GCE MARKING SCHEME

SUMMER 2018

MATHEMATICS – C4 (LEGACY)
0976-01

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INTRODUCTION

This marking scheme was used by WJEC for the 2018 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

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Mathematics C4 June 2018

Solutions and Mark Scheme

1.	(*a*)	$f(x) \equiv \frac{A}{x} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$	(correct form)	M1
		$3x^2 - 3x - 8 \equiv A(x-2)^2 + Bx(x-2) + Cx$		
		(correct clearing of fractions and genuine attempt to find coefficients)		m1
		$A = -2,\ C = -1,\ B = 5$	(all three coefficients correct)	A2
		(**If A2 not awarded**, award A1 for either 1 or 2 correct coefficients)		
	(*b*)	$\int f(x)\,dx = -2\ln x + 5\ln(x-2) - (-1) \times (x-2)^{-1}$		
			(f.t. candidate's values for *A*, *B*, *C*)	
			(**at least one** the first and second terms)	B1
			(third term)	B1
		$\int_6^9 f(x)\,dx = 1{\cdot}88$	(c.a.o.)	B1
		Note: Answer only with no working earns 0 marks		
2.	(*a*)	$2x - 3y^2\frac{dy}{dx} - 3x\frac{dy}{dx} - 3y = 0$	$\left(2x - 3y^2\frac{dy}{dx}\right)$	B1
			$\left(-3x\frac{dy}{dx} - 3y\right)$	B1
		Either $\frac{dy}{dx} = \frac{2x-3y}{3y^2+3x}$ **or** $\frac{dy}{dx} = \frac{1}{3}$	(o.e.) (c.a.o.)	B1
		Equation of tangent: $x = 3y + 1$	(convincing)	B1
	(*b*)	$(3y+1)^2 - y^3 - 3(3y+1)y + 1 = 0$		M1
		$y^3 - 3y - 2 = 0$		A1
		Either:		
		$(y + 1)$ must be a (repeated) factor of $y^3 - 3y - 2$		M1
		$y^3 - 3y - 2 = (y+1)(y+1)(y-2)$		A1
		At Q, $y = 2$, $x = 7$		A1
		Or:		
		Substitution of 2 for y in $y^3 - 3y - 2$		M1
		$y = 2$ is a root of $y^3 - 3y - 2 = 0$		A1
		At Q, $y = 2$, $x = 7$		A1

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3. (*a*) $2(2\cos^2\theta - 1) = 3(1 - \cos^2\theta) - 5\cos^2\theta + \cos\theta + 1$

(correct use of $\cos 2\theta = 2\cos^2\theta - 1$ and $\sin^2\theta = 1 - \cos^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = candidate's coefficient of $\cos^2\theta$ and $b \times d$ = candidate's constant m1

$12\cos^2\theta - \cos\theta - 6 = 0 \Rightarrow (4\cos\theta - 3)(3\cos\theta + 2) = 0$

$\Rightarrow \cos\theta = \frac{3}{4}, \quad \cos\theta = -\frac{2}{3},$ (c.a.o.) A1

$\theta = 41{\cdot}41°, 318{\cdot}59°$ B1

$\theta = 131{\cdot}81°, 228{\cdot}19°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\cos\theta$ = +, –, f.t. for 3 marks, $\cos\theta$ = –, –, f.t. for 2 marks
$\cos\theta$ = +, +, f.t. for 1 mark

(*b*) (i) $R = 13$ B1

Correctly expanding $\sin(\phi - \alpha)$, correctly comparing coefficients and using either $13\cos\alpha = 12$ **or** $13\sin\alpha = 5$ **or** $\tan\alpha = \frac{5}{12}$ to find α (f.t. candidate's value for R) M1

$\alpha = 22{\cdot}62°$ (c.a.o.) A1

(ii) $\sin(\phi - 22{\cdot}62°) = -\frac{2}{13}$

(f.t. candidate's values for R, α, $0° < \alpha < 90°$) B1

$\phi - 22{\cdot}62° = -8{\cdot}85°, 188{\cdot}85°, 351{\cdot}15°$ (at least one value)

(f.t. candidate's values for R, α, $0° < \alpha < 90°$) B1

$\phi = 13{\cdot}77°, 211{\cdot}47°$ (c.a.o.) B1

4. (*a*) $(1 + 2x)^{-2} = 1 - 4x + 12x^2$ $(1 - 4x)$ B1

$(12x^2)$ B1

(*b*) (i) $\left(\frac{1 + 3x}{1 + 2x}\right)^2 = (1 + 3x)^2 \times (1 - 4x + 12x^2)$ M1

$\left(\frac{1 + 3x}{1 + 2x}\right)^2 = 1 + (6x - 4x) + (9x^2 + 12x^2 - 24x^2)$

(at least two terms correct) A1

$\left(\frac{1 + 3x}{1 + 2x}\right)^2 = 1 + 2x - 3x^2$

(c.a.o.) A1

(ii) $|x| < {}^1/_2$ B1

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5. (*a*) The *x*-coordinate of *Q* is *a* — B1

(*b*) (i)

Volume = $\pi\int_0^a (a^2 - x^2)\,dx$ — M1

$\int (a^2 - x^2)\,dx = a^2x - \frac{x^3}{3}$ — B1

Volume = $\frac{2\pi a^3}{3}$ (c.a.o.) — A1

(ii) This is the volume of a hemisphere of radius *a* — E1

6. candidate's *x*-derivative = $-6t^{-3}$ (o.e.)

candidate's *y*-derivative = $12t^2$ (at least one term correct)

and use of

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ — M1

$\frac{dy}{dx} = -\frac{12t^5}{6}$ (o.e.) — A1

Equation of tangent at *P*: $y - 4p^3 = -\frac{12p^5}{6}\left(x - \frac{3}{p^2}\right)$

(f.t. candidate's expression for $\frac{dy}{dx}$) — m1

Equation of tangent at *P*: $y = -2p^5x + 10p^3$ (o.e) (c.a.o.) — A1

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7.	(*a*)	$u = 4x + 1 \Rightarrow \mathrm{d}u = 4\mathrm{d}x$	(o.e.)	B1
		$\mathrm{d}v = \mathrm{e}^{4x-5}\,\mathrm{d}x \Rightarrow v = \frac{1}{4}\mathrm{e}^{4x-5}$	(o.e.)	B1
		$\int (4x+1)\,\mathrm{e}^{4x-5}\,\mathrm{d}x = \frac{1}{4}\mathrm{e}^{4x-5} \times (4x+1) - \int \frac{1}{4}\mathrm{e}^{4x-5} \times 4\mathrm{d}x$	(o.e.)	M1
		$\int (4x+1)\,\mathrm{e}^{4x-5}\,\mathrm{d}x = \frac{1}{4}\mathrm{e}^{4x-5} \times (4x+1) - \frac{1}{4}\mathrm{e}^{4x-5} + c$		
		$\int (4x+1)\,\mathrm{e}^{4x-5}\,\mathrm{d}x = x\,\mathrm{e}^{4x-5} + c$		A1
	(*b*)	(i) $x = 2\sqrt{2} \Rightarrow \theta = \pi/4$		B1
		An attempt to express each of x^2, $\sqrt{(16 - x^2)}$ and $\mathrm{d}x$ in terms of θ only		M1
		$\int \frac{x^2}{\sqrt{(16-x^2)}}\,\mathrm{d}x = \int \frac{16\sin^2\theta \times 4\cos\theta\,\mathrm{d}\theta}{\sqrt{(16 - 16\sin^2\theta)}}$		A1
		$\int \frac{x^2}{\sqrt{(16-x^2)}}\,\mathrm{d}x = \int 16\sin^2\theta\,\mathrm{d}\theta$		A1
		(ii) Use of $\sin^2\theta = \frac{(1 - \cos 2\theta)}{2}$		B1
		$\int (p + q\cos 2\theta)\,\mathrm{d}\theta = p\theta + {}^1/_2\, q\sin 2\theta$	$(p \neq 0, q \neq 0)$	B1
		Correct substitution of candidate's upper limit and 0 in candidate's integrated expression of the form $m\theta + n\sin 2\theta$ $(m \neq 0, n \neq 0)$		M1
		$\int_0^{2\sqrt{2}} \frac{x^2}{\sqrt{(16-x^2)}}\,\mathrm{d}x = 2\pi - 4$	(c.a.o.)	A1

Note: Answer only with no working earns 0 marks

8.	(*a*)	$\frac{\mathrm{d}V}{\mathrm{d}t} = kV^{3/2}$		B1
	(*b*)	$\int \frac{\mathrm{d}V}{V^{3/2}} = \int k\,\mathrm{d}t$	(o.e.)	M1
		$\frac{V^{-1/2}}{-{}^1/_2} = kt + c$		A1
		Substituting $V = 900$, $t = 0$		M1
		$c = -\frac{1}{15}$	(c.a.o.)	A1
		Substituting $V = 1600$, $t = 3$		M1
		$k = \frac{1}{180}$	(f.t. one slip in the evaluation of c)	A1
		Substituting $t = 8$		M1
		$V = 8100$	(f.t. one slip in the evaluation of c and k)	A1

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9. (*a*) $\mathbf{p.q} = -37$ B1

$|\mathbf{p}| = \sqrt{62}$, $|\mathbf{q}| = \sqrt{53}$ (at least one correct) B1

Correctly substituting candidate's derived values in the formula

$\mathbf{p.q} = |\mathbf{p}| \times |\mathbf{q}| \times \cos\theta$ M1

$\theta = 130{\cdot}2°$ (c.a.o.) A1

(*b*) (i) **Either:**

Position vector of $E = \frac{3}{4}(4\mathbf{a} - \mathbf{b}) + \frac{1}{4}(-10\mathbf{a} + 5\mathbf{b})$

(allow $\frac{1}{4}(4\mathbf{a} - \mathbf{b}) + \frac{3}{4}(-10\mathbf{a} + 5\mathbf{b})$) M1

Position vector of $E = \frac{1}{4}(12\mathbf{a} - 3\mathbf{b} - 10\mathbf{a} + 5\mathbf{b})$ A1

Position vector of $E = \frac{1}{2}(\mathbf{a} + \mathbf{b})$ A1

Or:

An attempt to find **CD** and then use of position vector of E =

$\mathbf{OC} + \frac{1}{4}\mathbf{CD}$ (allow $\mathbf{OC} + \frac{3}{4}\mathbf{CD}$) M1

Position vector of $E = (4\mathbf{a} - \mathbf{b}) + \frac{1}{4}(-14\mathbf{a} + 6\mathbf{b})$ A1

Position vector of $E = \frac{1}{2}(\mathbf{a} + \mathbf{b})$ A1

Or:

An attempt to find **CD** and then use of position vector of E =

$\mathbf{OD} - \frac{3}{4}\mathbf{CD}$ (allow $\mathbf{OD} - \frac{1}{4}\mathbf{CD}$) M1

Position vector of $E = (-10\mathbf{a} + 5\mathbf{b}) - \frac{3}{4}(-14\mathbf{a} + 6\mathbf{b})$ A1

Position vector of $E = \frac{1}{2}(\mathbf{a} + \mathbf{b})$ A1

(ii) E is the midpoint of AB E1

10. Assume that there is a real and positive value of x such that $25x + \frac{4}{x} < 20$

$25x^2 - 20x + 4 < 0$ B1

$(5x - 2)^2 < 0$ B1

This is impossible since the square of a real number cannot be negative and thus $25x + \frac{4}{x} \geq 20$ B1

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