GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - C3
0975/01

INTRODUCTION

This marking scheme was used by WJEC for the 2017 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

Mathematics C3 June 2017

Solutions and Mark Scheme

1. (*a*)

5	3·258096538		
5·5	3·442019376		
6	3·610917913		
6·5	3·766997233		
7	3·912023005	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0{\cdot}5$ M1

$I \approx \frac{0{\cdot}5}{3} \times \{3{\cdot}258096538 + 3{\cdot}912023005 + 4(3{\cdot}442019376 + 3{\cdot}766997233) + 2(3{\cdot}610917913)\}$

$I \approx 43{\cdot}22802181 \times 0{\cdot}5 \div 3$

$I \approx 7{\cdot}204670301$

$I \approx 7{\cdot}2$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

(*b*)

$\int_5^7 \ln\left(\frac{3}{\sqrt{(1 + x^2)}}\right) dx = \int_5^7 \ln 3\, dx - \frac{1}{2}\int_5^7 \ln(1 + x^2)\, dx$ M1

$\frac{1}{2}\int_5^7 \ln(1 + x^2)\, dx \approx 3{\cdot}6$ (f.t. candidate's answer to (*a*)) B1

$\int_5^7 \ln\left(\frac{3}{\sqrt{(1 + x^2)}}\right) dx \approx 2{\cdot}2 - 3{\cdot}6 = -1{\cdot}4$

(f.t. candidate's answer to (*a*)) A1

2. (*a*) $6(\sec^2\theta - 1) - 6 = 4\sec^2\theta + 5\sec\theta$.

(correct use of $\tan^2\theta = \sec^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in $\sec\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sec\theta + b)(c\sec\theta + d)$, with $a \times c$ = candidate's coefficient of $\sec^2\theta$ and $b \times d$ = candidate's constant m1

$2\sec^2\theta - 5\sec\theta - 12 = 0 \Rightarrow (2\sec\theta + 3)(\sec\theta - 4) = 0$

$\Rightarrow \sec\theta = -\frac{3}{2}$, $\sec\theta = 4$

$\Rightarrow \cos\theta = -\frac{2}{3}$, $\cos\theta = \frac{1}{4}$ (c.a.o.) A1

$\theta = 131{\cdot}81°, 228{\cdot}19°$ B1 B1

$\theta = 75{\cdot}52°, 284{\cdot}48°$ B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\cos\theta$ = +, –, f.t. for 3 marks, $\cos\theta$ = –, –, f.t. for 2 marks
$\cos\theta$ = +, +, f.t. for 1 mark

(*b*) Correct use of $\sec\phi = \frac{1}{\cos\phi}$ and $\tan\phi = \frac{\sin\phi}{\cos\phi}$ (o.e.) M1

$\sin\phi = -\frac{3}{5}$ A1

$\phi = 323{\cdot}13°, 216{\cdot}87°$ (f.t. for $\sin\phi = -a$) A1

3. (*a*) $\frac{d}{dx}(2y^3) = 6y^2\frac{dy}{dx}$ B1

$\frac{d}{dx}(-3x^2y) = -3x^2\frac{dy}{dx} - 6xy$ B1

$\frac{d}{dx}(x^4) = 4x^3$, $\frac{d}{dx}(-4x) = -4$, $\frac{d}{dx}(7) = 0$ B1

$\frac{dy}{dx} = \frac{4 - 4x^3 + 6xy}{6y^2 - 3x^2}$ (o.e.) (c.a.o.) B1

(*b*) (i) candidate's *x*-derivative = $7 + 4t$ B1

candidate's *y*-derivative = $\frac{(7 + 4t)r - (4 + 3t)m}{(7 + 4t)^2}$,

where *r*, *m* are integers M1

candidate's *y*-derivative = $\frac{(7 + 4t)3 - (4 + 3t)4}{(7 + 4t)^2}$ A1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{5}{(7 + 4t)^3}$ (c.a.o.) A1

(ii) $\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{-3 \times 5 \times 4}{(7 + 4t)^4}$ (o.e.)

(f.t. candidate's expression of correct given form for $\frac{dy}{dx}$) B1

Use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div \frac{dx}{dt}$

(f.t. candidate's expression for $\frac{d}{dt}\left(\frac{dy}{dx}\right)$) M1

$\frac{d^2y}{dx^2} = \frac{-60}{(7 + 4t)^5}$ (c.a.o.) A1

4. (*a*) (i) $V(x) = 150 \Rightarrow x \times (x + 4) \times (x - 2) = 150$ M1

$x^3 + 2x^2 - 8x - 150 = 0$ (convincing) A1

(ii) Let $f(x) = x^3 + 2x^2 - 8x - 150$

Use of a correct method to find $f(x)$ when $x = 5$ and $x = 6$ M1

$f(5) = -15\ (< 0)$, $f(6) = 90\ (> 0)$

Change of sign $\Rightarrow 5 < x < 6$ A1

(*b*) $x_0 = 6$

$x_1 = 5{\cdot}013297935$ (x_1 correct, at least 2 places after the point) B1

$x_2 = 5{\cdot}190516135$

$x_3 = 5{\cdot}163166906$

$x_4 = 5{\cdot}167508826 = 5{\cdot}17$ (x_4 correct to 2 decimal places) B1

An attempt to check values or signs of $f(x)$ at $x = 5{\cdot}165$ and $x = 5{\cdot}175$ M1

$f(5{\cdot}165) = -0{\cdot}178\ (< 0)$, $f(5{\cdot}175) = 0{\cdot}751\ (> 0)$ A1

Change of sign $\Rightarrow x = 5{\cdot}17$ correct to two decimal places A1

5. (*a*) (i) $\dfrac{dy}{dx} = \dfrac{1}{2} \times (3x^2 + 5x)^{-1/2} \times f(x)$ $(f(x) \neq 1)$ M1

$\dfrac{dy}{dx} = \dfrac{1}{2} \times (3x^2 + 5x)^{-1/2} \times (6x + 5)$ A1

(ii) $\dfrac{dy}{dx} = \dfrac{\mathbf{3}}{\sqrt{(\mathbf{1} - (\mathbf{3}x)^2)}}$ **or** $\dfrac{1}{\sqrt{(1 - (3x)^2)}}$ **or** $\dfrac{3}{\sqrt{(1 - 3x^2)}}$ M1

$\dfrac{dy}{dx} = \dfrac{3}{\sqrt{(1 - 9x^2)}}$ A1

(*b*) $x = \cot y \Rightarrow \dfrac{dx}{dy} = -\operatorname{cosec}^2 y$ B1

$\dfrac{dx}{dy} = -(1 + \cot^2 y)$ B1

$\dfrac{dx}{dy} = -(1 + x^2)$ B1

$\dfrac{dy}{dx} = -\dfrac{1}{1 + x^2}$ (c.a.o.) B1

6. (*a*) (i) $\int 8e^{2-5x}dx = k \times 8 \times e^{2-5x} + c$ $(k = 1, -5, {}^{1}/_{5}, -{}^{1}/_{5})$ M1

$\int 8e^{2-5x}dx = -\frac{8}{5} \times e^{2-5x} + c$ A1

(ii) $\int 6(4x-7)^{-1/3} = \frac{6 \times k \times (4x-7)^{2/3}}{2/3} + c$ $(k = 1, 4, {}^{1}/_{4})$ M1

$\int 6(4x-7)^{-1/3} = \frac{6 \times 1/4 \times (4x-7)^{2/3}}{2/3} + c$

$\int 6(4x-7)^{-1/3} = \frac{9}{4} \times (4x-7)^{2/3} + c$ A1

(iii) $\int \cos\left(\frac{7x-9}{3}\right) dx = k \times \sin\left(\frac{7x-9}{3}\right) + c$ $(k = 1, {}^{7}/_{3}, {}^{3}/_{7}, -{}^{3}/_{7}, {}^{1}/_{7})$ M1

$\int \cos\left(\frac{7x-9}{3}\right) dx = \frac{3}{7} \times \sin\left(\frac{7x-9}{3}\right) + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) (i) $\frac{dy}{dx} = \frac{a+bx}{3x^2-8}$ (including $a = 1$, $b = 0$) M1

$\frac{dy}{dx} = \frac{6x}{3x^2-8}$ A1

(ii) $\int_2^6 \frac{3x}{3x^2-8}\,dx = r\left[\ln(3x^2-8)\right]_2^6$

where r is a constant M1

$\int_2^6 \frac{3x}{3x^2-8}\,dx = \frac{1}{2}\left[\ln(3x^2-8)\right]_2^6$ A1

$\int_2^6 \frac{3x}{3x^2-8}\,dx = r\,\{\ln(108-8)) - \ln(12-8)\}$ m1

$\int_2^6 \frac{3x}{3x^2-8}\,dx = \ln(5)$ (c.a.o.) A1

7. (*a*) Choice of negative x — M1

Correct verification that L.H.S. of inequality > 5 and a statement to the effect that this is in fact the case — A1

(*b*) $a = -\frac{1}{2}$ — B1

$b = -6$ — B1

8. (*a*) $y - 2 = \frac{3}{\sqrt{(5x - 4)}}$ — B1

An attempt to isolate $5x - 4$ by crossmultiplying and squaring — M1

$x = \frac{1}{5}\left\{4 + \frac{9}{(y-2)^2}\right\}$ (c.a.o.) — A1

$f^{-1}(x) = \frac{1}{5}\left\{4 + \frac{9}{(x-2)^2}\right\}$

(f.t. one slip in candidate's expression for x) — A1

(*b*) $D(f^{-1}) = (2, 2{\cdot}5]$ — B1 B1

9. (*a*) $R(f) = [8 + k, \infty)$ — B1

(*b*) $8 + k \geq -3$ — M1

$k \geq -11$ ($\Rightarrow$ least value of k is -11)

(f.t. candidate's $R(f)$ provided it is of form $[a, \infty)$) — A1

(*c*) (i) $gf(x) = (4x + k)^2 - 9$ — B1

(ii) $(4 \times 2 + k)^2 - 9 = 7$

(substituting 2 for x in candidate's expression for $gf(x)$ and putting equal to 7) — M1

Either $k^2 + 16k + 48 = 0$ or $(8 + k)^2 = 16$ (c.a.o.) — A1

$k = -4, -12$ (f.t. candidate's quadratic in k) — A1

$k = -4$ (c.a.o.) — A1