LEVEL 2 MARKING SCHEME

SUMMER 2022

LEVEL 2
ADDITIONAL MATHEMATICS
9550-01

INTRODUCTION

This marking scheme was used by WJEC for the 2022 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

LEVEL 2 ADDITIONAL MATHEMATICS

SUMMER 2022 MARK SCHEME

		Mark	Comment
1			*Penalise including '+c' -1 only throughout*
	(a) $32x^7$ (+) $-7x^{-8}$ (+0) or $32x^7$ (+) $-\frac{7}{x^8}$ (+0)	B3	B1 for $32x^7$ (not $4\times8x^7$), B1 for $-7x^{-8}$ and B1 for +0 (or blank) provided at least one other mark awarded. If B3 penalise further incorrect working -1, e.g. treat further incorrect work with term $-7x^{-8}$ as ISW unless B3
	(b) $\frac{1}{6}x^{-5/6}$ or equivalent	B1	Index needs to be simplified. ISW
	(c) $\frac{-5x^{-6}}{3}$ or $\frac{-5}{3x^6}$ or $\frac{-5}{3}(\times)x^{-6}$	B1	CAO. ISW
		5	
2	n(n + 1)(n – 1)	B2	(= (n-1)n(n+1) which are 3 consecutive numbers) B1 for $n(n^2 - 1)$
	n = 5	B2	Do not award any marks an answer of n = 5 from incorrect working Depends on at least B1 previously awarded Mark answer space, unless blank If answer space blank, allow for unique solution such as $5^3 - 5 = 120$ B1 for 4 (×) 5 (×) 6 (=120) identified as 'their solution', e.g. 4, 5, 6 leading to n=4 is awarded B1 If no marks, award SC1 for an unambiguous answer (n =) 5 provided not from incorrect working
		4	
3			***No marks if no working throughout***
	(a) $48 + 4\sqrt{3} + 4\sqrt{3} + 1$ (- 1)	M2	Must be convincing non calculator, e.g. $48 + 8\sqrt{3} + 1$ (-1) shown M1 for 3 or 4 terms correct in $48 + 4\sqrt{3} + 4\sqrt{3} + 1 - 1$, or 2 or 3 terms correct in $48 + 4\sqrt{3} + 4\sqrt{3} + 1$
	$48 + 8\sqrt{3}$	A1	CAO, or correct but partially factorised
	$8(6 + \sqrt{3})$	A1	FT from M1 for equivalent level of difficulty Mark final answer
	(a) Alternative method (difference of two squares)		
	$(4\sqrt{3} + 1 - 1)(4\sqrt{3} + 1 + 1)$	*M1*	
	$4\sqrt{3}(4\sqrt{3} + 2)$	*A1*	
	$48 + 8\sqrt{3}$	*A1*	*CAO, or correct but partially factorised*
	$8(6 + \sqrt{3})$	*A1*	*FT from M1 for 'their a(b + √c)'*
	(b) $\frac{1}{6 - \sqrt{7}} \times \frac{6 + \sqrt{7}}{6 + \sqrt{7}}$	M1	This step must be shown
	$= \frac{6 + \sqrt{7}}{29}$	A1	Mark final answer
		6	

4	(a) $6x^{5/8}$	B1	Mark final answer
	(b) x^3	B1	Allow $x^{3/1}$. Mark final answer
	(c) $5 + 7x^{½}$ or $5 + 7\sqrt{x}$	B2	Mark final answer. Allow $1(5 + 7x^{½})$ B1 for sight of $5(x^0)$ or $5x^0$ **and** $7x^{4/8}$ or $7x^{1/2}$
		4	
5	$(a^2 - b^2 =)\ (v + u)^2 - (v - u)^2$ or $a^2 = v^2 + 2uv + u^2$ and $b^2 = v^2 - 2uv + u^2$ or $a^2 = v^2 + uv+uv + u^2$ and $b^2 = v^2 - uv-uv + u^2$	B1	Ignore writing '= 4uv' or '= 4c'
	$(a^2 - b^2 =)\ v^2 + 2uv + u^2 - (v^2 - 2uv + u^2)$ or $(a^2 - b^2 =)\ v^2 + uv+uv + u^2 - (v^2 - uv-uv+ u^2)$	B1	Ignore writing '= 4uv' or '= 4c' Do not allow error with signs. Must show intention to subtract, do not allow missing brackets unless terms from within brackets treated correctly.
	= 4uv or uv + uv + uv + uv or 2uv + 2uv	B1	
	= 4c	B1	Convincing, must show 4uv = 4c
	Alternative method 1		
	$a^2 - b^2 = (v + u)^2 - (v - u)^2$	*B1*	*Ignore writing '= 4uv' or '= 4c'*
	$= (v + u - (v - u))\ (v + u + v - u)$	*B1*	*Ignore writing '= 4uv' or '= 4c'* *Do not allow error with signs. Must show intention of subtracting (v-u) as shown*
	$= 2u(\times)2v$ $(=4uv)$	*B1*	
	$=4c$	*B1*	*Convincing, must show 4uv = 4c*
	Alternative method 2		
	$a^2 - b^2 = (a + b)(a - b)$	*B1*	*Ignore writing '= 4uv' or '= 4c'*
	$= (v + u + v - u)(v + u - (v - u))$	*B1*	*Ignore writing '= 4uv' or '= 4c'* *Do not allow error with signs. Must show intention of subtracting (v-u) as shown*
	$= 2v(\times)2u$ $(=4uv)$	*B1*	
	$= 4c$	*B1*	*Convincing, must show 4uv = 4c*
		4	
6	(a) $3(2)^3 + 4(2)^2 + 3(2)+ 1$ $(= 24 + 16 + 6 + 1)$	M1	Or division method giving $3x^2 + 10x$ …
	47	A1	
	(b)(i) Substitute x = -6	M1	Or division method giving $x^2 - 5x$ …
	Showing f(-6) = 0	A1	Accept sight of **substitution** with '=0' shown
	(ii) $(x+6)(x^2 + bx + c)$ or intention to divide by $(x+6)$ with x^2 shown	M1	If any values are inserted at least 1 needs to be correct, appropriate sight of -5x or +6 implies M1 (and A1 to follow) A1 for -5x or +6
	$(x+6)\ (x^2 - 5x + 6)$	A2	Or use of factor theorem A1 (x - 2), A1 (x - 3)
	$(x+6)(x - 2)(x - 3)$	A1	CAO, with all 3 factors shown, ignore sight of "=0", ISW Must not be from previous incorrect work
		8	

7	(To find the diagonal of the base) $3^2 + 4^2 = 25$ or $3^2 + 4^2 = 5^2$ OR horizontal diagonal = 5 cm (To find the height) (height^2=)$13^2 - 25$ or $13^2 - 5^2$ or $13^2 - (3^2 + 4^2)$ *Alternative methods for M1 M1* *3D Pythagoras' Theorem* $4^2 + 3^2 + height^2 = 13^2$ *(height^2 =) $13^2 - 3^2 - 4^2$ (= 144)* *Diagonal of the right/left end face = d* *$d^2 + 4^2 = 13^2$ or $d^2 = 13^2 - 4^2$ or $d = 3\sqrt{17}$* ***provided** d labelled or used appropriately* *(height^2=) $d^2 - 3^2$ or $(3\sqrt{17})^2 - 3^2$* *Diagonal of the back/front face = c* *$c^2 + 3^2 = 13^2$ or $c^2 = 13^2 - 3^2$ or $c = 4\sqrt{10}$* ***provided** c labelled or used appropriately* *(height^2=) $c^2 - 4^2$ or $(4\sqrt{10})^2 - 4^2$* (Height of the cuboid) 12 (cm) (Surface area of the cuboid) 2(4 × 3 + 12 × 3 + 4 × 12) or equivalent 192 (cm^2) QWC2: • Candidates will be expected to present work clearly, with words explaining process or steps AND • make few if any mistakes in mathematical form, spelling, punctuation and grammar in their answer QWC1: Candidates will be expected to • present work clearly, with words explaining process or steps OR • make few if any mistakes in mathematical form, spelling, punctuation and grammar in their final answer	M1 M1 *M1* *M1* *M1* *M1* *M1* *M1* A1 M1 A1 QWC 2 7	**M0** if used as the height of the cuboid The award of 2nd M1 implies the award of 1st M1 *The award of 2nd M1 implies the award of 1st M1* ***M0** if used as the height of the cuboid* *The award of 2nd M1 **may** imply the award of 1st M1* ***M0** if used as the height of the cuboid* *The award of 2nd M1 **may** imply the award of 1st M1* CAO FT 'their derived 12' provided Pythagoras' Theorem has been attempted. (Note: surface area = 24 + 14 × height) If final M0, A0, award SC1 for a final answer of 96(cm^2) (from 4 × 3 + 12 × 3 + 4 × 12 or equivalent) QWC2 Presents relevant material in a coherent and logical manner, using acceptable mathematical form, and with few if any errors in spelling, punctuation and grammar. QWC1 Presents relevant material in a coherent and logical manner but with some errors in use of mathematical form, spelling, punctuation or grammar OR evident weaknesses in organisation of material but using acceptable mathematical form, with few if any errors in spelling, punctuation and grammar. QWC0 Evident weaknesses in organisation of material, and errors in use of mathematical form, spelling, punctuation or grammar.
8	$(x + 20)^2$ (± …) (Minimum value at x =) -20 (Minimum value is) - 300	M1 A1 A1 3	Ignore 'their (± …)' or '=0' Do not accept method dy/dx =2x+40 CAO CAO, from $(x + 20)^2$ - 300

9	$10x + 2 = 5x^2 - 8x - 6$ $5x^2 - 18x - 8 = 0$	M1 A1	Must be equated to zero. '=0' may be implied in further work to solve, if no further work and not '=0' then A0
	$x = \frac{-(-18) \pm\sqrt{((-18)^2 - 4\times5\times-8)}}{2\times5}$ or $(x \ldots4)(5x \ldots2)$	m1	Working **must** be seen for m1 to be awarded, **must show** use of correct quadratic formula, allow 1 slip in substitution (not a slip with the formula)
	$x = \frac{18 \pm\sqrt{484}}{10}$ or $(x - 4)(5x + 2)\ (=0)$	A1	
	$x = 4$ **with** $x = -0.4$	A1	Or equivalent
	$x = 4$ **and** $y = 42$ **with** $x = -0.4$ **and** $y = -2$	A1	Or equivalent. Both solutions are required FT provided M1, m1 previously awarded using their values of x in $10x + 2$ or equivalent to find y-values Accept answers given as coordinates No marks for a single point of intersection found from trial & improvement.
	Alternative method using $x = (y - 2)/10$		
	$y = 5\frac{(y - 2)^2}{10^2} - \frac{8(y - 2)}{10} - 6$ or equivalent	*M1*	
	$5y^2 - 200y - 420 = 0$ or $y^2 - 40y - 84 = 0$ or equivalent (equate to zero)	*A1*	
	$y = \frac{40\pm\sqrt{(40^2 - 4\times1\times-84)}}{2\times1}$ or $(y \ldots42)(y \ldots2)$	*m1*	*Allow 1 slip in substitution*
	$y = (40\pm\sqrt{1936})/2$ or $(y + 2)(y - 42)$	*A1*	
	*$y = 42$ **with** $y = -2$*	*A1*	*Both solutions are required*
	*$x = 4$ **and** $y = 42$ **with** $x = -0.4$ **and** $y = -2$*	*A1*	*Or equivalent* *FT provided M1, m1 previously awarded using their values of y in $(y - 2)/10$ or equivalent to find x-values*
		6	
10	(a) $270x^8$	B2	B1 for sight of $30x^9$. FT to 2nd B1 from $dy/dx = kx^n$ Ignore incorrect notation Allow B1 for $270x^8 + c$
	(b) For sight of $(dy/dx =) 3ax^2 + 2bx + c$ or $(y =) \frac{27x^3}{3} + \frac{8x^2}{2} + 13x$ (+ constant) or $d^2y/dx^2 = 54x + 8$	B1	May be implied by 2 or 3 correct values
	$a = 9$ $b = 4$ $c = 13$ $d = 9$	B3	B2 for any 2 or 3 values correct, or B1 for 1 value correct *Accept sight of correct answers from 'uncorrected' working* *Only accept embedded answers if clearly stated unambiguously*
		6	

11	(a)(i) $(AB^2 =) (12 - -3)^2 + (6 - 3)^2$ $(=15^2 + 3^2)$ $AB = \sqrt{234}$ $= 3\sqrt{26}$	M1 A1 B1	Or equivalent. Allow 1 slip in sign of substitution CAO. Allow for sight of 15.297... FT 'their AB' of equivalent difficulty expressed correctly, e.g. needs to be in the form $a\sqrt{b}$ where $a \neq 1$ and $b \neq 1$ or simpler Sight of $3\sqrt{26}$ implies previous $\sqrt{234}$
	(a)(ii) Gradient AB $(6 - 3) / (12 - -3)$ $= 3/15$ $(= ⅕)$ Perpendicular gradient is -5	M1 A1 B1	Or equivalent CAO. Mark final answer and then FT Must be simplified if FT is a whole number
	(a)(iii) $(12 + - 3)/2$, $(6 + 3)/2$ Midpoint AB $(9/2 , 9/2)$ or $(4.5, 4.5)$ or equivalent	M1 A1	Need to see working for x and y coordinate CAO
	(b) $15 = 6\times2 + c$ or $c = 3$ or $y - 15 = 6(x - 2)$ or $6 = \frac{y - 15}{x - 2}$ $y = 6x + 3$	M1 A1 10	
12	(a) $16x^8/8 + 15x^5/5 - 4x + 6x^{-3}/-3$ $2x^8 + 3x^5 - 4x - 2x^{-3}$ or $2x^8 + 3x^5 - 4x - 2/x^3$ $+ c$ (constant) (b) $3x^3/3 + 2x^2/2$ or $x^3 + x^2$ Use of correct limits 3 & 2 in correct order **and** intention to subtract 24	B4 B1 B1 M2 m1 A1 10	B1 for each term ISW from correct unsimplified form. CAO simplified form. Mark final answer Awarded only if at least B1 is awarded for integration *No workings, no marks* Ignore sight of '+c. M1 one term correct. FT from M2 or M1 CAO. Must be from correct working
13	$(dy/dx=) \frac{3x^2}{3} - \frac{14x}{2} + 10$ or $x^2 - 7x + 10$ $dy/dx = 0$ or $x^2 - 7x + 10 = 0$ $x = \{-(-7) \pm\sqrt{((-7)^2 - 4\times1\times10)}\} / 2$ or $(x ...2)(x...5) = 0$ $x = 2$ with $x = 5$ $d^2y/dx^2 = 2x - 7$ At $x = 2$, $d^2y/dx^2 < 0$, point is a maximum At $x = 5$, $d^2y/dx^2 > 0$, point is a minimum	B1 M1 m1 A1 M1 A1 A1 7	 FT their $dy/dx = ax^2 + bx$ $(+ c)$ throughout for equivalent level of difficulty Allow 1 slip in substitution. Working must be shown Both solutions are required *Method for determining min or max MUST be shown, final answer only is M0 here, then A0,A0* Or first derivative test, interpretation of first derivative test. Or alternative. FT 'their dy/dx' for M1 provided equivalent difficulty FT for 'their x value' FT for 'their other x value' provided this does not have the same interpretation as the first x value *If M0A0A0, award SC1 for correct FT from at least 'their $d^2y/dx^2 = ax + b$, $a>0$' applied correctly provided it leads to 1 maximum and 1 minimum* *Do not accept trial & improvement methods unless both stationary points are found correctly and confirmed as stated in the mark scheme*

14	$y+\delta y = (x+\delta x)^2 + 6(x+\delta x)$ **Intention** to subtract $(y=) x^2 + 6x$ to find δy $\delta y = 2x\delta x + (\delta x)^2 + 6\delta x$ Dividing by δx **and** (lim) $\delta x \to 0$ $dy/dx = \lim_{\delta x \to 0} \delta y/\delta x = 2x + 6$	B1 M1 A1 M1 A1 5	Or alternative notation. Allow if final bracket omitted Accept δx^2 as meaning $(\delta x)^2$ FT equivalent level of difficulty CAO. Must follow from correct working Use of dy/dx throughout or incorrect notation then possible maximum is only 4 marks, final A0 Allow if 'dy/dx' is implied at the start of each line consistently throughout
15	(a) General sine curve intersecting x-axis at (0°,0) , (90°,0) (180°,0) (270°,0) and (360°,0) Correct curve with 1 and -1 on y-axis (b) 14°, 76°, 194° and 256° alone	M1 A1 B2 4	Maxima should be between 0° to 90° and 180° to 270° Minima should be between 90° to 180° and 270 to 360° Allow general shape as the joining of key values, but straight rather than clearly curving towards a turn Must show a clear curve, not straight at turning points Do not award B2 or B1 for answers from incorrect working: $2x = 28.034\ldots$ leading to $x = 14(.017\ldots)°$ correct, but $x = \sin^{-1} 0.47/2 = \sin^{-1}0.235 = 13.59(1\ldots)°$ is incorrect B1 for sight of 14° or for the correct 4 angles but not given to nearest degree
16	Intention to integrate $-x^3/3 + 5x^2/2 + 6x$ Use of correct limits 6 & 2 in correct order **and** intention to subtract 104/3 or 34⅔ or 34.66(...) or 34.67 or 34.7 or equivalent	M1 A2 m1 A1 5	Intention to integrate, hence not using given or differentiated expression Ignore sight of '+c. A1 one term correct CAO. Do not accept 34⅔ + c or 34.6 *Correct answer only gets M1 A0 m0 A0 (for intention to integrate)* *No marks for use of the trapezium rule*
17	When x = 1, finding y = -1 $dy/dx = 8x - 6$ (when x = 1) gradient is 2 Use of $y - y_1 = m(x - x_1)$ or $y = mx + c$ or $m = \frac{y - y_1}{x - x_1}$ $y - -1 = 2(x-1)$ or $-1 = 2\times1 + c$, $c = -3$ or $y = 2x - 3$ $2x - y - 3 = 0$ or $-2x + y + 3 = 0$	B1 M1 A1 M1 A1 A1 6	 Must be from sight of $dy/dx = 8x - 6$ Method to form equation with appropriate substitution for at least two of x, y and m. FT 'their y value' (but not y=2) and 'their derived gradient'. Needs to be x = 1, do not FT 'their x' FT for x = 1, 'their y' and 'their derived m' CAO. Allow terms in other orders provided '= 0' Mark final answer

Differentiating from first principles. Marking guide.

Q14.

14	$y+\delta y = (x+\delta x)^2 + 6(x+\delta x)$ Intention to subtract $(y=)\ x^2 + 6x$ to find δy $\delta y = 2x\delta x + (\delta x)^2 + 6\delta x$ Dividing by δx and (lim) $\delta x \to 0$ $dy/dx = \lim_{\delta x \to 0} \delta y/\delta x = 2x + 6$	B1 M1 A1 M1 A1 5	Or alternative notation. Allow if final bracket omitted Accept δx^2 as meaning $(\delta x)^2$ FT equivalent level of difficulty CAO. Must follow from correct working Use of dy/dx throughout or incorrect notation then possible maximum is only 4 marks, final A0 Allow if 'dy/dx' is implied at the start of each line consistently throughout

B1 For sight of $(x+\delta x)^2 + 6(x+\delta x)$ or $(x+h)^2 + 6(x+h)$ or using alternative notation.
This mark is given whether $(x+\delta x)^2 + 6(x+\delta x)$ stands alone or is embedded in an expression or a formula.

M1 For the <u>intent</u> to subtract $x^2 + 6x$ from the above.
So $(x+\delta x)^2 + 6(x+\delta x) - x^2 + 6x$ will gain the M1 even though there are missing brackets.
It can also be awarded to those who have expanded $(x+\delta x)^2 + 6(x+\delta x)$ and then crossed out the x^2 term and the $+6x$ term.
Those who reverse the subtraction will gain M0 <u>unless</u> there is evidence later on of dividing by $-\delta x$.

A1 For sight of $2x\delta x + (\delta x)^2 + 6\delta x$ (Accept δx^2 as meaning $(\delta x)^2$) with no other terms.
Treat as a CAO.
$2x + \delta x + 6$ will imply the above if division by δx has already been done.

M1 A FT, if of equivalent difficulty, is possible for this M1 (but not the subsequent A1).
A <u>correct</u> division by δx has to be done
(so if a FT it has to be correct for their $2x\delta x + (\delta x)^2 + 6\delta x$)
<u>AND</u> we must see 'lim $\delta x \to 0$' OR '$\delta x \to 0$' OR 'δx tends to 0'.
It is M0 for '$\delta x = 0$' OR '$\delta x \approx 0$' OR 'δx is so small we can forget about it'.

All of the above marks can be gained even if there is no l.h.s. shown.

Final A1. Must be for a 'text book' quality presentation. E.g.
Has to be a correct l.h.s. for each line, 'δy' or '$\delta y/\delta x$'
AND at some point '$dy/dx = \lim_{\delta x \to 0} \delta y/\delta x$' or '$dy/dx = \lim_{\delta x \to 0} 2x + \delta x + 6$'