

C4

1. (a) $f(x) \equiv \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$ (correct form) M1

$$11 + x - x^2 \equiv A(x-2)^2 + B(x+1)(x-2) + C(x+1)$$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$$A = 1, C = 3, B = -2 \quad (2 \text{ correct coefficients}) \quad \text{A1}$$

(third coefficient, f.t. one slip in enumeration of other 2 coefficients) A1

(b) $f'(x) = -\frac{1}{(x+1)^2} + \frac{2}{(x-2)^2} - \frac{6}{(x-2)^3}$ (o.e.)

(f.t. candidate's values for A, B, C)

(at least one of the first two terms) B1

(third term) B1

$$f'(0) = 1/4 \quad (\text{c.a.o.}) \quad \text{B1}$$

2. $3y^2 \frac{dy}{dx} - 8x - 3x \frac{dy}{dx} - 3y = 0$ $\left[\begin{array}{l} 3y^2 \frac{dy}{dx} - 8x \\ \frac{dy}{dx} \end{array} \right]$ B1

$$\left[\begin{array}{l} -3x \frac{dy}{dx} - 3y \\ \frac{dy}{dx} \end{array} \right] \quad \text{B1}$$

Either $\frac{dy}{dx} = \frac{3y+8x}{3y^2-3x}$ **or** $\frac{dy}{dx} = \frac{1}{3}$ (o.e.) (c.a.o.) B1

Equation of tangent: $y - (-3) = \frac{1}{3}(x - 2)$ $\left[\begin{array}{l} \text{f.t. candidate's value for } \frac{dy}{dx} \\ \frac{dy}{dx} \end{array} \right]$ B1

3. (a) $4(1 - 2 \sin^2 \theta) = 1 - 2 \sin \theta$ (correct use of $\cos 2\theta = 1 - 2 \sin^2 \theta$) M1
 An attempt to collect terms, form and solve quadratic equation in $\sin \theta$, either by using the quadratic formula or by getting the expression into the form $(a \sin \theta + b)(c \sin \theta + d)$, with $a \times c =$ candidate's coefficient of $\sin^2 \theta$ and $b \times d =$ candidate's constant m1
 $8 \sin^2 \theta - 2 \sin \theta - 3 = 0 \Rightarrow (4 \sin \theta - 3)(2 \sin \theta + 1) = 0$
 $\Rightarrow \sin \theta = \frac{3}{4}, \sin \theta = -\frac{1}{2}$ (c.a.o.) A1
 $\theta = 48.59^\circ, 131.41^\circ$ B1
 $\theta = 210^\circ, 330^\circ$ B1 B1
 Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
 $\sin \theta = +, -, \text{ f.t. for 3 marks, } \sin \theta = -, -, \text{ f.t. for 2 marks}$
 $\sin \theta = +, +, \text{ f.t. for 1 mark}$

- (b) (i) $R = 17$ B1
 Correctly expanding $\sin(x + \alpha)$ and using either $17 \cos \alpha = 8$ or $17 \sin \alpha = 15$ or $\tan \alpha = \frac{15}{8}$ to find α
 (f.t. candidate's value for R) M1
 $\alpha = 61.93^\circ$ (c.a.o.) A1
 (ii) $\sin(x + \alpha) = \frac{11}{17}$ (f.t. candidate's value for R) B1
 $x + 61.93^\circ = 40.32^\circ, 139.68^\circ, 400.32^\circ,$
 (at least one value on R.H.S.,
 f.t. candidate's values for α and R) B1
 $x = 77.75^\circ, 338.39^\circ$ (c.a.o.) B1
 (iii) Greatest possible value for k is 17 since greatest possible value for \sin is 1 (f.t. candidate's value for R) E1

4. Volume = $\pi \int_3^4 \left(\sqrt{x + \frac{5}{\sqrt{x}}} \right)^2 dx$ B1
 $\left(\sqrt{x + \frac{5}{\sqrt{x}}} \right)^2 = \left(x + 10 + \frac{25}{x} \right)$ B1
 $\int \left(ax + b + \frac{c}{x} \right) dx = \frac{ax^2}{2} + bx + c \ln x$, where $c \neq 0$ and at least one of $a, b \neq 0$ B1
 Correct substitution of correct limits in candidate's integrated expression M1
 of form $\frac{ax^2}{2} + bx + c \ln x$, where $c \neq 0$ and at least one of $a, b \neq 0$
 Volume = $65(\cdot 0059 \dots)$ (c.a.o.) A1

5. $\left(\frac{1+x}{3} \right)^{-1/2} = 1 - \frac{x}{6} + \frac{x^2}{24}$ $\left(\frac{1-x}{6} \right)$ B1
 $\left(\frac{x^2}{24} \right)$ B1
 $|x| < 3$ or $-3 < x < 3$ B1
 $\left(\frac{16}{15} \right)^{-1/2} \approx 1 - \frac{1}{30} + \frac{1}{600}$ (f.t. candidate's coefficients) B1
 $\sqrt{15} \approx \frac{581}{150}$ (c.a.o.) B1

6. (a) candidate's x -derivative = $2t$
candidate's y -derivative = 2 (at least one term correct)
and use of
 $\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1
 $\frac{dy}{dx} = \frac{1}{t}$ (o.e.) (c.a.o.) A1
Use of $\text{grad}_{\text{normal}} \times \text{grad}_{\text{tangent}} = -1$ m1
Equation of normal at P : $y - 2p = -p(x - p^2)$
(f.t. candidate's expression for $\frac{dy}{dx}$) m1
 $y + px = p^3 + 2p$ (convincing) (c.a.o.) A1

(b) (i) Substituting $x = 9, y = 6$ in equation of normal M1
 $p^3 - 7p - 6 = 0$ (convincing) A1
(ii) A correct method for solving $p^3 - 7p - 6 = 0$ M1
 $p = -1$ A1
 $p = -2$ A1
 P is either $(1, -2)$ or $(4, -4)$ (c.a.o.) A1

7. (a) $u = x \Rightarrow du = dx$ (o.e.) B1
 $dv = e^{-2x} dx \Rightarrow v = -\frac{1}{2}e^{-2x}$ (o.e.) B1
 $\int x e^{-2x} dx = x \times -\frac{1}{2}e^{-2x} - \int -\frac{1}{2}e^{-2x} dx$ M1
 $\int x e^{-2x} dx = -\frac{x}{2}e^{-2x} - \frac{1}{4}e^{-2x} + c$ (c.a.o.) A1
- (b) $\int \frac{1}{x(1+3\ln x)} dx = \int \frac{k}{u} du$ ($k = 1/3$ or 3) M1
 $\int \frac{a}{u} du = a \ln u$ B1
 $\int_1^e \frac{1}{x(1+3\ln x)} dx = k [\ln u]_1^4$ or $k [\ln(1+3\ln x)]_1^e$ B1
 $\int_1^e \frac{1}{x(1+3\ln x)} dx = 0.4621$ (c.a.o.) A1
8. (a) $\frac{dV}{dt} = -kV^3$ (where $k > 0$) B1
- (b) $\int \frac{dV}{V^3} = - \int k dt$ (o.e.) M1
 $-\frac{V^{-2}}{2} = -kt + c$ A1
 $c = -\frac{1}{7200}$ (c.a.o.) A1
 $V^2 = \frac{3600}{7200kt + 1} = \frac{3600}{at + 1}$ (convincing)
where $a = 7200k$ A1
- (c) Substituting $t = 2$ and $V = 50$ in expression for V^2 M1
 $a = 0.22$ A1
Substituting $V = 27$ in expression for V^2 with candidate's value for a M1
 $t = 17.9$ (c.a.o.) A1

9. (a) An attempt to evaluate $\mathbf{a} \cdot \mathbf{b}$ M1
 Correct evaluation of $\mathbf{a} \cdot \mathbf{b}$ and $\mathbf{a} \cdot \mathbf{b} \neq 0 \Rightarrow \mathbf{a}$ and \mathbf{b} not perpendicular A1
- (b) (i) $\mathbf{AB} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ B1
 (ii) Use of $\mathbf{a} + \lambda\mathbf{AB}$, $\mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$, $\mathbf{b} + \lambda\mathbf{AB}$ or $\mathbf{b} + \lambda(\mathbf{b} - \mathbf{a})$ to find vector equation of AB M1
 $\mathbf{r} = 4\mathbf{i} + \mathbf{j} - 6\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ (o.e.)
 (f.t. if candidate uses his/her expression for \mathbf{AB}) A1
- (c) $4 + 2\lambda = 2 - 2\mu$
 $1 + \lambda = 6 + \mu$
 $-6 + 2\lambda = p + 3\mu$ (o.e.)
 (comparing coefficients, at least one equation correct) M1
 (at least two equations correct) A1
 Solving the first two of the equations simultaneously m1
 (f.t. for all 3 marks if candidate uses his/her expression for \mathbf{AB})
 $\lambda = 2, \mu = -3$ (o.e.) (c.a.o.) A1
 $p = 7$ from third equation
 (f.t. candidates derived values for λ and μ) A1
10. $a^2 = 5b^2 \Rightarrow (5k)^2 = 5b^2 \Rightarrow b^2 = 5k^2$ B1
 $\therefore 5$ is a factor of b^2 and hence 5 is a factor of b B1
 $\therefore a$ and b have a common factor, which is a contradiction to the original assumption B1