GCE AS/A level

975/01

MATHEMATICS C3
Pure Mathematics

A.M. THURSDAY, 26 May 2011

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. *(a)* Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_1^2 \ln(3+x^2)\,\mathrm{d}x.$$

Show your working and give your answer correct to four decimal places. [4]

(b) Use your answer to part *(a)* to deduce an approximate value for the integral

$$\int_1^2 \ln\left(\frac{1}{3+x^2}\right)\mathrm{d}x.$$

[1]

2. Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$2\operatorname{cosec}^2\theta + 3\cot^2\theta + 4\operatorname{cosec}\theta = 9.$$ [6]

3. *(a)* Given that

$$2x^3 + x^2\cos y + y^4 + 2x - 25 = 0,$$

find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [4]

(b) Given that

$$x = t^3, \quad y = 2t^2 + 5t^4,$$

(i) find and simplify an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t,

(ii) show that there is no real value of t for which $\frac{\mathrm{d}y}{\mathrm{d}x} = 5$. [7]

4. *(a)* Show that $f(x) = 11\tan^{-1}2x - 3x^2$ has a stationary value when x satisfies

$$12x^3 + 3x - 11 = 0\ .$$ [3]

(b) **You may assume** that the equation $12x^3 + 3x - 11 = 0$ has a root α between 0 and 1.

The recurrence relation

$$x_{n+1} = \left(\frac{11-3x_n}{12}\right)^{\frac{1}{3}}$$

with $x_0 = 0{\cdot}9$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and show this is the value of α correct to five decimal places. [5]

5. Differentiate each of the following with respect to x, simplifying your answer wherever possible.

(a) $(9-2x)^{\frac{1}{3}}$ *(b)* $\ln(\cos x)$ *(c)* $x^3 \tan 4x$ *(d)* $\dfrac{e^{6x}}{(3x+2)^4}$

[2], [3], [3], [3]

6. *(a)* Find

(i) $\displaystyle\int \frac{9}{4x+3}\,dx$, (ii) $\displaystyle\int 3e^{5-2x}\,dx$, (iii) $\displaystyle\int \frac{5}{(7x-1)^3}\,dx$. [6]

(b) Evaluate $\displaystyle\int_0^{\frac{\pi}{3}} \cos\left(3x-\frac{\pi}{6}\right)dx$. [4]

7. *(a)* Show, by counter-example, that the statement

$$|a+b| \equiv |a|+|b|$$

is false. [2]

(b) Solve the equation

$$|2x+1| = |3x-4|$$ [3]

8. Given that $f(x) = \ln x$, sketch, on the same diagram, the graphs of $y = f(x)$ and $y = \frac{1}{2}f(x+3)$. Label the coordinates of the point of intersection of each of the graphs with the x-axis. Indicate the behaviour of each of the graphs for large positive and negative values of y. [5]

9. The function f has domain $(-\infty, -\frac{1}{2})$ and is defined by

$$f(x) = e^{2x+1} - 3.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) Write down the domain of f^{-1}. [2]

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10. The functions f and g have domains $(-\infty, 0)$ and $(6, \infty)$ respectively and are defined by

$$f(x) = x^2 - 19,$$
$$g(x) = 1 - \frac{1}{2}x.$$

(a) Write down the range of f and the range of g. [2]

(b) Write down the domain and range of fg. [2]

(c) (i) Write down an expression for $fg(x)$.

(ii) Hence, solve the equation

$$fg(x) = 2x - 26.$$ [4]