



GCE AS/A Level – **LEGACY**

0976/01



MATHEMATICS – C4
Pure Mathematics

FRIDAY, 15 JUNE 2018 – AFTERNOON

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Given that $f(x) = \frac{3x^2 - 3x - 8}{x(x-2)^2}$,

(a) express $f(x)$ in terms of partial fractions, [4]

(b) evaluate

$$\int_6^9 f(x) dx,$$

giving your answer correct to two decimal places. [3]

2. The curve C has equation

$$x^2 - y^3 - 3xy + 1 = 0.$$

The point P has coordinates $(-2, -1)$ and lies on C .

(a) Show that the equation of the tangent to C at the point P is given by

$$x = 3y + 1. \quad [4]$$

(b) The tangent to C at the point P intersects C again at the point Q . Find the coordinates of Q . [5]

3. (a) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$2 \cos 2\theta = 3 \sin^2\theta - 5 \cos^2\theta + \cos \theta + 1. \quad [6]$$

(b) (i) Express $12 \sin \phi - 5 \cos \phi$ in the form $R \sin(\phi - \alpha)$, where R and α are constants with $R > 0$ and $0^\circ < \alpha < 90^\circ$.

(ii) Hence find all values of ϕ in the range $0^\circ \leq \phi \leq 360^\circ$ satisfying

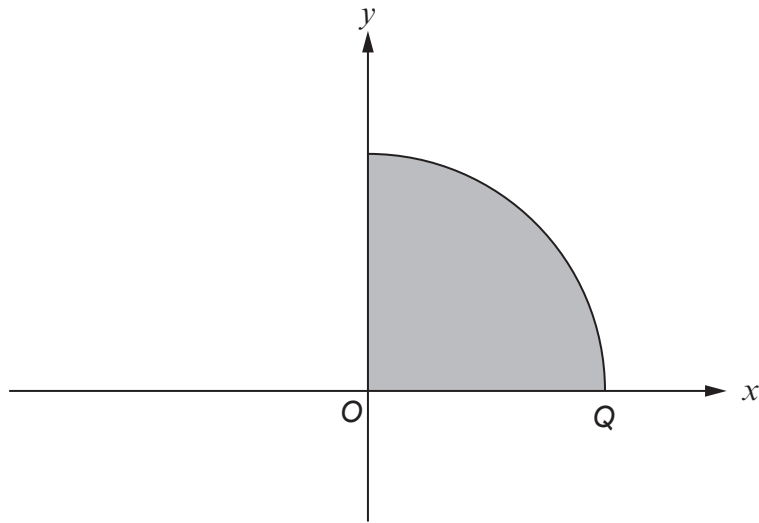
$$12 \sin \phi - 5 \cos \phi = -2. \quad [6]$$

4. (a) Expand $\frac{1}{(1+2x)^2}$ in ascending powers of x up to and including the term in x^2 . [2]

(b) (i) **Use your answer to part (a)** to expand $\left(\frac{1+3x}{1+2x}\right)^2$ in ascending powers of x up to and including the term in x^2 .

(ii) State the range of values of x for which your expansion is valid. [4]

5. The region shaded in the diagram below is bounded by the x -axis, the y -axis, and that part of the curve with equation $x^2 + y^2 = a^2$ ($a > 0$) lying in the first quadrant. The curve intersects the x -axis at the point Q .



- (a) Write down the x -coordinate of Q . [1]
- (b) (i) By carrying out an appropriate integration, find the volume generated when the region shaded in the diagram is rotated through four right-angles about the x -axis. [4]
- (ii) Give a geometrical interpretation of your answer. [4]
6. The curve C has the parametric equations

$$x = \frac{3}{t^2}, \quad y = 4t^3.$$

The point P lies on C and has parameter p . Find and simplify the equation of the tangent to C at the point P . [4]

7. (a) Find $\int (4x+1)e^{4x-5} dx$. Simplify your answer. [4]

- (b) (i) Use the substitution $x = 4 \sin \theta$ to show that

$$\int_0^{2\sqrt{2}} \frac{x^2}{\sqrt{(16-x^2)}} dx = \int_0^a b \sin^2 \theta d\theta,$$

where a and b are constants whose values are to be determined.

- (ii) Hence, evaluate

$$\int_0^{2\sqrt{2}} \frac{x^2}{\sqrt{(16-x^2)}} dx.$$

Give your answer in the form $c\pi + d$, where c and d are integers whose values are to be determined. [8]

TURN OVER

8. The value of a painting on January 1st 2000 was £900. The value, £ V , of the painting t years after this date may be modelled as a continuous variable. The rate of increase of V may be assumed to be directly proportional to $V^{\frac{3}{2}}$.

(a) Write down a differential equation satisfied by V . [1]

(b) The value of the painting on January 1st 2003 was £1600. Find its value on January 1st 2008. [8]

9. (a) The vectors \mathbf{p} and \mathbf{q} are given by

$$\begin{aligned}\mathbf{p} &= 3\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}, \\ \mathbf{q} &= \mathbf{i} + 6\mathbf{j} - 4\mathbf{k}.\end{aligned}$$

Find the angle between \mathbf{p} and \mathbf{q} . Give your answer in degrees, correct to one decimal place. [4]

- (b) The position vectors of the points A and B are denoted by \mathbf{a} and \mathbf{b} respectively. The points C and D have position vectors $4\mathbf{a} - \mathbf{b}$ and $-10\mathbf{a} + 5\mathbf{b}$ respectively. The point E lies on CD and is such that $CE : ED = 1 : 3$.

(i) Find and simplify an expression for the position vector of the point E in terms of \mathbf{a} and \mathbf{b} .

(ii) Interpret your result geometrically. [4]

10. Prove by contradiction the following proposition.

When x is real and positive,

$$25x + \frac{4}{x} \geq 20.$$

The first line of the proof is given below.

Assume that there is a real and positive value of x such that

$$25x + \frac{4}{x} < 20. \quad [3]$$

END OF PAPER