DIFFERENTIAL EQUATIONS: A2

A2 Unit 3: Pure Mathematics B

WJEC past paper questions: 2010 – 2019

1. The value, £V, of a car may be modelled as a continuous variable. At time t years, the rate of decrease of V is directly proportional to V^2.

   (a) Write down a differential equation satisfied by V. [1]

   (b) Given that $V = 12000$ when $t = 0$, show that

   $$V = \frac{12000}{at+1},$$

   where a is a constant. [4]

   (c) The value of the car at the end of two years is £9000. Find the value of the car at the end of four years. [4]

(Summer 10)

2. The size N of the population of a small island may be modelled as a continuous variable. At time t, the rate of increase of N is directly proportional to the value of N.

   (a) Write down the differential equation that is satisfied by N. [1]

   (b) Show that $N = Ae^{kt}$, where A and k are constants. [3]

   (c) Given that $N = 100$ when $t = 2$ and that $N = 160$ when $t = 12$,

   (i) show that $k = 0{\cdot}047$, correct to three decimal places,

   (ii) find the size of the population when $t = 20$. [7]

(Summer 11)

3. Water is leaking from a hole at the bottom of a large tank. The volume of the water in the tank at time t hours is $V\,\mathrm{m}^3$. The rate of decrease of V is directly proportional to V^3.

   (a) Write down a differential equation satisfied by V. [1]

   (b) Given that $V = 60$ when $t = 0$, show that

   $$V^2 = \frac{3600}{at+1},$$

   where a is a constant. [4]

   (c) When $t = 2$, the volume of the water in the tank is $50\,\mathrm{m}^3$. Find the value of t when the volume of the water in the tank is $27\,\mathrm{m}^3$. Give your answer correct to one decimal place. [4]

(Summer 12)

4. Part of the surface of a small lake is covered by green algae. The area of the lake covered by the algae at time t years is $A\,\text{m}^2$. The rate of increase of A is directly proportional to $\sqrt{A}$.

   (a) Write down a differential equation satisfied by A. [1]

   (b) The area of the lake covered by the algae at time $t = 3$ is $64\,\text{m}^2$ and the area covered at time $t = 5{\cdot}5$ is $196\,\text{m}^2$. Find an expression for A in terms of t. [6]

(Summer 13)

5. The value £V of a long term investment may be modelled as a continuous variable. At time t years, the rate of increase of V is directly proportional to the value of V.

   (a) Write down a differential equation satisfied by V. [1]

   (b) Show that $V = Ae^{kt}$, where A and k are constants. [3]

   (c) The value of the investment after 2 years is £292 and its value after 28 years is £637.

   (i) Show that $k = 0{\cdot}03$, correct to two decimal places.

   (ii) Find the value of A correct to the nearest integer.

   (iii) Find the initial value of the investment. Give your answer correct to the nearest pound. [6]

(Summer 14)

6. A bookseller values a rare book at £A on August 1st 2010. The value, £P, of the book t years after this date may be modelled as a continuous variable. The rate of increase of P may be assumed to be directly proportional to P^2.

   (a) Write down a differential equation satisfied by P. [1]

   (b) Show that

$$\frac{1}{k}\left(\frac{P-A}{PA}\right) = t,$$

   where k is a constant. [4]

   (c) The value of the book is £800 on August 1st 2013 and £900 on August 1st 2014. Find the value of A. [3]

(Summer 15)

7\.

The value, £V, of a particular car may be modelled as a continuous variable. At time t years, the rate of decrease of V is directly proportional to V^3.

(a) Write down a differential equation satisfied by V. [1]

(b) Given that the initial value of the car is £A, show that

$$V^2 = \frac{A^2}{bt+1},$$

where b is a constant. [4]

(c) When $t = 2$, the value of the car has fallen to a half of its initial value. Find the value of t when the value of the car will have fallen to a quarter of its initial value. [4]

(Summer 16)

8\.

The size N of the population of a small island may be modelled as a continuous variable. At time t years, the rate of increase of N is assumed to be directly proportional to the value of $\sqrt{N}$.

(a) Write down a differential equation satisfied by N. [1]

(b) When $t = 5$, the size of the population was 256. When $t = 7$, the size of the population was 400. Find an expression for N in terms of t. [6]

(Summer 17)

9\.

The value of a painting on January 1st 2000 was £900. The value, £V, of the painting t years after this date may be modelled as a continuous variable. The rate of increase of V may be assumed to be directly proportional to $V^{\frac{3}{2}}$.

(a) Write down a differential equation satisfied by V. [1]

(b) The value of the painting on January 1st 2003 was £1600. Find its value on January 1st 2008. [8]

(Summer 18)

10\. The variable y satisfies the differential equation

$$2\frac{\mathrm{d}y}{\mathrm{d}x} = 5 - 2y, \qquad \text{where } x \geqslant 0.$$

Given that $y = 1$ when $x = 0$, find an expression for y in terms of x. [5]

(Summer 18)

11. (a) A cylindrical water tank has base area 4 m^2. The depth of the water at time t seconds is h metres. Water is poured in at the rate 0.004 m^3 per second. Water leaks from a hole in the bottom at a rate of $0.0008h$ m^3 per second. Show that

$$5000\frac{dh}{dt} \equiv 5 - h.$$ [2]

[Hint: the volume, V, of the cylindrical water tank is given by $V = 4h$.]

(b) Given that the tank is empty initially, find h in terms of t. [7]

(c) Find the depth of the water in the tank when $t = 3600$ s, giving your answer correct to 2 decimal places. [1]

(WJEC Sample)

12. Wildflowers grow on the grass verge by the side of a motorway. The area populated by wildflowers at time t years is A m^2. The rate of increase of A is directly proportional to A.

a) Write down a differential equation that is satisfied by A. [1]

b) At time t = 0, the area populated by wildflowers is 0·2 m^2. One year later, the area has increased to 1·48 m^2. Find an expression for A in terms of t in the form pq^t, where p and q are rational numbers to be determined. [7]

(Summer 19)