WJEC – AS unit 2 Applied Mathematics A

8th June 2022 1 hour 45 minutes

PAPER WITH TEACHER'S SOLUTIONS

Section A Statistics 40 marks

Section B Mechanics 35 marks **Total marks available: 75**

Reminder: *Sufficient working must be shown to demonstrate the **mathematical** method employed.*

Section A: Statistics

0 1 The events A and B are such that

$P(A) = 0{\cdot}3, \quad P(B) = 0{\cdot}6, \quad P(A \cup B) = 0{\cdot}82.$

a) Determine whether or not A and B are independent. [4]

b) Find the probability that exactly one of A and B occurs. [3]

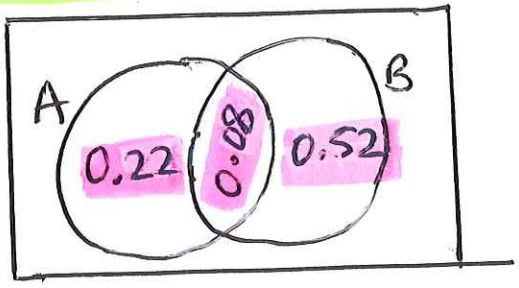
a) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$ ← Addition Law

$= 0{\cdot}3 + 0{\cdot}6 - 0{\cdot}82 = 0{\cdot}08$

If independent, $P(A) \times P(B) = P(A \cap B)$ $0{\cdot}3 \times 0{\cdot}6 = 0{\cdot}18$

Clearly, $0{\cdot}18 \neq 0{\cdot}08$ ∴ A and B are not independent

b)



∴ Probability exactly one occurs

$= 0.52 + 0.22 = 0.74$

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0 2 The probability distribution for X, the lifetime of a light bulb, in hours, is given below.

X	$256 \leqslant x < 259$	$259 \leqslant x < 262$	$262 \leqslant x < 265$	$265 \leqslant x < 267$	$267 \leqslant x < 300$
Probability	0·05	0·25	0·45	0·20	0·05

a) Suppose that a random sample of 40 light bulbs is tested, and a histogram is drawn of their lifetimes. Calculate the expected height of the bar for the interval $262 \leqslant x < 265$. [2]

b) Now suppose that the last two intervals are changed to $265 \leqslant x < 268$ and $268 \leqslant x < 300$. Explain why it is not possible to tell what will happen to the expected heights of the last two bars. [1]

c) Celyn collects a different random sample of 40 light bulbs to test. She draws a histogram of their lifetimes and finds that it is different to the histogram referred to in part (a). Should Celyn be concerned that the two histograms are different? [1]

a) Expected number, $262 \leq x < 265$

→ $0.45 \times 40 = 18$ lightbulbs

→ Area = 18, Width = 3 ∴ Height = $18 \div 3 = 6$ units

b) We don't know how the probability is distributed within the two groups.

c) • Different samples will lead to different results

• The lifetime of the lightbulbs that Celyn samples will be different to those considered in part a).

or • If the differences are big enough, it would imply that something might have gone wrong.

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0 3 In a study, samples of soil were collected during the summer. Soil samples of dimensions 25 cm × 25 cm × 40 cm were collected for analysis. The study found that there were, on average, 11 earthworms per sample.

a) Explain briefly the conditions under which a Poisson distribution could be used to model the number of earthworms per sample. [1]

b) In July, pupils at a primary school are asked to dig a smaller hole, 25 cm × 25 cm × 10 cm, and to count the number of earthworms they find. Calculate the probability that the pupils find exactly 5 earthworms. [3]

c) In the autumn, the average number of earthworms per sample is greater than in the summer. The probability that, in the autumn, there are fewer than 13 earthworms in a soil sample of dimensions 25 cm × 25 cm × 40 cm is close to 36%. Find the mean number of earthworms, to the nearest whole number, per 25 cm × 25 cm × 40 cm soil sample in the autumn. [2]

a) • The average rate of occurence of earthworms must be constant

• The occurence of each earthworm is independent of other earthworms occuring.

b) $X \sim Po(2.75)$ because $\lambda = \frac{11}{4} = 2.75$ ($\frac{1}{4}$ of volume)

$$P(X=x) = e^{-\lambda}\frac{\lambda^x}{x!}$$

$$P(X=5) = e^{-2.75}\frac{2.75^5}{5!} = 0.0837861 = 0.0838 \text{ (3sf)}$$

c) Fewer than 13, $\Rightarrow P(X \leq 12) \approx 0.36$

using statistics tables

$P(X \leq 12) = 0.3585$ when $\lambda = 14$

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0 4 Jessica is studying the relationship between hip girth, h cm, and thigh girth, t cm, for American adults who are physically active. She takes a random sample of 11 people from a very large dataset which she has downloaded into a spreadsheet software package. The results are shown below.

h (cm)	98·6	112·1	97·9	110·2	89·2	111·7	87·0	94·7	100·4	104·0	88·4
t (cm)	48·3	87·2	55·2	68·0	48·5	63·2	49·5	55·7	59·1	64·0	52·4

a) Jessica notes that, for the thigh girth data, the lower quartile is 49·5 and the upper quartile is 64·0.

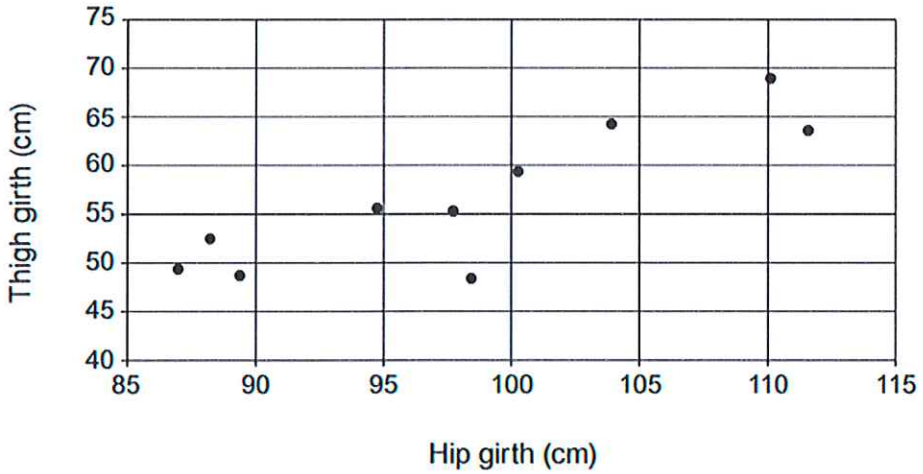
i) Show that 87·2 should be classified as an outlier for t.

ii) Give a reason why Jessica might exclude the outlier.

iii) Give a reason why Jessica might include the outlier. [4]

Jessica decides to exclude the outlier and produces the following scatter diagram.

Thigh girth versus Hip girth



b) Interpret, in context, the correlation in the data shown in the diagram. [1]

The equation of the regression line of t on h for this sample is

$$t = 0{\cdot}69h - 11{\cdot}26.$$

c) Interpret the gradient of the regression line in this context. [1]

d) Use your knowledge of large data sets and spreadsheet software packages to suggest a way in which Jessica could improve her investigation. [1]

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a)i) Outlier if bigger than $Q_3 + 1.5 \times I.Q.R.$

$Q_3 + 1.5 \times IQR = 64.0 + (1.5 \times (64.0 - 49.5))$

$= 85.75$

$87.2 > 85.75$ ∴ it is an outlier

ii)
- Inclusion might distort calculations and therefore inferences about the population
- So that the mean won't be affected by outliers
- Because it might be a data collection error
- It may skew the data

iii)
- It is still a valid measurement (only just an outlier)
- To use all of the data

b) The larger the hip girth, the larger the thigh girth, on average

c) An increase in hip girth of 1cm, corresponds to an increase in thigh girth of 0.69cm, on average

d)
- Using all of the data, instead of a sample would lead to more accurate results
- Increase the sample size

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0 5 A company, *Run4Lyfe*, sponsors an athletic event. The organisers of the event claim that 70% of the participants know the name of the sponsoring company. *Run4Lyfe* is concerned that the proportion, p, of participants knowing the name of the sponsoring company is less than 70%. They decide to survey 60 randomly selected participants to carry out a significance test.

a) State suitable hypotheses for carrying out the test. [1]

b) **i)** Explain what is meant by the critical region for this test.

ii) Determine the critical region if the test is to be carried out at a significance level as close as possible to, but not exceeding, 5%.

iii) Given that 40 participants out of the 60 in the sample know the name of the company, complete the significance test. [6]

c) State, with a reason, how you would advise *Run4Lyfe* with regards to sponsoring the event next year. [1]

a) $H_0: p = 0.7$ $H_1: p < 0.7$

b) i) The critical region, is the region of values of the number of people that know the name of the company, that would lead us to reject H_0.

ii) Under the null hypothesis, $X \sim B(60, 0.7)$

Require 5% significance

$P(X \leq 35) = 0.03624$ (3.6%)

Critical region is $X \leq 35$ participants

iii) 40 is clearly not in the critical region. Therefore, there is insufficient evidence to reject H_0. Run4Lyfe should continue to assume that 70% of participants know their name.

c) • It is worth sponsoring the event as the result of the hypothesis test suggests that it

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is an effective way to get brand recognition.

- The hypothesis test implies that it's likely that a reasonable proportion of participants know the name of the sponsor, so it may be worth doing.
- Suggest to Run4Lyfe that they continue sponsoring the event as the hypothesis test revealed insufficient evidence to substantiate their concern.

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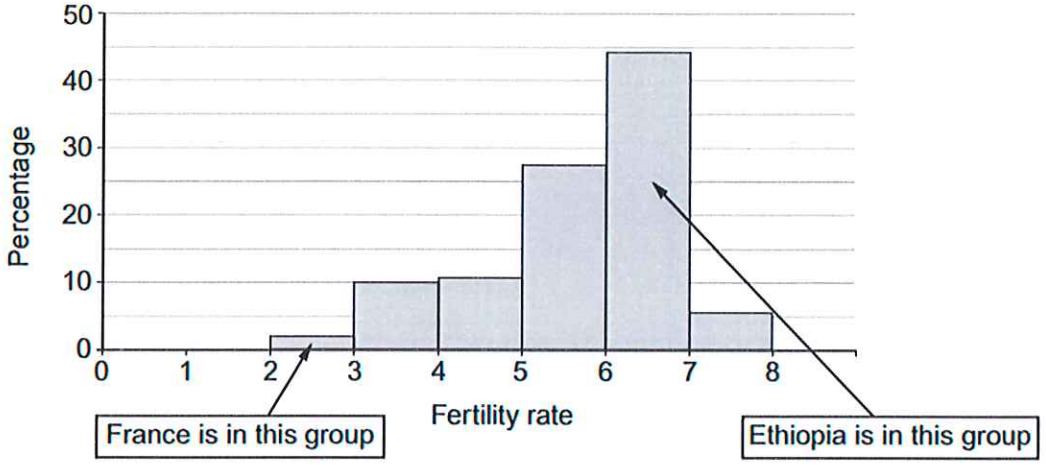
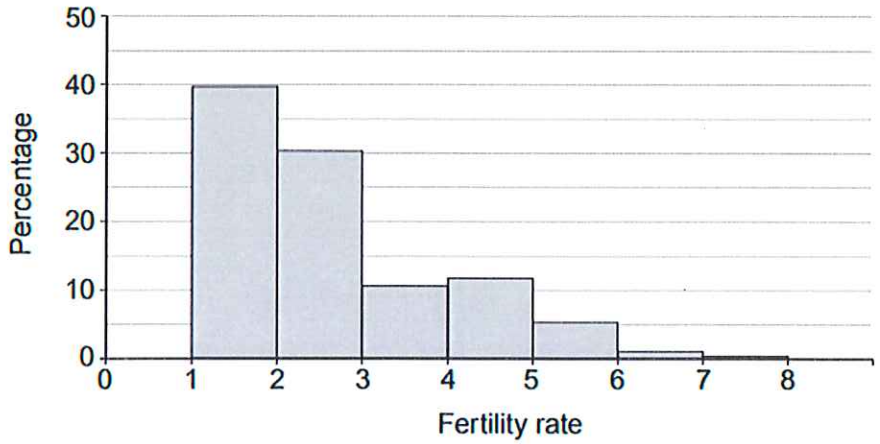
0 6 The fertility rate for a country is the average number of children that are born to a woman over her lifetime. The graphs and table below show some data on the fertility rates for 197 countries in the years 1914 and 2014.

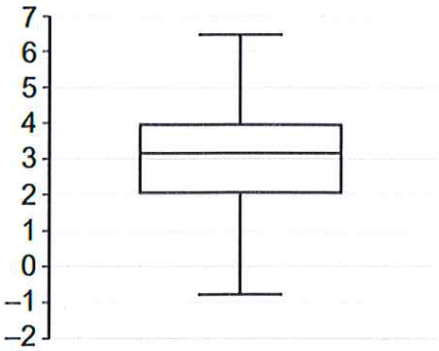
Fertility rates in 1914



Fertility rates in 2014



Decreases in fertility rates from 1914 to 2014



Decreases in fertility rates from 1914 to 2014, n = 197	
Minimum value	–0·71
Lower quartile	2·08
Median	3·19
Upper quartile	3·94
Maximum value	6·49

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a) Comment on the shapes of the distributions of fertility rates for 1914 and 2014. [2]

b) Interpret the minimum value, –0·71, in the boxplot. [1]

You are also given the following information:

Country	Fertility rate for 1914	Fertility rate for 2014
France	Between 2 and 3	1·98
Ethiopia	Between 6 and 7	4·4

c) **i)** Find the best possible estimate for the decrease in the fertility rate from 1914 to 2014 for France.

ii) Find the best possible estimate for the decrease in the fertility rate from 1914 to 2014 for Ethiopia.

iii) Give one possible reason why the answers to **i)** and **ii)** are so different.

iv) Explain why these estimates may not be very accurate. [5]

a) Examiner expects to see two valid comments

- 1914 negatively skewed
- 2014 positively skewed
- Fertility rates were greater in 1914 than 2014 (or people tended to have more children in 1914 than 2014.)

b) A decrease of '-0.71' must mean that in at least one country, fertility rates are actually increasing. This means, that in at least one country, women on average had more babies in 2014 compared to 1914.

ci) Mean of 2 and 3 = 2·5

∴ Decrease for France = 2·5 − 1·98 = 0.52

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ii) Decrease for Ethiopia = 6·5 − 4·4 = 2·1
(using mean of 6 and 7 = 6·5)

iii) • Countries with a higher fertility rate in 1914 have more of an opportunity for it to decrease
• Because Ethiopia is a developing country, its fertility rate is likely to have decreased more rapidly than France which is a developed country.

iv) • We have used the midpoints to estimate the fertility rates in 1914.
• We don't know the exact fertility rates in 1914.

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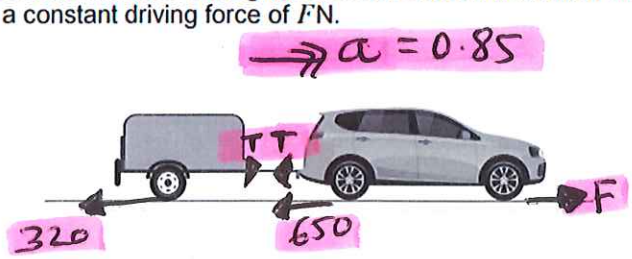
Section B: Mechanics

0 7 The diagram below shows a vehicle of mass 1300 kg towing a trailer of mass 500 kg by means of a light horizontal tow bar. The vehicle is moving forward along a straight horizontal road such that a constant resistance of magnitude 650 N acts on the vehicle and a constant resistance of magnitude 320 N acts on the trailer. The vehicle's engine produces a constant driving force of F N.



Given that the acceleration of the vehicle and trailer is $0{\cdot}85\,\text{ms}^{-2}$, show that $F = 2500$ and determine the tension in the tow bar. [6]

Applying Newton's second law to trailer & vehicle combined

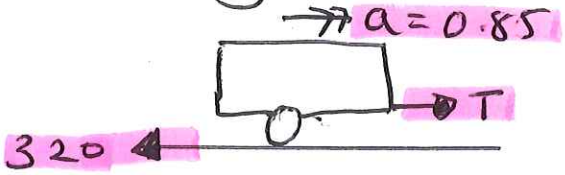
$F = ma$

$F - (650 + 320) = (1300 + 500)(0{\cdot}85)$

$F - 970 = 1530$

$F = 2500\ \text{N}$

Applying Newton's second law to trailer



$F = ma$

$T - 320 = 500(0{\cdot}85)$

$T = 425 + 320$

$T = 745\ \text{N}$

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0 8 An aircraft moves along a straight horizontal runway with a constant acceleration of $1{\cdot}5\,\text{ms}^{-2}$. Points A and B lie on the runway. The aircraft passes A with speed $4\,\text{ms}^{-1}$ and its speed at B must be at least $78\,\text{ms}^{-1}$ if it is to take off successfully.

a) Find the speed of the aircraft 8 seconds after it passes A. [2]

b) Determine the minimum value of the distance AB for the aircraft to take off successfully. [3]

a)

$s = X$
$u = 4$
$v = ?$
$a = 1{\cdot}5$
$t = 8$

Use $v = u + at$

$v = 4 + 1{\cdot}5(8)$

$v = 16\,\text{ms}^{-1}$

b) Minimum distance of AB will occur when $v = 78$

$s = ?$
$u = 4$
$v = 78$
$a = 1{\cdot}5$
$t = X$

Use $v^2 = u^2 + 2as$

$78^2 = 4^2 + 2(1{\cdot}5)s$

$6084 = 16 + 3s$

$s = \frac{6068}{3}$

$s = 2022{\cdot}67\,\text{m}$ (2dp)

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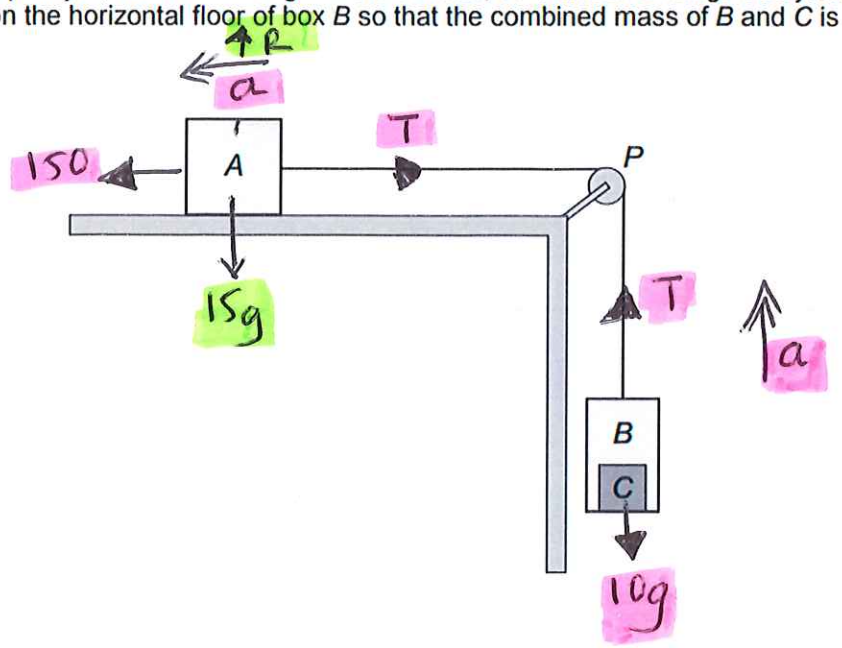
0 9 The diagram below shows an object *A*, of mass 15 kg, lying on a smooth horizontal surface. It is connected to a box *B* by a light inextensible string which passes over a smooth pulley *P*, fixed at the edge of the surface, so that box *B* hangs freely. An object *C* lies on the horizontal floor of box *B* so that the combined mass of *B* and *C* is 10 kg.



Initially, the system is held at rest with the string just taut. A horizontal force of magnitude 150 N is then applied to *A* in the direction *PA* so that box *B* is raised.

a) Find the magnitude of the acceleration of *A* and the tension in the string. [6]

b) Given that object *C* has mass 4 kg, calculate the reaction of the floor of the box on object *C*. [3]

a) Applying Newton's Second Law ($F=ma$) to both A and B

A :- $150 - T = 15a$ (1)

B :- $T - 10g = 10a$ (2) +

(1) + (2) $150 - 10g = 25a$

$$\frac{150 - 10(9.8)}{25} = a$$

$a = 2.08\ ms^{-2}$

in (2) $T = 10(2.08) + 10(9.8)$

$T = 118.8\ N$

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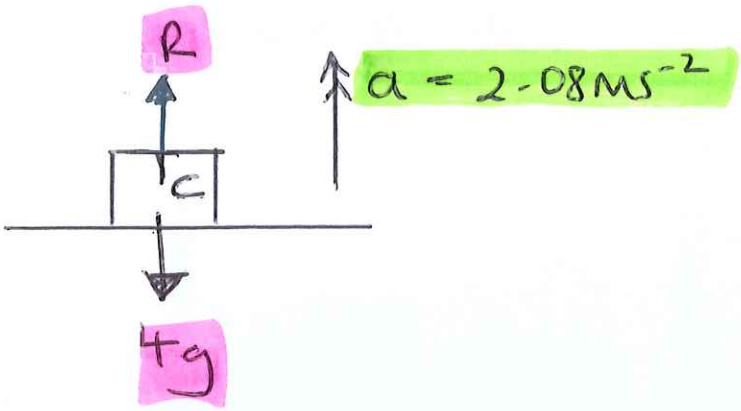
b)



$F = ma$ for object C gives

$$R - 4g = 4(2.08)$$

$$R = 4(2.08) + 4(9.8)$$

$$R = 47.52 \text{ N}$$

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10 In this question, **i** and **j** represent unit vectors due east and due north respectively.

Sarah is going for a walk. She leaves her house and walks directly to the shop. She then walks directly from the shop to the park. Relative to her house:

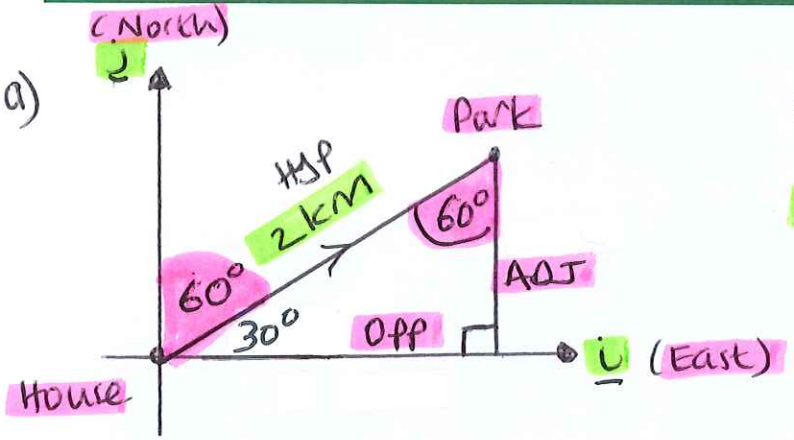
- the shop has position vector $\left(-\frac{2}{3}\mathbf{j}\right)$ km,
- the park is 2 km away on a bearing of 060°.

a) Show that the position vector of the park relative to the house is $\left(\sqrt{3}\mathbf{i}+\mathbf{j}\right)$ km. [2]

b) Determine the total distance walked by Sarah from her house to the park. [3]

c) By considering a modelling assumption you have made, explain why the answer you found in part (b) may not be the actual distance that Sarah walked. [1]

a)



$\cos 60 = \frac{ADJ}{2}$

$ADJ = 2\cos 60$

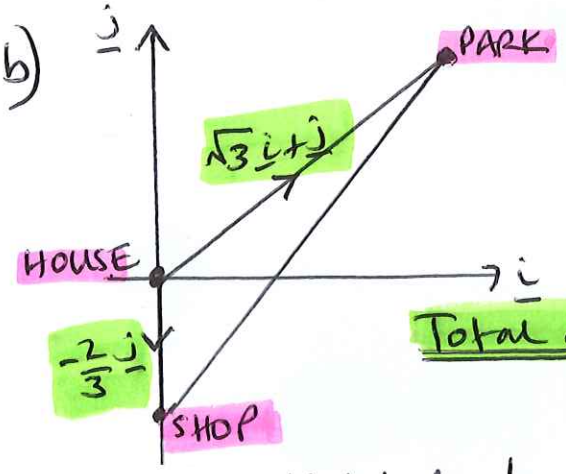
$\sin 60 = \frac{OPP}{2}$

$OPP = 2\sin 60$

$\underline{r}_{park} = (2\sin 60)\,\underline{i} + (2\cos 60)\,\underline{j} = \left(2\left(\frac{\sqrt{3}}{2}\right)\right)\underline{i} + \left(2\left(\frac{1}{2}\right)\right)\underline{j}$

$\underline{r}_{park} = (\sqrt{3}\,\underline{i} + \underline{j})$ km

b)



$\overrightarrow{SP} = -\left(-\frac{2}{3}\underline{j}\right) + \sqrt{3}\,\underline{i} + \underline{j}$

$= \sqrt{3}\,\underline{i} + \frac{5}{3}\underline{j}$

$|\overrightarrow{SP}| = \sqrt{(\sqrt{3})^2 + \left(\frac{5}{3}\right)^2} = \frac{2\sqrt{13}}{3}$ km

Total distance walked $= \frac{2}{3} + \frac{2\sqrt{13}}{3}$

$= 3.07$ km (3sf)

c) e.g.

- Unlikely to walk in straight lines
- Route unlikely to be flat (hills)
- Actual route unlikely to be straight lines

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1 1 A particle P moves along the x-axis so that its velocity v ms^{-1} at time t seconds ($t \geqslant 0$) is given by

$$v = 3t^2 - 24t + 36.$$

a) Find the values of t when P is instantaneously at rest. [2]

b) Calculate the total distance travelled by the particle P whilst its velocity is decreasing. [7]

a) At rest $\Rightarrow$ velocity is zero

$3t^2 - 24t + 36 = 0 \quad (\div 3)$

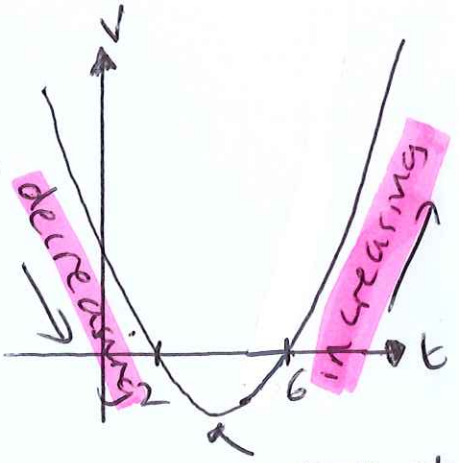
$t^2 - 8t + 12 = 0$

$(t-6)(t-2) = 0$

$t = 2$ seconds and $t = 6$ seconds

b) Consider a sketch of $V = 3t^2 - 24t + 36$

The velocity is clearly decreasing when $t < 4$



$\therefore$ Need to integrate to find distance travelled

careful here

$$\int_0^2 (3t^2 - 24t + 36)\,dt - \int_2^4 (3t^2 - 24t + 36)\,dt$$

$$= \left[\frac{3t^3}{3} - \frac{24t^2}{2} + 36t\right]_0^2 - \left[t^3 - 12t^2 + 36t\right]_2^4$$

$$= \left[(8 - 12(4) + 36(2)) - 0\right] - \left[(64 - 192 + 144) - 32\right]$$

$$= 32 - (-16)$$

$$= 48 \text{ metres}$$

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