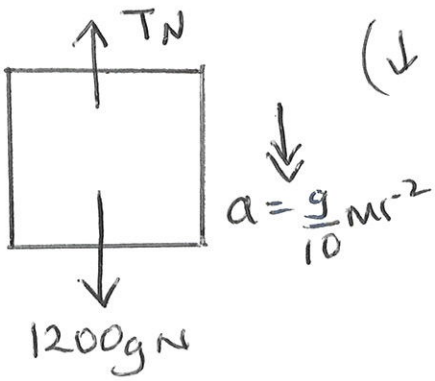
Section B: Mechanics

0 7 A person is standing in a lift which is descending with a constant acceleration of $\frac{g}{10}\text{ms}^{-2}$. The lift is supported by a single cable.

a) The total mass of the lift and the person is 1200 kg. Find the tension in the lift cable. [3]

b) The mass of the person is M kg. Given that the magnitude of the reaction of the floor of the lift on the person is $63g$ N, find the value of M. [3]

a)



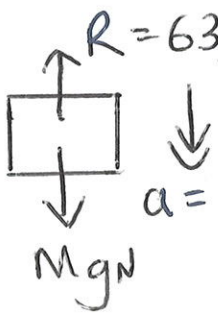
$(\downarrow \text{N2L})$ $F = ma$

$1200g - T = 1200\left(\frac{9.8}{10}\right)$

$1200 \times 9.8 - 1200 \times \frac{9.8}{10} = T$

$T = 10,584\text{ N}$

b)



$(\downarrow \text{N2L})$ $F = ma$

$Mg - 63g = M\left(\frac{g}{10}\right)$

$M - 63 = \frac{M}{10}$

$M - \frac{M}{10} = 63$

$M\left(1 - \frac{1}{10}\right) = 63$

$M = \frac{63}{9/10}$

$M = 70$

©MathsDIY.com

0 8 A particle P moves in a straight line such that its displacement, s metres, at time t seconds, relative to a fixed point is given by

$$s = 4t^2 - t^3 \qquad \text{for } t \geqslant 0.$$

Find the speed of P when it returns to its starting position. [5]

$$V = \frac{ds}{dt} = \frac{d(4t^2 - t^3)}{dt}$$

$$V = (8t - 3t^2)\ ms^{-1}$$

when particle returns to starting point, displacement is zero

$$4t^2 - t^3 = 0$$

$$t^2(4 - t) = 0 \qquad t = 0s \text{ and } t = 4s$$

at $t = 4$, $V = 8(4) - 3(4)^2$

$$V = 32 - 48$$

velocity → $V = -16\ ms^{-1}$

speed $= |v| = 16\ ms^{-1}$

©MathsDIY.com

0 9 Three forces **F**, **G** and **H** are such that

$$\mathbf{F} = (2\mathbf{i} - 13\mathbf{j})\,\text{N}, \qquad \mathbf{G} = (-6\mathbf{i} + 8\mathbf{j})\,\text{N}, \qquad \mathbf{H} = (12\mathbf{i} + k\mathbf{j})\,\text{N},$$

where k is a constant. The resultant of the three forces, **R**, has magnitude 17 N.

a) Find the two possible values of the constant k. [4]

b) For the larger of the two values of k, calculate the angle that **R** makes with the **i** vector. [2]

a) Resultant force $= (2\underline{i} - 13\underline{j}) + (-6\underline{i} + 8\underline{j}) + (12\underline{i} + k\underline{j})$

$= \underline{8\underline{i} + (k-5)\underline{j}}$

Now, $17 = \sqrt{8^2 + (k-5)^2}$

$\therefore \quad 289 = 64 + (k-5)^2$

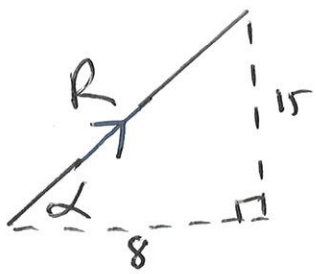
$225 = k^2 - 10k + 25$

$k^2 - 10k - 200 = 0$

$(k-20)(k+10) = 0$

$\underline{k = 20}$, $\underline{k = -10}$

b) If $\underline{k = 20}$, $\underline{R} = 8\underline{i} + 15\underline{j}$



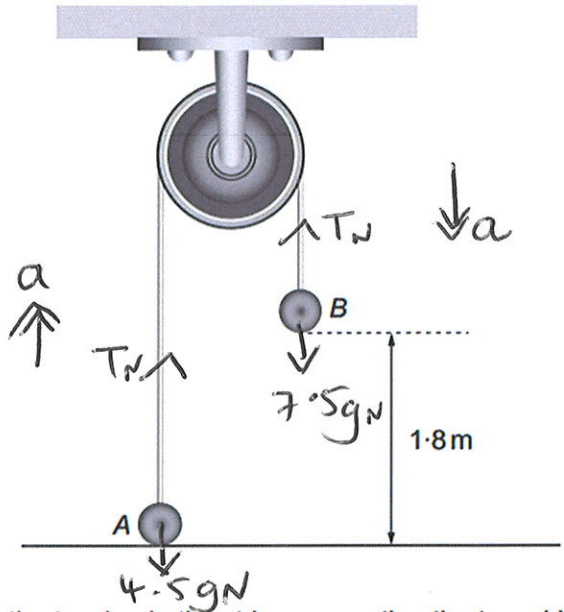
$\tan\alpha = \frac{15}{8}$

$\alpha = \tan^{-1}\left(\frac{15}{8}\right)$

$\underline{\alpha = 61.9^\circ}$

©MathsDIY.com

1 0 The diagram below shows two objects *A* and *B* connected by a light inextensible string passing over a smooth fixed pulley. Object *A* has mass 4·5 kg and object *B* has mass 7·5 kg. Initially, object *A* is held on a horizontal surface so that object *B* hangs freely with the string taut. The distance of object *B* above the horizontal surface is 1·8 m.



a) Write down the tension in the string connecting the two objects. [1]

b) Object *A* is then released.

i) Calculate the magnitude of the acceleration of *A*.

ii) Hence, find the time taken for the objects to be at the same height above the horizontal surface.

iii) In reality, there will be air resistance acting on the objects as they move. State how this fact will affect your answer to (ii). [9]

a) $T = 7{\cdot}5g = 73{\cdot}5\,N$

b) i) A :- ($\uparrow$ N2L) $F = ma$

$T - 4{\cdot}5g = 4{\cdot}5a$ (1)

B :- ($\downarrow$ N2L) $F = ma$

$7{\cdot}5g - T = 7{\cdot}5a$ (2)

(1) + (2) $\quad 3g = 12a$

$\frac{g}{4} = a, \quad a = 2{\cdot}45\,ms^{-2}$

©MathsDIY.com

ii) Objects are at the same height when B has travelled $\frac{1.8}{2}$ m = 0.9m

$s = ut + \frac{1}{2}at^2$

$0.9 = 0 + \frac{1}{2}(2.45)t^2$

$\frac{0.9}{1.225} = t^2$

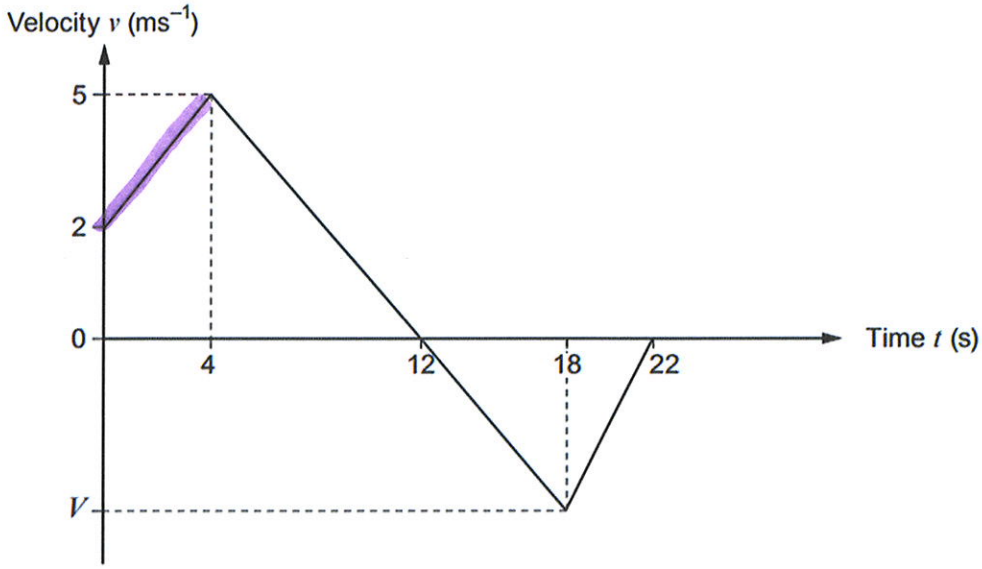
$t = \frac{6}{7}$ s

$s = 0.9$
$u = 0$
~~v~~
$a = 2.45$
$t = ?$

iii) The time would be greater.

©MathsDIY.com

1 1 A particle, which is initially at the point A, travels in a straight line during the interval $0 \leqslant t \leqslant 22$, where t is the time in seconds. The diagram below shows a velocity-time graph for the motion of the particle. The velocity of the particle when $t = 18$ is V ms^{-1}.



a) Find the acceleration of the particle when $t = 1$. [2]

b) Calculate the distance travelled by the particle during the interval $0 \leqslant t \leqslant 12$. [2]

c) When $t = 22$, the displacement of the particle from A is 14 m. Determine the value of V. [3]

d) Write down the time interval during which the particle is moving towards A and its speed is increasing. [1]

a) acceleration = $\frac{\text{change in velocity}}{\text{time}} = \frac{5-2}{4} = \frac{3}{4}\ \text{ms}^{-1}$

b) distance travelled = area trapezium $(0 \leq t \leq 4)$ + area triangle $(4 \leq t \leq 12)$

$= \frac{1}{2}(2+5)(4) + \frac{1}{2}(8)(5)$

$= 14 + 20 = 34\text{m}$

©MathsDIY.com

c) displacement:- $12 \leq t \leq 22$ = area triangle

$= \frac{1}{2}(10)v$

$= 5v$

and, at $t = 22s$, displacement from A = 14m

$\therefore \quad 34 + 5v = 14$

$5v = -20$

$v = -4$

d) $12 < t < 18$ or 12 to 18

$12 \rightarrow 18$

©MathsDIY.com