



GCE AS/A Level – **LEGACY**

0975/01



MATHEMATICS – C3
Pure Mathematics

WEDNESDAY, 6 JUNE 2018 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. (a) Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_0^1 e^{-x^3} dx.$$

Show your working and give your answer correct to five decimal places. [4]

- (b) Use your answer to part (a) to deduce an approximate value for the integral

$$\int_0^1 e^{x^3-1} dx. [2]$$

2. (a) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$3 \operatorname{cosec}^2 \theta + 6 \cot \theta = 8 - 5 \cot^2 \theta. [6]$$

- (b) Without carrying out any calculations, explain why there are no values of ϕ in the range $0^\circ \leq \phi \leq 90^\circ$ which satisfy the equation

$$\sec \phi + 2 \tan \phi = 0.8. [1]$$

3. Given that

$$x^5 + 4xy^2 - 2y^3 - 17 = 0,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y . [4]

4. A function is defined parametrically by

$$x = \ln t, y = 4t^4 - 3t^2, \quad \text{for } t > 0.$$

- (a) Find and simplify an expression for $\frac{dy}{dx}$ in terms of t . [4]

- (b) (i) Find and simplify an expression for $\frac{d^2y}{dx^2}$ in terms of t .

- (ii) Show that there is only one value of t such that $\frac{d^2y}{dx^2} = 1$.

Write down this value of t . [6]

5. (a) Show that $f(x) = (2x - 5)e^{2x} + 12x + 7$ has a stationary value when x satisfies

$$(x - 2)e^{2x} + 3 = 0. \quad [4]$$

- (b) **You may assume** that the equation

$$(x - 2)e^{2x} + 3 = 0$$

has a root α between 1 and 2.
The recurrence relation

$$x_{n+1} = 2 - 3e^{-2x_n}$$

with $x_0 = 2$ may be used to find α . Find and record the values of x_1, x_2, x_3, x_4 . Write down the value of x_4 correct to four decimal places and prove that this is the value of α correct to four decimal places. [5]

6. (a) Differentiate each of the following with respect to x , simplifying your answer wherever possible.

(i) $\ln(8 + 7x - 4x^3)$

(ii) $\sqrt[3]{(5 - 9x^2)}$

(iii) $\frac{2 + 5 \sin x}{4 - 3 \cos x}$ [7]

- (b) By first writing $y = \tan^{-1} 5x$ as $5x = \tan y$, find $\frac{dy}{dx}$ in terms of x . [4]

7. (a) Find each of the following integrals, simplifying your answer wherever possible.

(i) $\int \frac{5}{e^{3-4x}} dx$ (ii) $\int \frac{6}{9x-4} dx$ [4]

(b) (i) Evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos\left(2x - \frac{\pi}{6}\right) dx$.

Give your answer in the form $-\frac{a}{b}$, where a, b are positive integers whose values are to be found.

- (ii) Explain briefly why you would expect your answer to be negative. [5]

TURN OVER

8. (a) Show, by counter-example, that the statement

$$\text{'If } \sin \theta = \sin \phi \text{ then } \sin 2\theta = \sin 2\phi \text{'}$$

is false.

[2]

- (b) A straight railway line starts at station O . The next two stations on the line are station S and station T . Station S is 7 km from O and station T is 16 km from O . A footbridge F crosses over the line at a distance x km from O . The distance from F to T is 5 times the distance from F to S .

- (i) Write down the given information in an equation of the form

$$|x - a| = b|x - c|,$$

where the values of the constants a, b, c are to be found.

- (ii) Solve your equation, and hence show that F can be at one of two points on the line. Find the distance of each of these points from T . [5]

9. The function f has domain $(-\infty, 0]$ and is defined by

$$f(x) = 4 - \frac{7}{2 - 3x}.$$

- (a) Find an expression for $f^{-1}(x)$. [4]

- (b) Write down the domain of f^{-1} . [2]

10. The functions f and g have domains $(-\infty, \infty)$ and $(0, \infty)$ respectively and are defined by

$$f(x) = x^2 + 2x - 24,$$

$$g(x) = 5 - 3x.$$

- (a) Write down the domain of fg . [1]

- (b) (i) Write down an expression for $fg(x)$.

- (ii) Hence, solve the equation

$$fg(x) = 200.$$

[5]

END OF PAPER