C4

1. (*a*) $f(x) \equiv \frac{A}{x^2} + \frac{B}{x} + \frac{C}{(x+2)}$ (correct form) M1

$6 + x - 9x^2 \equiv A(x+2) + Bx(x+2) + Cx^2$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = 3, C = -8, B = -1$ (all three coefficients correct) A2

If A2 not awarded, award A1 for at least one correct coefficient

(*b*) (i) $f'(x) = \frac{-6}{x^3} + \frac{1}{x^2} + \frac{8}{(x+2)^2}$ (o.e.)

(f.t. candidate's values for *A*, *B*, *C*)

(first term) B1

(at least one of last two terms) B1

(ii) $f'(2) = 0 \Rightarrow$ stationary value when $x = 2$ (c.a.o.) B1

2. $3x^2 - 2x \times 2y\frac{dy}{dx} - 2y^2 + 3y^2\frac{dy}{dx} = 0$ $\left(-2x \times 2y\frac{dy}{dx} - 2y^2\right)$ B1

$\left(3x^2, 3y^2\frac{dy}{dx}\right)$ B1

Either $\frac{dy}{dx} = \frac{2y^2 - 3x^2}{3y^2 - 4xy}$ **or** $\frac{dy}{dx} = 2$ (o.e.) (c.a.o.) B1

Use of $\text{grad}_{normal} \times \text{grad}_{tangent} = -1$ M1

Equation of normal: $y - 1 = -\frac{1}{2}(x - 2)$ $\left(\text{f.t. candidate's value for } \frac{dy}{dx}\right)$ A1

3. (*a*) $8(2\cos^2\theta - 1) + 6 = \cos^2\theta + \cos\theta$

(correct use of $\cos 2\theta = 2\cos^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = candidate's coefficient of $\cos^2\theta$ and $b \times d$ = candidate's constant m1

$15\cos^2\theta - \cos\theta - 2 = 0 \Rightarrow (5\cos\theta - 2)(3\cos\theta + 1) = 0$

$\Rightarrow \cos\theta = \frac{2}{5}, \quad \cos\theta = -\frac{1}{3},$ (c.a.o.) A1

$\theta = 66{\cdot}42°, 293{\cdot}58°$ B1

$\theta = 109{\cdot}47°, 250{\cdot}53°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\cos\theta = +, -$, f.t. for 3 marks, $\cos\theta = -, -$, f.t. for 2 marks

$\cos\theta = +, +$, f.t. for 1 mark

(*b*) $R = 4$ B1

Correctly expanding $\cos(\theta + \alpha)$, correctly comparing coefficients and using either $4\cos\alpha = \sqrt{15}$ **or** $4\sin\alpha = 1$ **or** $\tan\alpha = \frac{1}{\sqrt{15}}$ to find α

(f.t. candidate's value for *R*) M1

$\alpha = 14{\cdot}48°$ (c.a.o.) A1

$\cos(\theta + 14{\cdot}48°) = \frac{3}{4} = 0{\cdot}75$

(f.t. candidate's values for *R*, α, $0° < \alpha < 90°$) B1

$\theta + 14{\cdot}48° = 41{\cdot}41°, 318{\cdot}59°$

(at least one value, f.t. candidate's values for *R*, α, $0° < \alpha < 90°$) B1

$\theta = 26{\cdot}93°, 304{\cdot}11°$ (c.a.o.) B1

4.

$\text{Volume} = \pi\int_{\pi/6}^{\pi/2} \sin^2 2x \, dx$ B1

$\sin^2 2x = \frac{(1 - \cos 4x)}{2}$ B1

$\int(a + b\cos 4x)\,dx = ax + \frac{1}{4}b\sin 4x$, $a \neq 0, b \neq 0$ B1

Correct substitution of candidate's limits in candidate's integrated expression of form $mx + n\sin 4x$ $m \neq 0, n \neq 0$ M1

Volume = $1{\cdot}985$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks

5. (*a*) (i) $(1 + 6x)^{1/3} = 1 + 2x - 4x^2$ $(1 + 2x)$ B1

$(-4x^2)$ B1

(ii) $|x| < 1/6$ or $-1/6 < x < 1/6$ B1

(*b*) $2 + 4x - 8x^2 = 2x^2 - 15x \Rightarrow 10x^2 - 19x - 2 = 0$ M1

(An attempt to set up and use a correct method to solve quadratic using candidate's expansion for $(1 + 6x)^{1/3}$)

$x = -0{\cdot}1$ (f.t. only candidate's range for *x* in (*a*)) A1

6. (*a*) candidate's x-derivative = a

candidate's y-derivative = $-\frac{b}{t^2}$ (at least one term correct) B1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = -\frac{b}{at^2}$ (c.a.o.) A1

Tangent at P: $y - \frac{b}{p} = -\frac{b}{ap^2}(x - ap)$ (o.e.)

(f.t. candidate's expression for $\frac{dy}{dx}$) M1

$ap^2y - abp = -bx + abp$

$ap^2y + bx - 2abp = 0.$ (convincing) A1

(*b*) $y = 0 \Rightarrow x = 2ap$ (o.e.) B1

$x = 0 \Rightarrow y = 2b/p$ (o.e.) B1

Area of triangle $AOB = 2ab$ (c.a.o.) B1

(*c*) $p^2 - 2p + 2 = 0$ $(abp^2 - 2abp + 2ab = 0)$ B1

Attempting **either** to use the formula to solve the candidate's quadratic in p **or** to find the discriminant of the candidate's quadratic **or** to complete the square M1

Either discriminant < 0 ($\Rightarrow$ no real roots) $\Rightarrow$ no such P can exist **or** $(p - 1)^2 + 1 = 0$ ($\Rightarrow (p - 1)^2 = -1$)) $\Rightarrow$ no such P can exist

(c.a.o.) A1

7. (*a*) $u = 3x - 1 \Rightarrow du = 3dx$ (o.e.) B1

$dv = \cos 2x\, dx \Rightarrow v = \frac{1}{2}\sin 2x$ (o.e.) B1

$\int(3x - 1)\cos 2x\, dx = (3x - 1) \times \frac{1}{2}\sin 2x - \int\frac{1}{2}\sin 2x \times 3dx$ M1

$\int(3x - 1)\cos 2x\, dx = \frac{1}{2}(3x - 1)\sin 2x + \frac{3}{4}\cos 2x + c$ (c.a.o.) A1 (*b*) $\displaystyle\int \frac{x}{(2x+1)^3}\,dx = \int \frac{f(u)}{u^3} \times k\,du$

($f(u) = pu + q$, $p \neq 0$, $q \neq 0$ and $k = \frac{1}{2}$ or 2) M1

$\displaystyle\int \frac{x}{(2x+1)^3}\,dx = \int \frac{(u-1)}{2} \times \frac{1}{u^3} \times \frac{du}{2}$ A1

$\displaystyle\int (au^{-2} + bu^{-3})\,du = \frac{au^{-1}}{-1} + \frac{bu^{-2}}{-2}$ ($a \neq 0$, $b \neq 0$) B1

Either: Correctly inserting limits of 1, 3 in candidate's $cu^{-1} + du^{-2}$
($c \neq 0$, $d \neq 0$)

or: Correctly inserting limits of 0, 1 in candidate's
$c(2x+1)^{-1} + d(2x+1)^{-2}$ ($c \neq 0$, $d \neq 0$) m1

$\displaystyle\int_0^1 \frac{x}{(2x+1)^3}\,dx = \frac{1}{18}$ (= 0·055 . . .) (c.a.o.) A1

Note: Answer only with no working earns 0 marks

8. (*a*) $\dfrac{dA}{dt} = k\sqrt{A}$ B1

(*b*) $\displaystyle\int \frac{dA}{\sqrt{A}} = \int k\,dt$ M1

$\dfrac{A^{1/2}}{1/2} = kt + c$ A1

Substituting 64 for A and 3 for t and 196 for A and 5·5 for t in candidate's derived equation m1

$16 = 3k + c$, $28 = 5{\cdot}5k + c$ (both equations) (c.a.o.) A1

Attempting to solve candidate's derived simultaneous linear equations in k and c m1

$A = (2{\cdot}4t + 0{\cdot}8)^2$ (o.e.) (c.a.o.) A1

9. (*a*) $\mathbf{AB} = 8\mathbf{i} - 4\mathbf{j} + 12\mathbf{k}$ B1

(*b*) $\mathbf{OC} = -\mathbf{i} + 3\mathbf{j} - 7\mathbf{k} + \frac{3}{4}(8\mathbf{i} - 4\mathbf{j} + 12\mathbf{k})$ (o.e.) M1

$\mathbf{OC} = 5\mathbf{i} + 2\mathbf{k}$ A1

(*c*) (i) Use of $\mathbf{OA} + \mu(-4\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ on r.h.s. M1

$\mathbf{r} = -\mathbf{i} + 3\mathbf{j} - 7\mathbf{k} + \mu(-4\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ (all correct) A1

(ii) $-1 + \lambda \times (-4) = 7$
(an equation in λ from one set of coefficients) M1

$\lambda = -2$ A1

$1 + (-2) \times 1 = -1$

$11 + (-2) \times 3 = 5$ (both verifications) A1

An attempt to evaluate $\mathbf{AB}.(-4\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ M1

$\mathbf{AB}.(-4\mathbf{i} + \mathbf{j} + 3\mathbf{k}) = 0$ (convincing) A1

B lies on L, AB is perpendicular to L and thus B is the foot of the perpendicular from A to L (c.a.o.) A1

10. Assume that there is a real value of x such that

$$(5x - 3)^2 + 1 < (3x - 1)^2.$$

$25x^2 - 30x + 9 + 1 < 9x^2 - 6x + 1 \Rightarrow 16x^2 - 24x + 9 < 0$ B1

$(4x - 3)^2 < 0$ B1

This contradicts the fact that x is real and thus $(5x - 3)^2 + 1 \geq (3x - 1)^2$. B1