1. (*a*) $f(x) \equiv \dfrac{A}{(x+3)^2} + \dfrac{B}{(x+3)} + \dfrac{C}{(x-1)}$ (correct form) M1

$2x^2 + 5x + 25 \equiv A(x-1) + B(x+3)(x-1) + C(x+3)^2$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = -7$, $C = 2$, $B = 0$ (all three coefficients correct) A2

If A2 not awarded, award A1 for at least one correct coefficient

(*b*) $\int f(x)\,dx = \dfrac{7}{(x+3)} + 2\ln(x-1)$ B1 B1

(f.t. candidate's values for *A*, *B*, *C*)

$\int_3^{10} f(x)\,dx = \left[\dfrac{7}{13} + 2\ln 9\right] - \left[\dfrac{7}{6} + 2\ln 2\right] = 2{\cdot}38$ (c.a.o.) B1

Note: Answer only with no working earns 0 marks

2. (*a*) $4x^3 + 3x^2\dfrac{dy}{dx} + 6xy - 4y\dfrac{dy}{dx} = 0$ $\left[3x^2\dfrac{dy}{dx} + 6xy\right]$ B1

$\left[4x^3 - 4y\dfrac{dy}{dx}\right]$ B1

$\dfrac{dy}{dx} = \dfrac{4x^3 + 6xy}{4y - 3x^2}$ (convincing) B1

(*b*) $4y - 3x^2 = 0$ M1

Either: Substituting $\dfrac{3x^2}{4}$ for y in the equation of C and an attempt to collect terms m1

$x^4 = 16 \Rightarrow x = (\pm)\,2$ A1

$y = 3$ (for both values of x)

(f.t. $x^4 = a$, $a \neq 16$, provided both x values are checked) A1

Or: Substituting $\dfrac{4y}{3}$ for x^2 in the equation of C and an attempt to collect terms m1

$y^2 = 9 \Rightarrow y = (\pm)\,3$ A1

$y = 3 \Rightarrow x = \pm\,2$ (f.t. $y^2 = b$, $b \neq 9$) A1

3. (*a*) $\frac{\tan x + \tan 45°}{1 - \tan x \tan 45°} = 8 \tan x$ (correct use of formula for $\tan(x + 45°)$) M1

Use of $\tan 45° = 1$ and an attempt to form a quadratic in $\tan x$ by cross multiplying and collecting terms M1

$8\tan^2 x - 7\tan x + 1 = 0$ (c.a.o.) A1

Use of a correct method to solve the candidate's derived quadratic in $\tan x$ m1

$x = 34·8°, 10·2°$ (both values)

(f.t. one slip in candidate's derived quadratic in $\tan x$ provided all three method marks have been awarded) A1

(*b*) (i) $R = 7$ B1

Correctly expanding $\sin(\theta - \alpha)$, correctly comparing coefficients and using either $7\cos\alpha = \sqrt{13}$ **or** $7\sin\alpha = 6$ **or** $\tan\alpha = \frac{6}{\sqrt{13}}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 59°$ (c.a.o) A1

(ii) $\sin(\theta - \alpha) = -\frac{4}{7}$

(f.t. candidate's values for R, α) B1

$\theta - 59° = -34·85°, 214·85°, 325·15°,$

(at least one value, f.t. candidate's values for R, α) B1

$\theta = 24·15°, 273·85°$ (c.a.o.) B1

4. (*a*)

$V = \pi\int_0^a (mx)^2\,dx$ M1

$\int (mx)^2\,dx = \frac{m^2x^3}{3}$ B1

$V = \pi\frac{m^2a^3}{3}$ (c.a.o.) A1

(*b*) (i) Substituting $\frac{b}{a}$ for m in candidate's derived expression for V M1

$V = \pi\frac{b^2a}{3}$ (c.a.o.) A1

(ii) This is the volume of a cone of (vertical) height a and (base) radius b E1

5. $\left(1 + \frac{x}{8}\right)^{-1/2} = 1 - \frac{x}{16} + \frac{3x^2}{512}$ — $\left(1 - \frac{x}{16}\right)$ B1; $\left(\frac{3x^2}{512}\right)$ B1

$|x| < 8$ or $-8 < x < 8$ — B1

$\frac{2}{3}\sqrt{2} \approx 1 - \frac{1}{16} + \frac{3}{512}$ (f.t. candidate's coefficients) — B1

Either: $\sqrt{2} \approx \frac{1449}{1024}$ (c.a.o.)

Or: $\sqrt{2} \approx \frac{2048}{1449}$ (c.a.o.) — B1

6. (*a*) (i) candidate's *x*-derivative = $2at$

candidate's *y*-derivative = $2a$ (at least one term correct)

and use of

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ — M1

$\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$

Gradient of tangent at $P = \frac{1}{p}$ (c.a.o.) — A1

(ii) Equation of tangent at *P*: $y - 2ap = \frac{1}{p}(x - ap^2)$

(f.t. candidate's expression for $\frac{dy}{dx}$) — m1

Equation of tangent at *P*: $py = x + ap^2$ — A1

(*b*) (i) Gradient $PQ = \frac{2ap - 2aq}{ap^2 - aq^2}$ — B1

Use of $ap^2 - aq^2 = a(p + q)(p - q)$ — B1

Gradient $PQ = \frac{2}{p + q}$ (c.a.o.) — B1

(ii) As the point *Q* approaches *P*, *PQ* becomes a tangent

$\lim_{q \to p}$ (gradient PQ) $= \frac{2}{2p} = \frac{1}{p}$. — E1

7. (*a*) $\displaystyle\int \frac{x^2}{(12-x^3)^2}\,dx = \int \frac{k}{u^2}\,du$ $\quad(k = {}^1/_3, -{}^1/_3, 3 \text{ or } -3)$ M1

$\displaystyle\int \frac{a}{u^2}\,du = a \times \frac{u^{-1}}{-1}$ B1

Either: Correctly inserting limits of 12, 4 in candidate's bu^{-1}

or: Correctly inserting limits of 0, 2 in candidate's $b(12-x^3)^{-1}$

M1

$\displaystyle\int_0^2 \frac{x^2}{(12-x^3)^2}\,dx = \frac{1}{18}$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks

(*b*) (i) $u = x \Rightarrow du = dx$ (o.e.) B1

$dv = \cos 2x\,dx \Rightarrow v = \frac{1}{2}\sin 2x$ (o.e.) B1

$\displaystyle\int x\cos 2x\,dx = x \times \frac{1}{2}\sin 2x - \int \frac{1}{2}\sin 2x \times dx$ M1

$\displaystyle\int x\cos 2x\,dx = \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$ (c.a.o.) A1

(ii) $\displaystyle\int x\sin^2 x\,dx = \int x\left(\frac{k}{2} - \frac{m}{2}\cos 2x\right)dx$ (o.e.)

$(k = \mathbf{1}, -1, m = \mathbf{1}, -1)$ M1

$\displaystyle\int x\sin^2 x\,dx = \frac{1}{2}\int x\,dx - \frac{1}{2}\int x\cos 2x\,dx$ A1

$\displaystyle\int x\sin^2 x\,dx = \frac{x^2}{4} - \frac{1}{4}x\sin 2x - \frac{1}{8}\cos 2x + c$

(f.t. only candidate's answer to (*b*)(*i*)) A1

8. (*a*) (i) $\mathbf{AB} = -\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$ B1

(ii) Use of $\mathbf{a} + \lambda\mathbf{AB}$, $\mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$, $\mathbf{b} + \lambda\mathbf{AB}$ or $\mathbf{b} + \lambda(\mathbf{b} - \mathbf{a})$ to find vector equation of *AB* M1

$\mathbf{r} = 5\mathbf{i} - \mathbf{j} - \mathbf{k} + \lambda(-\mathbf{i} - 2\mathbf{j} + 7\mathbf{k})$ (o.e.)

(f.t. if candidate uses his/her expression for **AB**) A1

(*b*)

$5 - \lambda = 2 + \mu$

$-1 - 2\lambda = -3 + \mu$

$-1 + 7\lambda = -4 - \mu$ (o.e.)

(comparing coefficients, at least one equation correct) M1

(at least two equations correct) A1

Solving two of the equations simultaneously m1

(f.t. for all 3 marks if candidate uses his/her equation of *AB*)

$\lambda = -1$, $\mu = 4$ (o.e.) (c.a.o.) A1

Correct verification that values of λ and μ satisfy third equation A1

Position vector of point of intersection is $6\mathbf{i} + \mathbf{j} - 8\mathbf{k}$

(f.t. one slip) A1

9. (*a*) $\frac{dP}{dt} = kP^2$ B1

(*b*) $\int \frac{dP}{P^2} = \int k \, dt$ M1

$-\frac{1}{P} = kt + c$ (o.e.) A1

$c = -\frac{1}{A}$ (c.a.o.) A1

$-\frac{1}{P} = kt - \frac{1}{A} \Rightarrow kt = \frac{1}{A} - \frac{1}{P} \Rightarrow \frac{1}{k}\left(\frac{P - A}{PA}\right) = t$ (convincing) A1

(*c*) $\frac{1}{k}\left(\frac{800 - A}{800A}\right) = 3, \quad \frac{1}{k}\left(\frac{900 - A}{900A}\right) = 4$ (both equations) B1

An attempt to solve these equations simultaneously by eliminating k

M1

$A = 600$ (c.a.o.) A1

10. Assume that 4 is a factor of $a + b$.
Then there exists an integer c such that $a + b = 4c$.
Similarly, there exists an integer d such that $a - b = 4d$. B1
Adding, we have $2a = 4c + 4d$. B1
Therefore $a = 2c + 2d$, an even number, which contradicts the fact that a is odd. B1