

C4

1. (a) $f(x) \equiv \frac{A}{(x+2)^2} + \frac{B}{(x+2)} + \frac{C}{(x-3)}$ (correct form) M1

$x^2 + x + 13 \equiv A(x-3) + B(x+2)(x-3) + C(x+2)^2$
 (correct clearing of fractions and genuine attempt to find coefficients)

$A = -3, C = 1, B = 0$ (all three coefficients correct) A2
 (at least one coefficient correct) A1

(b) $\int f(x) dx = \frac{3}{(x+2)} + \ln(x-3)$ B1 B1
 (f.t. candidates values for A, B, C)

$\int_6^7 f(x) dx = \left[\frac{3}{9} - \frac{3}{8} \right] - [\ln 4 - \ln 3] = 0.246(015405)$ (c.a.o.) B1

Note: Answer only with no working earns 0 marks

2. $4x^3 - 2x^2 \frac{dy}{dx} - 4xy + 2y \frac{dy}{dx} = 0$ $\left[-2x^2 \frac{dy}{dx} - 4xy \right]$ B1
 $\left[\frac{dy}{dx} \right]$

$\left[4x^3 + 2y \frac{dy}{dx} \right]$ B1
 $\left[\frac{dy}{dx} \right]$

Either $\frac{dy}{dx} = \frac{4xy - 4x^3}{2y - 2x^2}$ **or** $\frac{dy}{dx} = 2$ (o.e.) (c.a.o.) B1

Attempting to substitute $x = 1$ and $y = 3$ in candidate's expression **and** the use of $\text{grad}_{normal} \times \text{grad}_{tangent} = -1$ M1

Equation of normal: $y - 3 = -\frac{1}{2}(x - 1)$ $\left[\text{f.t. candidate's value for } \frac{dy}{dx} \right]$ A1
 $\left[\frac{dy}{dx} \right]$

3. (a) $\frac{2 \tan x}{1 - \tan^2 x} = 4 \tan x$ (correct use of formula for $\tan 2x$) M1
 $\tan x = 0$ A1
 $2 \tan^2 x - 1 = 0$ A1
 $x = 0^\circ, 180^\circ$ (both values) A1
 $x = 35.26^\circ, 144.74^\circ$ (both values) A1
- (b) $R = 25$ B1
Expanding $\cos(\theta - \alpha)$ and using either $25 \cos \alpha = 7$
or $25 \sin \alpha = 24$ **or** $\tan \alpha = \frac{24}{7}$ to find α
 $\alpha = 73.74^\circ$ (c.a.o.) A1
 $\cos(\theta - \alpha) = \frac{16}{25} = 0.64$ (f.t. candidate's value for R) B1
 $\theta - \alpha = 50.21^\circ, -50.21^\circ$
(at least one value, f.t. candidate's value for R) B1
 $\theta = 23.53^\circ, 123.95^\circ$ (c.a.o.) B1
4. (a) candidate's x -derivative = $-3 \sin t$
candidate's y -derivative = $4 \cos t$ (at least one term correct) B1
 $\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1
 $\frac{dy}{dx} = \frac{-4 \cos t}{3 \sin t}$ (o.e.) (c.a.o.) A1
- At P , $y - 4 \sin p = -\frac{4 \cos p}{3 \sin p} (x - 3 \cos p)$ (o.e.)
(f.t. candidate's expression for $\frac{dy}{dx}$) M1
- $(3 \sin p)y - 12 \sin^2 p = (-4 \cos p)x + 12 \cos^2 p$
 $(3 \sin p)y = (-4 \cos p)x + 12 \cos^2 p + 12 \sin^2 p$
 $(3 \sin p)y + (4 \cos p)x - 12 = 0$ (convincing) A1
- (b) (i) $A = (2\sqrt{3}, 0)$ B1
 $B = (0, 8)$ B1
(ii) Correct use of Pythagoras Theorem to find AB M1
 $AB = 2\sqrt{19}$ (convincing) A1

5. (a) $A(-3, 0), B(3, 0), C(0, 3)$ B1

(b) (i)
$$\text{Volume} = \pi \int_{-3}^3 (9 - x^2) dx$$
 (f.t candidate's x -coordinates for A, B) M1

$$\int (9 - x^2) dx = 9x - \frac{x^3}{3} \quad \text{B1}$$

$$\text{Volume} = 36\pi \quad (\text{c.a.o.}) \quad \text{A1}$$

Note: Answer only with no working earns 0 marks

(ii) This is the volume of a sphere of radius 3 E1

6. $(1 + 2x)^{1/2} = 1 + (1/2) \times (2x) + \frac{(1/2) \times (1/2 - 1) \times (2x)^2}{1 \times 2} + \dots$
(-1 each incorrect term) B2

$\frac{1}{(1 + 3x)^2} = 1 + (-2) \times (3x) + \frac{(-2) \times (-3) \times (3x)^2}{1 \times 2} + \dots$
(-1 each incorrect term) B2

$4(1 + 2x)^{1/2} - \frac{1}{(1 + 3x)^2} = 3 + 10x - 29x^2 + \dots$
(-1 each incorrect term) B2

Expansion valid for $|x| < 1/3$ B1

7. (a) $\int x \sin 2x \, dx = x \times k \times \cos 2x - \int k \times \cos 2x \times g(x) \, dx$ (k = ±1/2, ±2 or ±1) M1
 $k = -\frac{1}{2}, g(x) = 1$ A1, A1

$\int x \sin 2x \, dx = -\frac{1}{2} \times x \times \cos 2x + \frac{1}{4} \times \sin 2x + c$ (c.a.o.) A1

(b) $\int \frac{x}{(5-x^2)^3} \, dx = \int \frac{k}{u^3} \, du$ (k = ±1/2 or ±2) M1
 $\int \frac{a}{u^3} \, du = -\frac{a}{2} u^{-2}$ B1

$\int_0^2 \frac{x}{(5-x^2)^3} \, dx = -\frac{k}{2} \left[u^{-2} \right]_5^1$ or $-\frac{k}{2} \left[\frac{1}{(5-x^2)^2} \right]_0^2$

(f.t. candidate's value for k, k = ±1/2 or ±2) A1

$\int_0^2 \frac{x}{(5-x^2)^3} \, dx = \frac{6}{25}$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks

8. (a) $\frac{dN}{dt} = kN$ B1

(b) $\int \frac{dN}{N} = \int k \, dt$ M1
 $\ln N = kt + c$ A1
 $N = e^{kt+c} = Ae^{kt}$ (convincing) A1

(c) (i) $100 = Ae^{2k}$
 $160 = Ae^{12k}$ (both values) B1
 Dividing to eliminate A M1
 $1.6 = e^{10k}$ A1
 $k = \frac{1}{10} \ln 1.6 = 0.047$ (convincing) A1

(ii) $A = 91(.0283)$ (o.e.) B1
 When $t = 20, N = 91(.0283) \times e^{0.94}$
 (f.t. candidate's derived value for A) M1
 $N = 233$ (c.a.o.) A1

9. (a) Use of $(5\mathbf{i} - 8\mathbf{j} + 4\mathbf{k}) \cdot (4\mathbf{i} + 6\mathbf{j} + a\mathbf{k}) = 0$ M1
 $5 \times 4 + (-8) \times 6 + 4 \times a = 0$ m1
 $a = 7$ A1
- (b) (i) $\mathbf{r} = 8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ (o.e.) B1
(ii) $8 + 2\lambda = 4 - 2\mu$
 $3 + \lambda = 7 + \mu$
 $-7 + 2\lambda = 5 + 3\mu$ (o.e.)
(comparing coefficients, at least one equation correct) M1
(at least two equations correct) A1
Solving two of the equations simultaneously m1
 $\lambda = 1, \mu = -3$ (o.e.) (c.a.o.) A1
Correct verification that values of λ and μ do not satisfy third equation B1

10. Assume that there is a real and positive value of x such that $4x + \frac{9}{x} < 12$
- $4x^2 - 12x + 9 < 0$ B1
 $(2x - 3)^2 < 0$ B1
This contradicts the fact that x is real and thus $4x + \frac{9}{x} \geq 12$ B1