

**C4**

1. (a)  $f(x) \equiv \frac{A}{x} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$  (correct form) M1  
 $8 - x - x^2 \equiv A(x-2)^2 + Bx(x-2) + Cx$   
 (correct clearing of fractions and genuine attempt to find coefficients) m1  
 $A = 2, C = 1, B = -3$  (2 coefficients, c.a.o.) A1  
 (third coefficient, f.t. one slip in enumeration of other 2 coefficients) A1

- (b)  $f'(x) = \frac{-2}{x^2} + \frac{3}{(x-2)^2} - \frac{2}{(x-2)^3}$  (at least one of first two terms) B1  
 (third term) B1  
 (f.t. candidates values for A, B, C)  
 $f'(1) = 3$  (c.a.o.) B1

2.  $10x + 4x \frac{dy}{dx} + 4y - 3y^2 \frac{dy}{dx} = 0$   $\left[ \begin{array}{l} 4x \frac{dy}{dx} + 4y \\ \left[ \frac{\quad}{dx} \right] \end{array} \right]$  B1  
 $\left[ \begin{array}{l} -3y^2 \frac{dy}{dx} \\ \left[ \frac{\quad}{dx} \right] \end{array} \right]$  B1

- $\frac{dy}{dx} = \frac{1}{4}$  (o.e.) (c.a.o.) B1  
 Use of  $\text{grad}_{\text{normal}} \times \text{grad}_{\text{tangent}} = -1$  M1  
 Equation of normal:  $y - (-2) = -4(x - 1)$   
 $\left[ \begin{array}{l} \text{f.t. candidate's value for } \frac{dy}{dx} \\ \left[ \frac{\quad}{dx} \right] \end{array} \right]$  A1

3. (a)  $2(2 \cos^2 \theta - 1) = 9 \cos \theta + 7$  (correct use of  $\cos 2\theta = 2 \cos^2 \theta - 1$ ) M1

An attempt to collect terms, form and solve quadratic equation in  $\cos \theta$ , either by using the quadratic formula or by getting the expression into the form  $(a \cos \theta + b)(c \cos \theta + d)$ , with  $a \times c = \text{coefficient of } \cos^2 \theta$  and  $b \times d = \text{constant}$  m1  
 $4 \cos^2 \theta - 9 \cos \theta - 9 = 0 \Rightarrow (4 \cos \theta + 3)(\cos \theta - 3) = 0$   
 $\Rightarrow \cos \theta = \frac{-3}{4}, (\cos \theta = 3)$  (c.a.o.) A1

$\theta = 138.59^\circ, 221.41^\circ$  B1 B1  
 Note: Subtract (from final two marks) 1 mark for each additional root in range from  $4 \cos \theta + 3 = 0$ , ignore roots outside range.  
 $\cos \theta = -$ , f.t. for 2 marks,  $\cos \theta = +$ , f.t. for 1 mark

- (b) (i)  $R = 13$  B1  
 Correctly expanding  $\sin(x - \alpha)$  and using either  $13 \cos \alpha = 5$   
 or  $13 \sin \alpha = 12$  or  $\tan \alpha = \frac{12}{5}$  to find  $\alpha$   
 (f.t. candidate's value for  $R$ ) M1  
 $\alpha = 67.38^\circ$  (c.a.o) A1
- (ii) Least value of  $\frac{1}{5 \sin x - 12 \cos x + 20} = \frac{1}{13 \times (\pm 1) + 20}$   
 (f.t. candidate's value for  $R$ ) M1  
 Least value =  $\frac{1}{33}$  (f.t. candidate's value for  $R$ ) A1  
 Corresponding value for  $x = 157.38^\circ$  (o.e.)  
 (f.t. candidate's value for  $\alpha$ ) A1

4.

$$\text{Volume} = \pi \int_{\pi/6}^{\pi/3} \sin^2 x \, dx \quad \text{B1}$$

Use of  $\sin^2 x = \frac{(\pm 1 \pm \cos 2x)}{2}$  M1

Correct integration of candidate's  $\frac{(\pm 1 \pm \cos 2x)}{2}$  A1

Correct substitution of correct limits in candidate's integrated expression M1

$$\text{Volume} = \frac{\pi^2}{12} = 0.822(467\dots) \quad \text{(c.a.o.)} \quad \text{A1}$$

5.

$$\left( \frac{1-x}{4} \right)^{1/2} = 1 - \frac{x}{8} - \frac{x^2}{128} \quad \left( \frac{1-x}{8} \right) \quad \text{B1}$$

$$\left( \frac{1-x}{4} \right)^{1/2} = 1 - \frac{x}{8} - \frac{x^2}{128} \quad \left( -\frac{x^2}{128} \right) \quad \text{B1}$$

$|x| < 4$  or  $-4 < x < 4$  B1

$$\frac{\sqrt{3}}{2} \approx 1 - \frac{1}{8} - \frac{1}{128} \quad \text{(f.t. candidate's coefficients)} \quad \text{B1}$$

$$\sqrt{3} \approx \frac{111}{64} \quad \text{(convincing)} \quad \text{B1}$$

6. (a) Use of  $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$  and at least one of  $\frac{dx}{dt} = -\frac{2}{t^2}$ ,  $\frac{dy}{dt} = 4$  correct M1  
 $\frac{dy}{dx} = -2t^2$  (o.e.) A1  
Equation of tangent at P:  $y - 4p = -2p^2 \left[ \begin{matrix} x - 2 \\ p \end{matrix} \right]$   
(f.t. candidate's expression for  $\frac{dy}{dx}$ ) m1  
 $y = -2p^2x + 8p$  (convincing) A1
- (b) Substituting  $x = 2, y = 3$  in equation of tangent M1  
 $4p^2 - 8p + 3 = 0$  A1  
 $p = \frac{1}{2}, \frac{3}{2}$  (both values, c.a.o.) A1  
Points are  $(4, 2), (\frac{4}{3}, 6)$  (f.t. candidate's values for  $p$ ) A1
7. (a)  $\int x^3 \ln x \, dx = f(x) \ln x - \int f(x) g(x) \, dx$  M1  
 $f(x) = \frac{x^4}{4}, g(x) = \frac{1}{x}$  A1, A1  
 $\int x^3 \ln x \, dx = \frac{x^4}{4} \ln x - \frac{x^4}{16} + c$  (c.a.o.) A1
- (b)  $\int x(2x-3)^4 \, dx = \int f(u) \times u^4 \times k \, du$  M1  
 $(f(u) = pu + q, p \neq 0, q \neq 0 \text{ and } k = \frac{1}{2} \text{ or } 2)$   
 $\int x(2x-3)^4 \, dx = \int \frac{(u+3)}{2} \times u^4 \times \frac{du}{2}$  A1  
 $\int (au^5 + bu^4) \, du = \frac{au^6}{6} + \frac{bu^5}{5}$  ( $a \neq 0, b \neq 0$ ) B1
- Either:** Correctly inserting limits of  $-1, 1$  in candidate's  $\frac{au^6}{6} + \frac{bu^5}{5}$
- or:** Correctly inserting limits of  $1, 2$  in candidate's  
 $\frac{a(2x-3)^6}{6} + \frac{b(2x-3)^5}{5}$  m1
- $\int_1^2 x(2x-3)^4 \, dx = \frac{3}{10}$  (c.a.o.) A1

8. (a)  $\frac{dV}{dt} = -kV^2$  B1
- (b)  $\int \frac{dV}{V^2} = - \int k dt$  (o.e.) M1  
 $-\frac{1}{V} = -kt + c$  A1  
 $c = -\frac{1}{12000}$  (c.a.o.) A1  
 $V = \frac{12000}{12000kt + 1} = \frac{12000}{at + 1}$  (convincing) A1
- (c) Substituting  $t = 2$  and  $V = 9000$  in expression for  $V$  M1  
 $a = \frac{1}{6}$  A1  
Substituting  $t = 4$  in expression for  $V$  with candidate's value for  $a$  M1  
 $V = 7200$  (c.a.o.) A1
9. (a)  $(2\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \cdot (\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}) = 18$  B1  
 $|2\mathbf{i} - 2\mathbf{j} + \mathbf{k}| = \sqrt{9}, |\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}| = \sqrt{81}$  (one correct) B1  
Correctly substituting in the formula  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \times |\mathbf{b}| \times \cos \theta$  M1  
 $\theta = 48.2^\circ$  (c.a.o.) A1
- (b) (i)  $\mathbf{AB} = -\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$  B1  
(ii) Use of  $\mathbf{a} + \lambda\mathbf{AB}, \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}), \mathbf{b} + \lambda\mathbf{AB}$  or  $\mathbf{b} + \lambda(\mathbf{b} - \mathbf{a})$  to find vector equation of  $AB$  M1  
 $\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} - 2\mathbf{j} + 7\mathbf{k})$  (o.e.)  
(f.t. if candidate uses his/her expression for  $\mathbf{AB}$ ) A1
- (c)  $2 - \lambda = -1 + \mu$   
 $-2 - 2\lambda = -4 + \mu$   
 $1 + 7\lambda = -2 - \mu$  (o.e.)  
(comparing coefficients, at least one equation correct) M1  
(at least two equations correct) A1  
Solving two of the equations simultaneously m1  
(f.t. for all 3 marks if candidate uses his/her expression for  $\mathbf{AB}$ )  
 $\lambda = -1, \mu = 4$  (o.e.) (c.a.o.) A1  
Correct verification that values of  $\lambda$  and  $\mu$  satisfy third equation A1  
Position vector of point of intersection is  $3\mathbf{i} - 6\mathbf{k}$  (f.t. one slip) A1
10. Assume that positive real numbers  $a, b$  exist such that  $a + b < 2\sqrt{ab}$ .  
Squaring both sides we have:  $(a + b)^2 < 4ab \Rightarrow a^2 + b^2 + 2ab < 4ab$  B1  
 $a^2 + b^2 - 2ab < 0 \Rightarrow (a - b)^2 < 0$  B1  
This contradicts the fact that  $a, b$  are real and thus  $a + b \geq 2\sqrt{ab}$  B1