WJEC – AS unit 1 Pure Mathematics A

19th May 2022 2 hour 30 minutes

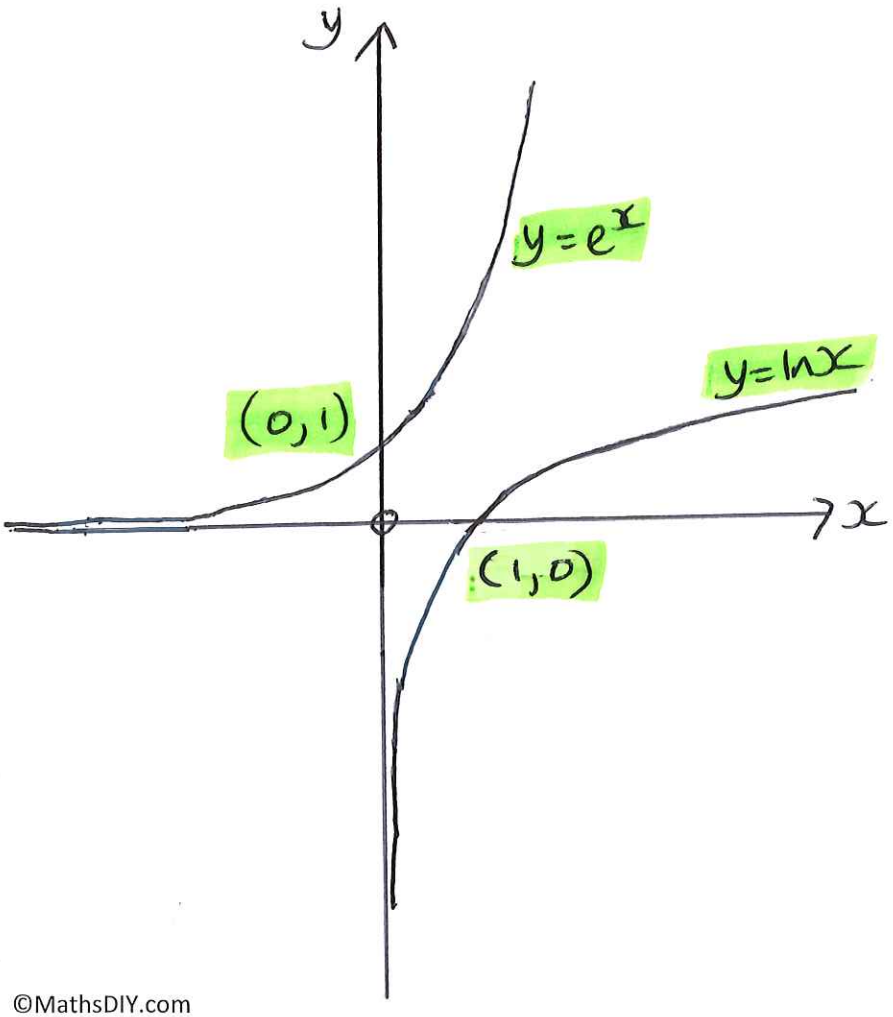
PAPER WITH TEACHER'S SOLUTIONS

Total marks available: 120

Reminder: *Sufficient working must be shown to demonstrate the **mathematical** method employed.*

0	1	Write down the inverse function of $y = e^x$. On the same set of axes, sketch the graphs of $y = e^x$ and its inverse function, clearly labelling the coordinates of the points where the graphs cross the x and y axes. [3]

$y = \ln x$



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0 2 Showing all your working, simplify the following expression. [6]

$$5\sqrt{48} + \frac{2+5\sqrt{3}}{5+3\sqrt{3}} - (2\sqrt{3})^3$$

$5\sqrt{48} = 5\sqrt{16}\sqrt{3}$

$= 5 \times 4\sqrt{3} = 20\sqrt{3}$

$(2\sqrt{3})^3 = 2\sqrt{3} \times 2\sqrt{3} \times 2\sqrt{3}$

$= 8 \times 3\sqrt{3} = 24\sqrt{3}$

$$\frac{2+5\sqrt{3}}{5+3\sqrt{3}} = \frac{(2+5\sqrt{3})(5-3\sqrt{3})}{(5+3\sqrt{3})(5-3\sqrt{3})}$$

$$= \frac{10-6\sqrt{3}+25\sqrt{3}-15(3)}{25-9(3)}$$

$$= \frac{-35+19\sqrt{3}}{-2} = \frac{35-19\sqrt{3}}{2}$$

Expression :-

$$20\sqrt{3} + \frac{35-19\sqrt{3}}{2} - 24\sqrt{3}$$

$$= \frac{40\sqrt{3} + 35 - 19\sqrt{3} - 48\sqrt{3}}{2}$$

$$= \frac{35-27\sqrt{3}}{2}$$

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0 3 The line L_1 passes through the points $A(0, 5)$ and $B(3, -1)$.

a) Find the equation of the line L_1. [3]

The line L_2 is perpendicular to L_1 and passes through the origin O.

b) Write down the equation of L_2. [1]

The lines L_1 and L_2 intersect at the point C.

c) Calculate the area of triangle OAC. [4]

d) Find the equation of the line L_3 which is parallel to L_1 and passes through the point $D(4, 2)$. [2]

e) The line L_3 intersects the y-axis at the point E. Find the area of triangle ODE. [1]

a) Gradient of $L_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{-1-5}{3-0} = \frac{-6}{3} = -2$

$(y-y_1) = m(x-x_1)$ gives

$(y-5) = -2(x-0)$

$y = -2x+5$

b) $y = \frac{1}{2}x$ (product of gradients is -1 $(-2 \times \frac{1}{2} = -1)$ and intercept $= 0$)

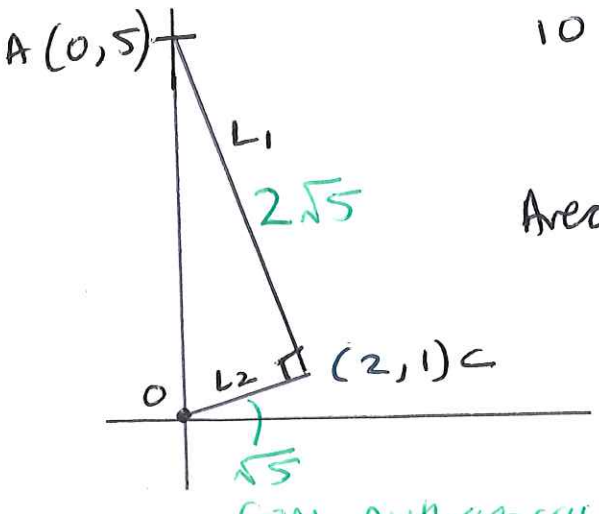
c) At C

$-2x+5 = \frac{1}{2}x$

$-4x+10 = x$

$10 = 5x$, $x=2$, $y=1$

$C = (2,1)$



Area $\Delta OAC = \frac{1}{2} \times$ base $\times$ perp. height

base $= \sqrt{2^2+1^2} = \sqrt{5}$

perp. height $= \sqrt{(0-2)^2+(5-1)^2}$

$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

from pythagoras

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area $\Delta OAC = \frac{1}{2} \times \sqrt{5} \times 2\sqrt{5} = 5$

d) L_3 parallel to $L_1 \rightarrow$ gradient $= -2$

$(4, 2)$ on L_3

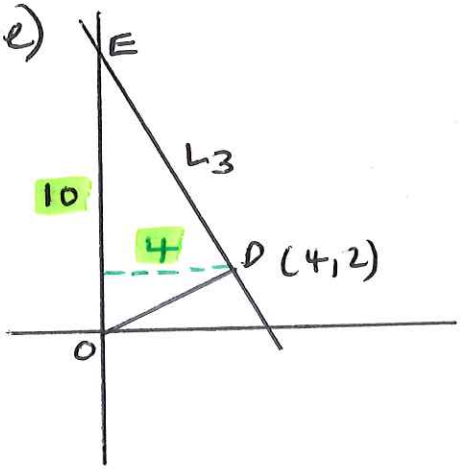
$(y - y_1) = m(x - x_1)$

$(y - 2) = -2(x - 4)$

$y - 2 = -2x + 8$

$y = -2x + 10$

e)



$E = (0, 10)$

area $\Delta ODE = \frac{1}{2} \times 10 \times 4 = 20$

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0	4	Solve the inequality $x^2 + 3x - 6 > 4x - 4$.	[4]

$x^2 + 3x - 6 > 4x - 4$

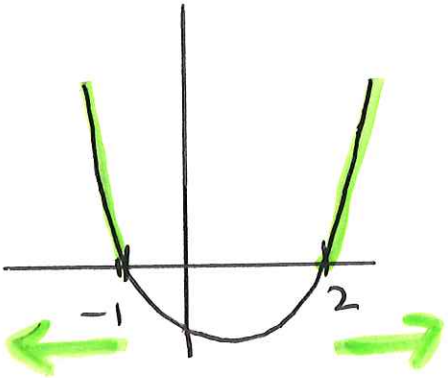
$x^2 - x - 2 > 0$

$(x-2)(x+1) > 0$

Consider the graph $y = (x-2)(x+1)$

cuts x axis when $y = 0$

$x = 2$ & $x = -1$



we require region when $y > 0$

$x < -1$ or $x > 2$

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0 5 The curve C_1 has equation $y = -x^2 + 2x + 3$ and the curve C_2 has equation $y = x^2 - x - 6$. The two curves intersect at the points A and B.

a) Determine the coordinates of A and B. [4]

b) On the same set of axes, sketch the graphs of C_1 and C_2. Clearly label the points where the two curves intersect. [3]

c) In the diagram drawn in part (b), shade the region satisfying the following inequalities: [2]

$$x > 0,$$

$$y < -x^2 + 2x + 3,$$

$$y > x^2 - x - 6.$$

a) $-x^2 + 2x + 3 = x^2 - x - 6$

$0 = 2x^2 - 3x - 9$

$2x^2 - 3x - 9 = 0$

x ac	$+b$
-18	-3

$-6, 3$

$2x^2 - 6x + 3x - 9 = 0$

$2x(x - 3) + 3(x - 3) = 0$

$(x - 3)(2x + 3) = 0$

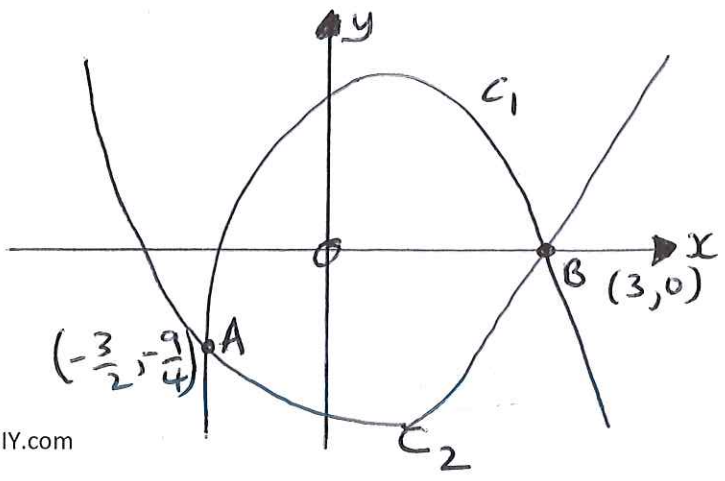
$x = 3$ and $x = -\frac{3}{2}$

when $x = 3$, $y = 3^2 - 3 - 6$, $y = 0$

when $x = -\frac{3}{2}$, $y = (-\frac{3}{2})^2 - (-\frac{3}{2}) - 6$, $y = -\frac{9}{4}$

$A = (-\frac{3}{2}, -\frac{9}{4})$ and $B = (3, 0)$ (or vice versa)

b)



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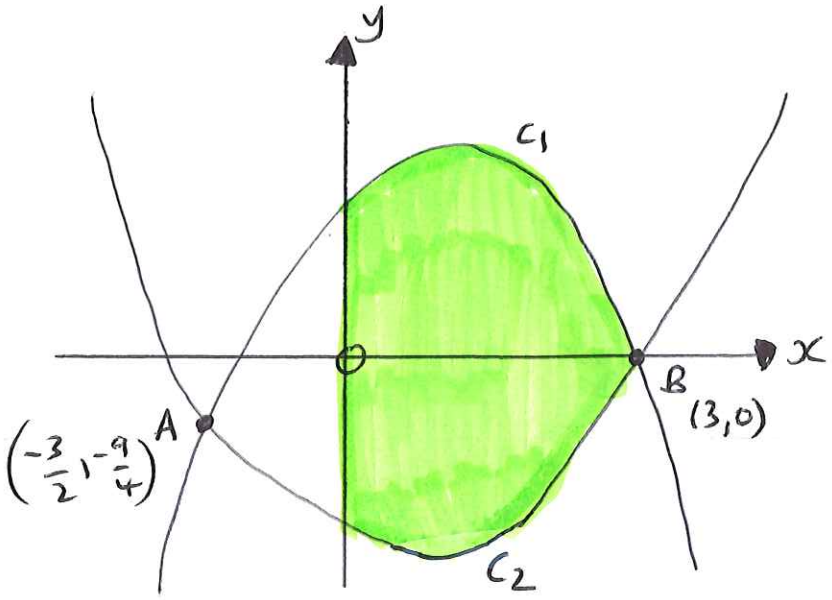
c)



$x > 0$ (to the right of the y axis)

$y < -x^2 + 2x + 3$ (below C_1)

$y > x^2 - x - 6$ (above C_2)

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0 6 In each of the two statements below, x and y are real numbers. One of the statements is true while the other is false.

A : $x^2 + y^2 \geqslant 2xy$, for all real values of x and y.

B : $x + y \geqslant 2\sqrt{xy}$, for all real values of x and y.

a) Identify the statement which is false. Find a counter example to show that this statement is in fact false. [3]

b) Identify the statement which is true. Give a proof to show that this statement is in fact true. [2]

a) Statement B is false

Consider $x = -3$, $y = -27$

$x + y = (-3) + (-27) = -30$

$2\sqrt{xy} = 2\sqrt{(-3)(-27)} = 2\sqrt{81} = 2 \times 9 = 18$

Since $-30 < 18$, statement B is false

b) Statement A states $x^2 + y^2 \geqslant 2xy$

$x^2 - 2xy + y^2 \geqslant 0$

factorising gives

$(x-y)(x-y) \geqslant 0$

$(x-y)^2 \geqslant 0$, which is always true

∴ Statement A is true

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0 7 A circle C has centre A and equation $x^2 + y^2 - 4x - 6y = 3$.

a) Find the coordinates of A and the radius of C. [3]

The line L with equation $y = x + 5$ intersects C at the points P and Q.

b) Determine the coordinates of P and Q. [4]

c) The point B is on PQ and is such that AB is perpendicular to PQ. Find the length of PB. [2]

d) Show that the area of the smaller segment enclosed by C and L is $4\pi - 8$. [2]

a) Need to re-write in format $(x-a)^2+(y-b)^2=r^2$

$(x^2-4x)+(y^2-6y)=3$

$(x-2)^2-4+(y-3)^2-9=3$

$(x-2)^2+(y-3)^2=4^2$ (centre $(2,3)$, Radius $=4$)

b) Substitute $y=x+5$ into $x^2+y^2-4x-6y=3$

$x^2+(x+5)^2-4x-6(x+5)=3$

$x^2+x^2+10x+25-4x-6x-30-3=0$

$2x^2-8=0$

$2x^2=8$

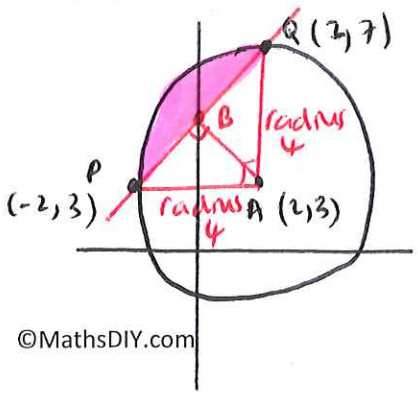
$x^2=4$, $x=2$ and $x=-2$

when $x=2$, $y=2+5$, $y=7$

when $x=-2$, $y=-2+5$, $y=3$

$P=(-2,3)$ and $Q=(2,7)$ (or vice versa)

c)



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B is the midpoint of PQ, by inspection

$B=(0,5)$

$PQ=\sqrt{4^2+4^2}=\sqrt{32}=4\sqrt{2}$

$PB=\frac{1}{2}$ of $PQ=2\sqrt{2}$

d) Required Area = $\frac{1}{4}$ area circle − area ΔPAQ

Area $\Delta PAQ = \frac{1}{2} \times 4 \times 4 = 8$

Area $\frac{1}{4}$ circle $= \frac{\pi r^2}{4} = \frac{\pi \times 4^2}{4} = \frac{16\pi}{4} = 4\pi$

Required Area $= 4\pi - 8$

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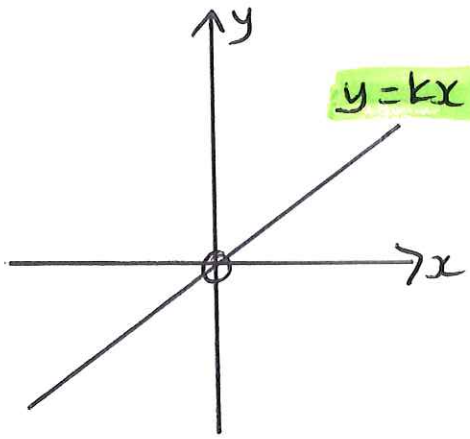
0 8 **a)** The graph G shows the relationship between the variables y and x, where $y \propto x$. Sketch the graph G. [1]

b) Mary and Jeff work for a company which pays its employees by hourly rates. Mary's hourly rate is twice Jeff's hourly rate. On a certain day, Jeff worked three times as long as Mary and was paid £120. Calculate Mary's earnings on that day. [3]

c) Atmospheric pressure, P units, decreases as the height, H metres, above sea level increases. The rate of decrease is 12% for every 1000 m. At sea level, the pressure P is 1013 units. Write down the model for P in terms of H and find the pressure at the top of Mount Everest, which is 8848 m above sea level. [3]

a)



$y \propto x$

$y = kx$

b) Mary's rate = 'M' Jeff's rate = 'J'

$M = 2J$

Jeff works 3x as long → £120 (÷3)

∴ If they worked the same hours Jeff → £40 (×2)

∴ May earn £80

c) (This can be thought of as a depreciation GCSE type of question)

12% rate of decrease → multiplier = 0.88

power = $\frac{8848}{1000}$ (in general → power is $\frac{H}{1000}$)

$P = 1013 \times 0.88^{\frac{H}{1000}}$

when $H = 8848$,

$P = 1013 \times 0.88^{\frac{8848}{1000}}$

$P = 326.88$

$P = 327$ units (3sf)

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0	9

Find the range of values of k for which the quadratic equation $x^2 + 2kx + 8k = 0$ has no real roots. [4]

No real roots $\Rightarrow$ $b^2 - 4ac < 0$

$a = 1$

$b = 2k$

$c = 8k$

$(2k)^2 - 4(1)(8k) < 0$

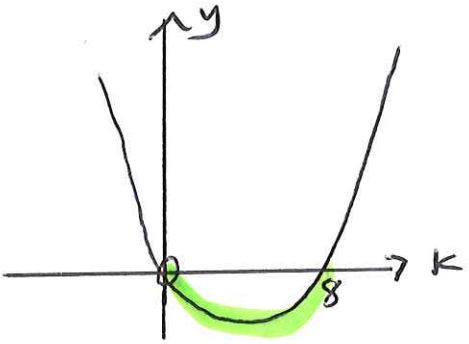
$4k^2 - 32k < 0$ $\quad (\div 4)$

$k^2 - 8k < 0$

$k(k-8) < 0$

If we drew $y = k(k-8)$ where does it cut the k axis?

$k = 0$ and $k = 8$



We want $k(k-8) < 0$

$\Rightarrow$ $0 < k < 8$

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1	0

Showing all your working, solve the equation $2^x = 53$. Give your answer correct to two decimal places. [3]

$2^x = 53$

Taking logs of both sides

$\ln 2^x = \ln 53$

Power law $\Rightarrow$ $x \ln 2 = \ln 53$

$$x = \frac{\ln 53}{\ln 2}$$

$x = 5.7279\ldots$

$x = 5.73$ (2dp)

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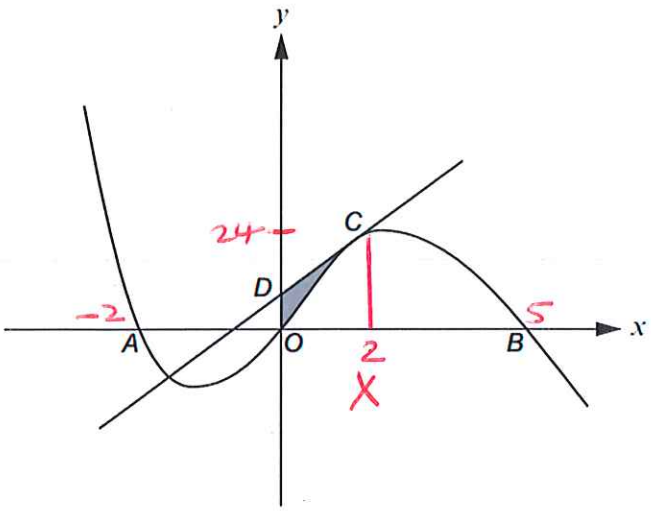
1	1

The diagram below shows a sketch of the curve $y = f(x)$, where $f(x) = 10x + 3x^2 - x^3$. The curve intersects the x-axis at the origin O and at the points $A(-2, 0)$, $B(5, 0)$. The tangent to the curve at the point $C(2, 24)$ intersects the y-axis at the point D.



a) Find the coordinates of D. [5]

b) Find the area of the shaded region. [6]

c) Determine the range of values of x for which $f(x)$ is an increasing function. [4]

a) Need to find equation of tangent

$\frac{dy}{dx} = 10 + 6x - 3x^2$

At C, $\frac{dy}{dx} = 10 + 6(2) - 3(2)^2 = 10$

$\therefore$ Gradient of tangent $= 10$

Using $(y - y_1) = m(x - x_1)$

$y - 24 = 10(x - 2)$

$y = 10x - 20 + 24$

Equation of tangent is $y = 10x + 4$

At D, $x = 0$ $\therefore$ $y = 4$, $D = (0, 4)$

b) Required area = Area of Trapezium $OADC$ $- \int_0^2 (10x + 3x^2 - x^3)\,dx$

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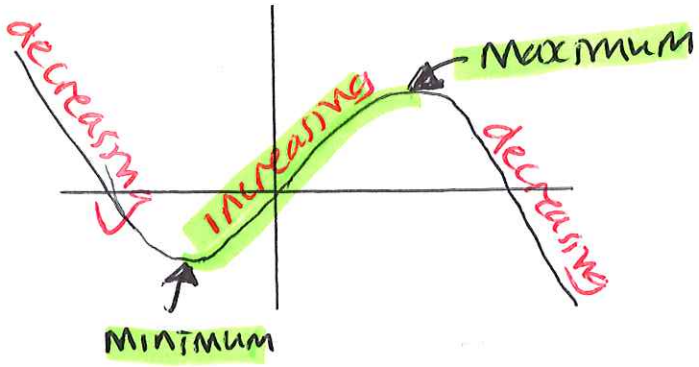
Area trapezium = $\frac{1}{2}(a+b)h = \frac{1}{2}(4+24)\times 2 = 28$

$$\int_0^2 (10x + 3x^2 - x^3)\,dx = \left[\frac{10x^2}{2} + \frac{3x^3}{3} - \frac{x^4}{4}\right]_0^2$$

$$= [5(4) + 8 - 4] - [0] = 24$$

Shaded Area = $28 - 24 = 4$

c) Conside the curve $f(x)$ below



We need to establish the x coordinate of the maximum and minimum to evaluate the range of values of x for which the function is increasing.

$$\frac{dy}{dx} = 10 + 6x - 3x^2$$

Stationary points at $\frac{dy}{dx} = 0$

$10 + 6x - 3x^2 = 0$

$3x^2 - 6x - 10 = 0$

$\times$	$+b$
-30	-6

won't factorise

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(3)(-10)}}{6} = \frac{3 \pm \sqrt{39}}{3}$$

$x = -1.08$ and $x = 3.08$

$f(x)$ is an increasing function in the range

$-1.08 < x < 3.08$

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1	2

a) Solve the equation $2x^3 - x^2 - 5x - 2 = 0$. [6]

b) Find all values of θ in the range $0° < \theta < 180°$ satisfying

$$\cos(2\theta - 51°) = 0{\cdot}891.$$ [3]

a) $f(x) = 2x^3 - x^2 - 5x - 2$

$f(1) = 2 - 1 - 5 - 2 = -6$

$f(-1) = -2 - 1 + 5 - 2 = 0$ ∴ $(x+1)$ is a factor

$$x+1 \overline{)\,2x^3 - x^2 - 5x - 2}$$

$2x^2 - 3x - 2$

$-(2x^3 + 2x^2)$

$-3x^2 - 5x$

$-(-3x^2 - 3x)$

$-2x - 2$

$-(-2x - 2)$

0

$2x^2 - 3x - 2 = 0$

x ac	+b	
−4	−3	−4, +1

$2x^2 - 4x + x - 2 = 0$

$2x(x-2) + 1(x-2) = 0$

$(x-2)(2x+1) = 0$

$f(x) = (x+1)(2x^2 - 3x - 2)$

$f(x) = (x+1)(x-2)(2x+1)$

$x = -1, \; x = -\frac{1}{2}, \; x = 2$

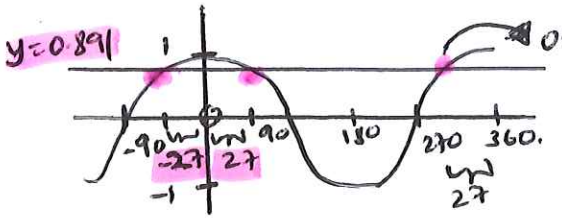
b) $\cos(2\theta - 51°) = 0{\cdot}891$

$2\theta - 51° = 27.0°$

$0° < \theta < 180$

$0° < 2\theta < 360$

$-51° < 2\theta - 51 < 309$



be very careful examiner penalises extra roots.

$2\theta - 51 = 27.0$

$2\theta = 78.0$

$\theta = 39.0°$

$2\theta - 51 = -27.0$

$2\theta = 24°$

$\theta = 12°$

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1	3	Find the term which is independent of x in the expansion of $\frac{(2-3x)^5}{x^3}$.	[4]

We require x^3 term of $(2-3x)^5$ expansion

x^3 term of $(a+x)^n$ is given by $\binom{n}{3}a^{n-3}x^3$

$\therefore$ x^3 term of $(2-3x)^5$ is given by $\binom{5}{3}2^2(-3x)^3$

$= -1080x^3$

$\Rightarrow \frac{-1080x^3}{x^3} \Rightarrow$ **-1080 is the term independent of x**

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1	4

A curve C has equation $f(x) = 3x^3 - 5x^2 + x - 6$.

a) Find the coordinates of the stationary points of C and determine their nature. [8]

b) **Without solving the equations**, determine the number of distinct real roots for each of the following:

i) $3x^3 - 5x^2 + x + 1 = 0$,

ii) $6x^3 - 10x^2 + 2x + 1 = 0$. [4]

a) $f'(x) = 9x^2 - 10x + 1$

Stationary points occur when $f'(x) = 0$

$9x^2 - 10x + 1 = 0$

$9x^2 - x - 9x + 1 = 0$

$x(9x - 1) - 1(9x - 1) = 0$

$(9x - 1)(x - 1) = 0$

$x = \frac{1}{9}$, $x = 1$

x ac	+b
9	-10

$-9, -1$

at $x = \frac{1}{9}$, $y = 3\left(\frac{1}{9}\right)^3 - 5\left(\frac{1}{9}\right)^2 + \left(\frac{1}{9}\right) - 6 = \frac{-1445}{243}$ (-5.9465)

at $x = 1$, $y = 3(1)^3 - 5(1)^2 + (1) - 6 = -7$

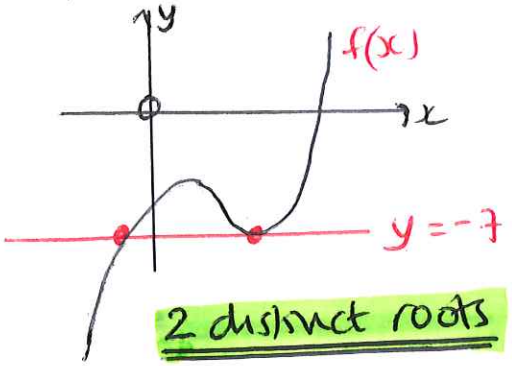
$f''(x) = 18x - 10$

at $(1, -7)$ $f''(x) = 18 - 10 > 0 \Rightarrow$ MINIMUM

at $\left(\frac{1}{9}, \frac{-1445}{243}\right)$ $f''(x) = \frac{18}{9} - 10 < 0 \Rightarrow$ MAXIMUM

bi) $3x^3 - 5x^2 + x + 1 = 0$

$\Rightarrow 3x^3 - 5x^2 + x - 6 = -7$

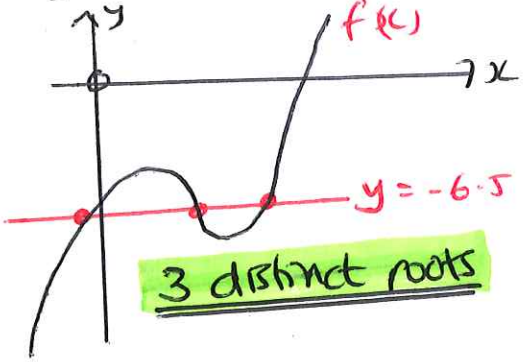


2 distinct roots

ii) $6x^3 - 10x^2 + 2x + 1 = 0$

$3x^3 - 5x^2 + x + \frac{1}{2} = 0$

$3x^3 - 5x^2 + x - 6 = -6.5$



3 distinct roots

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1 5 Solve the simultaneous equations

$$3\log_a(x^2y) - \log_a(x^2y^2) + \log_a\left(\frac{9}{x^2y^2}\right) = \log_a 36, \quad (1)$$

$$\log_a y - \log_a(x+3) = 0. \quad (2)$$

[8]

(1) gives $\log_a(x^2y)^3 - \log_a(x^2y^2) + \log_a\left(\frac{9}{x^2y^2}\right) = \log_a 36$ power law

$\log_a\left(\frac{9(x^2y)^3}{x^2y^2}\right) - \log_a(x^2y^2) = \log_a 36$ addition law

$\log_a\left(\frac{9(x^2y)^3}{(x^2y^2)^2}\right) = \log_a 36$ subtraction law

$\frac{9x^6y^3}{x^4y^4} = 36$, $x^2 = 4y$ (3)

(2) gives $\log_a\left(\frac{y}{x+3}\right) = \log_a 1$ subtraction law & $\log_a 1 = 0$

$\frac{y}{x+3} = 1$, $y = x+3$ (4)

(4) in (3) gives $x^2 = 4(x+3)$

$x^2 - 4x - 12 = 0$

$(x+2)(x-6) = 0$

$x = -2$, $x = 6$

in (4) when $x = -2$, $y = 1$

when $x = 6$, $y = 9$

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1 6 The vectors **a** and **b** are defined by $\mathbf{a} = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{b} = \mathbf{i} - 3\mathbf{j}$.

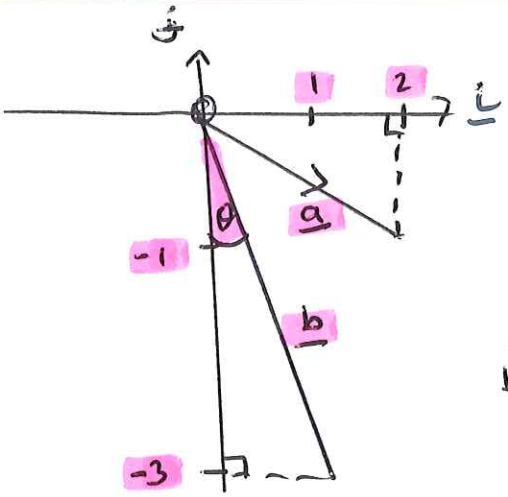
a) Find a unit vector in the direction of **a**. [2]

b) Determine the angle **b** makes with the x-axis. [2]

c) The vector $\mu\mathbf{a} + \mathbf{b}$ is parallel to $4\mathbf{i} - 5\mathbf{j}$.

i) Find the vector $\mu\mathbf{a} + \mathbf{b}$ in terms of μ, **i** and **j**. [1]

ii) Determine the value of μ. [4]



a) $|\underline{a}| = \sqrt{2^2 + (-1)^2} = \sqrt{5}$

∴ Required unit vector

$= \frac{1}{\sqrt{5}}(2\underline{i} - \underline{j})$

b) $\tan\theta = \frac{1}{3}$ $\theta = 18.4°$

Require angle **b** makes with x axis

$= 90 - 18.4° = 71.6°$

c) i) $\mu\underline{a} + \underline{b} = \mu(2\underline{i} - \underline{j}) + (\underline{i} - 3\underline{j})$

$= (2\mu + 1)\underline{i} - (\mu + 3)\underline{j}$

ii) given parallel to $4\underline{i} - 5\underline{j}$

$\therefore (2\mu + 1)\underline{i} - (\mu + 3)\underline{j} = k(4\underline{i} - 5\underline{j})$

$2\mu + 1 = 4k$ (1) $\quad \mu + 3 = 5k$

$2\mu + 6 = 10k$ (2)

(2) − (1) gives $5 = 6k$ $k = \frac{5}{6}$

in (1) $2\mu + 1 = \frac{20}{6}$

$2\mu = \frac{14}{6}$

$\mu = \frac{7}{6}$

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