

C4

$$1. \quad 9x^2 - 5x \times 2y \frac{dy}{dx} - 5y^2 + 8y^3 \frac{dy}{dx} = 0 \quad \left[\begin{array}{l} -5x \times 2y \frac{dy}{dx} - 5y^2 \\ \frac{dy}{dx} \end{array} \right] \quad \text{B1}$$

$$\left[\begin{array}{l} 9x^2 + 8y^3 \frac{dy}{dx} \\ \frac{dy}{dx} \end{array} \right] \quad \text{B1}$$

Either $\frac{dy}{dx} = \frac{9x^2 - 5y^2}{10xy - 8y^3}$ **or** $\frac{dy}{dx} = \frac{1}{4}$ (o.e.) (c.a.o.) B1

Attempting to substitute $x = 1$ and $y = 2$ in candidate's expression **and** the use of $\text{grad}_{\text{normal}} \times \text{grad}_{\text{tangent}} = -1$ M1

Equation of normal: $y - 2 = -4(x - 1)$

$$\left[\begin{array}{l} \text{f.t. candidate's value for } \frac{dy}{dx} \\ \frac{dy}{dx} \end{array} \right] \quad \text{A1}$$

2. (a) $f(x) \equiv \frac{A}{(x+1)^2} + \frac{B}{(x+1)} + \frac{C}{(x-4)}$ (correct form) M1

$$5x^2 + 7x + 17 \equiv A(x-4) + B(x+1)(x-4) + C(x+1)^2$$

(correct clearing of fractions and genuine attempt to find coefficients)

m1

$A = -3, C = 5, B = 0$ (all three coefficients correct) A2

(If A2 not awarded, award A1 for either 1 or 2 correct coefficients)

(b) $\frac{5x^2 + 9x + 9}{(x+1)^2(x-4)} = \frac{5x^2 + 7x + 17}{(x+1)^2(x-4)} + \frac{2}{(x+1)^2}$ M1

$$\frac{5x^2 + 9x + 9}{(x+1)^2(x-4)} = \frac{-1}{(x+1)^2} + \frac{5}{(x-4)}$$

(f.t. candidates values for A, B, C) A1

3. (a) $\frac{2 \tan x}{1 - \tan^2 x} = 3 \cot x$ (correct use of formula for $\tan 2x$) M1
- $\frac{2 \tan x}{1 - \tan^2 x} = \frac{3}{\tan x}$ (correct use of $\cot x = \frac{1}{\tan x}$) M1
- $\tan^2 x = \frac{3}{5}$ (o.e.) A1
- $x = 37.76^\circ, 142.24^\circ$ (both values) A1
- (f.t. $a \tan^2 x = b$ provided both M1's are awarded)
- (b) (i) $R = 29$ B1
- Correctly expanding $\sin(\theta - \alpha)$ and using either $29 \cos \alpha = 21$
 or $29 \sin \alpha = 20$ or $\tan \alpha = \frac{20}{21}$ to find α
- $\alpha = 43.6^\circ$ (c.a.o.) A1
- (ii) Greatest value of $\frac{1}{21 \sin \theta - 20 \cos \theta + 31} = \frac{1}{29 \times (\pm 1) + 31}$ M1
- (f.t. candidate's value for R)
- Greatest value = $\frac{1}{2}$ (f.t. candidate's value for R) A1
- Corresponding value for $\theta = 313.6^\circ$ (o.e.)
- (f.t. candidate's value for α) A1

4. Volume = $\pi \int_0^{\pi/4} (3 + 2 \sin x)^2 dx$ B1
- Correct use of $\sin^2 x = \frac{(1 - \cos 2x)}{2}$ M1
- Integrand = $(9 + 2 + 12 \sin x - 2 \cos 2x)$ (c.a.o.) A1
- $\int (a + b \sin x + c \cos 2x) dx = (ax - b \cos x + \frac{c}{2} \sin 2x)$ B1
- ($a \neq 0, b \neq 0, c \neq 0$)
- Correct substitution of correct limits in candidate's integrated expression
 of form $(ax - b \cos x + \frac{c}{2} \sin 2x)$ ($a \neq 0, c \neq 0$) M1
- Volume = 35 (c.a.o.) A1

Note: Answer only with no working earns 0 marks

5. $(1 - 2x)^{1/2} = 1 + (1/2) \times (-2x) + \frac{(1/2) \times (1/2 - 1) \times (-2x)^2}{1 \times 2} + \dots$
 (–1 each incorrect term) B2

$\frac{1}{1 + 4x} = 1 + (-1) \times (4x) + \frac{(-1) \times (-2) \times (4x)^2}{1 \times 2} + \dots$
 (–1 each incorrect term) B2

$6\sqrt{1 - 2x} - \frac{1}{1 + 4x} = 5 - 2x - 19x^2 + \dots$
 (–1 each incorrect term) B2

Expansion valid for $|x| < 1/4$ (o.e.) B1

6. (a) candidate's x -derivative = 2
 candidate's y -derivative = $15t^2$ (at least one term correct)
 and use of
 $\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1
 $\frac{dy}{dx} = \frac{15t^2}{2}$ (o.e.) (c.a.o.) A1
 Equation of tangent at P : $y - 5p^3 = \frac{15p^2}{2}(x - 2p)$
 (f.t. candidate's expression for $\frac{dy}{dx}$) m1
 $2y = 15p^2x - 20p^3$ (convincing) A1

(b) Substituting $p = 1, x = 2q, y = 5q^3$ in equation of tangent M1
 $q^3 - 3q + 2 = 0$ (convincing) A1
 Putting $f(q) = q^3 - 3q + 2$
Either $f(q) = (q - 1)(q^2 + q - 2)$ **or** $f(q) = (q + 2)(q^2 - 2q + 1)$ M1
Either $f(q) = (q - 1)(q - 1)(q + 2)$ **or** $q = 1, q = -2$ A1
 $q = -2$ A1

7. (a) $u = \ln 2x \Rightarrow du = 2 \times \frac{1}{2x} dx$ (o.e.) B1
 $dv = x^4 dx \Rightarrow v = \frac{1}{5} x^5$ (o.e.) B1
 $\int x^4 \ln 2x dx = \ln 2x \times \frac{1}{5} x^5 - \int \frac{1}{5} x^5 \times \frac{1}{x} dx$ (o.e.) M1
 $\int x^4 \ln 2x dx = \ln 2x \times \frac{1}{5} x^5 - \frac{1}{25} x^5 + c$ (c.a.o.) A1
- (b) $\int \sqrt{(10 \cos x - 1)} \sin x dx = \int k \times u^{1/2} du$ ($k = -1/10, 1/10$ or ± 10) M1
 $\int a \times u^{1/2} du = a \times \frac{u^{3/2}}{3/2}$ B1
 $\int_0^{\pi/3} \sqrt{(10 \cos x - 1)} \sin x dx = k \left[\frac{u^{3/2}}{3/2} \right]_9^4$ or $k \left[\frac{(10 \cos x - 1)^{3/2}}{3/2} \right]_0^{\pi/3}$ B1
 $\int_0^{\pi/3} \sqrt{(10 \cos x - 1)} \sin x dx = \frac{19}{15} = 1.27$ (c.a.o.) A1
8. (a) $\frac{dV}{dt} = kV$ B1
- (b) $\int \frac{dV}{V} = \int k dt$ M1
 $\ln V = kt + c$ A1
 $V = e^{kt+c} = Ae^{kt}$ (convincing) A1
- (c) (i) $292 = Ae^{2k}$
 $637 = Ae^{28k}$ (both values) B1
Dividing to eliminate A M1
 $\frac{637}{292} = e^{26k}$ A1
 $k = \frac{1}{26} \ln \left[\frac{637}{292} \right] = 0.03$ A1
- (ii) $A = 275$ B1
- (iii) When $t = 0$, initial value of investment = £275
(f.t. candidate's derived value for A) B1

9. (a) $\mathbf{p} \cdot \mathbf{q} = -18$ B1
 $|\mathbf{p}| = \sqrt{14}, |\mathbf{q}| = \sqrt{105}$ (at least one correct) B1
 Correctly substituting candidate's derived values in the formula
 $\mathbf{p} \cdot \mathbf{q} = |\mathbf{p}| \times |\mathbf{q}| \times \cos \theta$ M1
 $\theta = 118^\circ$ (c.a.o.) A1
- (b) (i) Use of $\mathbf{CD} = \mathbf{CO} + \mathbf{OD}$ and the fact that $\mathbf{OC} = \frac{1}{2}\mathbf{b}$ and
 $\mathbf{OD} = 2\mathbf{a}$, leading to printed answer $\mathbf{CD} = 2\mathbf{a} - \frac{1}{2}\mathbf{b}$
 (convincing) B1
 Use of $\frac{1}{2}\mathbf{b} + \lambda\mathbf{CD}$ (o.e.) to find vector equation of CD M1
 Vector equation of CD : $\mathbf{r} = 2\lambda\mathbf{a} + \frac{1}{2}(1 - \lambda)\mathbf{b}$
 (convincing) A1
- (ii) **Either:**
 Either substituting $\frac{1}{3}$ for λ in the vector equation of CD
 or substituting 2 for μ in the vector equation of L M1
 At least one of these position vectors = $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ A1
 Both position vectors = $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} \Rightarrow$ this must be the position
 vector of the point of intersection E A1
Or:
 $2\lambda = \frac{\mu}{3}$
 $\frac{1}{2}(1 - \lambda) = \frac{1}{3}(\mu - 1)$
 (comparing candidate's coefficients of \mathbf{a} and \mathbf{b} and an attempt
 to solve) M1
 $\lambda = \frac{1}{3}$ or $\mu = 2$ A1
 $\mathbf{OE} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ (convincing) A1
- (iii) **Either:** E lies on AB and is such that $AE : EB = 1 : 2$ (o.e.)
Or: E is the point of intersection of AB and CD B1
10. Squaring both sides we have
 $1 + 2 \sin \theta \cos \theta > 2$ B1
 $\sin 2\theta > 1$ B1
 Contradiction, since the sine of any angle ≤ 1 B1