GCE MARKING SCHEME

SUMMER 2016

Mathematics – C3
0975/01

INTRODUCTION

This marking scheme was used by WJEC for the Summer 2016 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCE MATHEMATICS – C3

SUMMER 2016 MARK SCHEME

1. (*a*)

0	1		
$\pi/20$	$1{\cdot}025402923$		
$\pi/10$	$1{\cdot}111347018$		
$3\pi/20$	$1{\cdot}296432399$		
$\pi/5$	$1{\cdot}695307338$	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = \pi/20$ M1

$$I \approx \frac{\pi/20}{3} \times \{1 + 1{\cdot}695307338 + 4(1{\cdot}025402923 + 1{\cdot}296432399) + 2(1{\cdot}111347018)\}$$

$I \approx 14{\cdot}20534263 \times (\pi/20) \div 3$

$I \approx 0{\cdot}7437900006$

$I \approx 0{\cdot}74379$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

(*b*)

$$\int_0^{\pi/5} e^{\sec^2 x}\,dx = e^1 \times \int_0^{\pi/5} e^{\tan^2 x}\,dx$$ M1

$$\int_0^{\pi/5} e^{\sec^2 x}\,dx \approx 2{\cdot}02183$$ (f.t. candidate's answer to (*a*)) A1

Note: Answer only with no working shown earns 0 marks

2. (*a*) $3\operatorname{cosec}\theta(\operatorname{cosec}\theta - 1) = 5(\operatorname{cosec}^2\theta - 1) - 9$

(correct use of $\cot^2\theta = \operatorname{cosec}^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in cosec θ, either by using the quadratic formula or by getting the expression into the form $(a\operatorname{cosec}\theta + b)(c\operatorname{cosec}\theta + d)$, with $a \times c$ = candidate's coefficient of $\operatorname{cosec}^2\theta$ and $b \times d$ = candidate's constant m1

$2\operatorname{cosec}^2\theta + 3\operatorname{cosec}\theta - 14 = 0 \Rightarrow (\operatorname{cosec}\theta - 2)(2\operatorname{cosec}\theta + 7) = 0$

$\Rightarrow \operatorname{cosec}\theta = 2$, $\operatorname{cosec}\theta = -\frac{7}{2}$

$\Rightarrow \sin\theta = \frac{1}{2}$, $\sin\theta = -\frac{2}{7}$ (c.a.o.) A1

$\theta = 30°, 150°$ B1

$\theta = 196{\cdot}6°, 343{\cdot}4°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\sin\theta$ = +, –, f.t. for 3 marks, $\sin\theta$ = –, –, f.t. for 2 marks
$\sin\theta$ = +, +, f.t. for 1 mark

(*b*) Correct use of $\operatorname{cosec}\phi = \frac{1}{\sin\phi}$ and $\sec\phi = \frac{1}{\cos\phi}$ (o.e.) M1

$\tan\phi = -\frac{2}{3}$ A1

$\phi = 146{\cdot}31°, 326{\cdot}31°$ (f.t. for negative $\tan\phi$) A1

3. $\frac{d}{dx}(x^2) = 2x$ $\frac{d}{dx}(2x) = 2$ $\frac{d}{dx}(21) = 0$ B1

$\frac{d}{dx}(3xy) = 3x\frac{dy}{dx} + 3y$ B1

$\frac{d}{dx}(2y^3) = 6y^2\frac{dy}{dx}$ B1

$\frac{dy}{dx} = \frac{6}{9} = \frac{2}{3}$ (c.a.o.) B1

4. (*a*) candidate's *x*-derivative = $12\cos 3t$ B1

candidate's *y*-derivative = $-6\sin 3t$ B1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = -\frac{1}{2}\tan 3t$ (c.a.o.) A1

(*b*) (i) $\frac{d}{dt}\left(\frac{dy}{dx}\right) = -\frac{3}{2}\sec^2 3t$ (f.t. $\frac{dy}{dx} = k\tan 3t$ or $k\frac{\sin 3t}{\cos 3t}$ only) B1

Use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div$ candidate's *x*-derivative M1

$\frac{d^2y}{dx^2} = -\frac{1}{8}\sec^3 3t$ or $\frac{-1}{8\cos^3 3t}$ (c.a.o.) A1

(ii) $\frac{d^2y}{dx^2} = -\frac{1}{y^3}$ (f.t. $\frac{d^2y}{dx^2} = m\sec^3 3t$ or $\frac{m}{\cos^3 3t}$ only) B1

5. (*a*) Denoting the end points of the chord by *A*, *B*

Length of arc $AB = 3\theta$ B1

Length of chord $AB = 2 \times 3 \times \sin(\theta/2)$ (convincing) B1

$3\theta + 6\sin(\theta/2) = 13\cdot5 \Rightarrow \theta + 2\sin(\theta/2) = 4\cdot5$

(convincing) B1

(*b*) $\theta_0 = 2\cdot5$

$\theta_1 = 2\cdot602030761$ (θ_1 correct, at least 2 places after the point) B1

$\theta_2 = 2\cdot572341396$

$\theta_3 = 2\cdot580466315 = 2\cdot58$ (θ_3 correct to 2 decimal places) B1

Let $f(\theta) = \theta + 2\sin(\theta/2) - 4\cdot5$

An attempt to check values or signs of $f(\theta)$ at $\theta = 2\cdot575$, $\theta = 2\cdot585$

M1

$f(2\cdot575) = -4\cdot72 \times 10^{-3} < 0$, $f(2\cdot585) = 8\cdot05 \times 10^{-3} > 0$ A1

Change of sign $\Rightarrow \theta = 2\cdot58$ correct to two decimal places A1

6. (*a*) $\frac{dy}{dx} = \frac{f(x)}{\cos x}$ (including $f(x) = 1$) M1

$\frac{dy}{dx} = -\frac{\sin x}{\cos x}$ A1

$\frac{dy}{dx} = -\tan x$ (f.t. only for $\tan x$ from $\frac{dy}{dx} = \frac{\sin x}{\cos x}$) A1

(*b*) $\frac{dy}{dx} = \frac{\mathbf{1/3}}{\mathbf{1 + (x/3)^2}}$ **or** $\frac{1}{1 + (x/3)^2}$ **or** $\frac{1/3}{1 + (1/3)x^2}$ M1

$\frac{dy}{dx} = \frac{1/3}{1 + (x/3)^2}$ A1

$\frac{dy}{dx} = \frac{3}{9 + x^2}$ (f.t. only for $\frac{dy}{dx} = \frac{9}{9 + x^2}$ from $\frac{1}{1 + (x/3)^2}$) A1

(*c*) $\frac{dy}{dx} = e^{6x} \times f(x) + (3x - 2)^4 \times g(x)$ M1

$\frac{dy}{dx} = e^{6x} \times f(x) + (3x - 2)^4 \times g(x)$
(either $f(x) = 4 \times 3 \times (3x - 2)^3$ or $g(x) = 6e^{6x}$) A1

$\frac{dy}{dx} = e^{6x} \times 12 \times (3x - 2)^3 + (3x - 2)^4 \times 6e^{6x}$ (all correct) A1

$\frac{dy}{dx} = e^{6x} \times 18x \times (3x - 2)^3$ (c.a.o.) A1

7. (*a*) (i) $\int 7e^{5-\frac{3}{4}x}\,dx = k \times 7e^{5-\frac{3}{4}x} + c$ ($k = 1, -\frac{3}{4}, \frac{4}{3}, \mathbf{-\frac{4}{3}}$) M1

$\int 7e^{5-\frac{3}{4}x}\,dx = -\frac{28}{3}e^{5-\frac{3}{4}x} + c$ A1

(ii) $\int \sin(2x/3 + 5)\,dx = k \times \cos(2x/3 + 5) + c$

($k = -1, -\frac{2}{3}, \frac{3}{2}, \mathbf{-\frac{3}{2}}$) M1

$\int \sin(2x/3 + 5)\,dx = -\frac{3}{2} \times \cos(2x/3 + 5) + c$ A1

(iii) $\int \frac{8}{(9-10x)^3}\,dx = \frac{8}{-2k} \times (9-10x)^{-2} + c$

($k = 1, 10, \mathbf{-10}, -\frac{1}{10}$) M1

$\int \frac{8}{(9-10x)^3}\,dx = \frac{2}{5} \times (9-10x)^{-2} + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int \frac{1}{4x+3}\,dx = k \times \ln(4x + 3)$ ($k = 1, 4, \mathbf{\frac{1}{4}}$) M1

$\int \frac{1}{4x+3}\,dx = 1/4 \times \ln(4x + 3)$ A1

$k \times [\ln(6 \times 4 + 3) - \ln(4a + 3)] = 0{\cdot}1986$ ($k = 1, 4, \mathbf{\frac{1}{4}}$) m1

$\frac{27}{4a+3} = e^{0{\cdot}7944}$ (o.e.) (c.a.o.) A1

$a = 2{\cdot}3$ (f.t. $a = 4{\cdot}8$ for $k = 1$ and $a = 5{\cdot}7$ for $k = 4$) A1

8. (*a*) Choice of *a*, *b*, *c*, *d* such that *a* is a factor of *c* and *b* is a factor of *d* M1

Correctly verifying that the candidate's *a*, *b*, *c*, *d* are such that $(a + b)$ is **not** a factor of $(c + d)$ and a statement to the effect that this is the case A1

(*b*) Trying to solve $5x + 4 = -7x$ M1

Trying to solve $5x + 4 = 7x$ M1

$x = -1/3, x = 2$ (c.a.o.) A1

$x = -1/3$ (c.a.o.) A1

Alternative mark scheme

$(5x + 4)^2 = (-7x)^2$ (squaring both sides) M1

$24x^2 - 40x - 16 = 0$ (at least two coefficients correct) A1

$x = -1/3, x = 2$ (c.a.o.) A1

$x = -1/3$ (c.a.o.) A1

(*c*) (i) $a = 5, -3$ B1

(ii) $b = -\frac{2}{3}$ B1

9. (*a*) $y - 8 = e^{4 - x/3}$. B1

An attempt to express equation as a logarithmic equation and to isolate x M1

$x = 3[4 - \ln(y - 8)]$ (c.a.o.) A1

$f^{-1}(x) = 3[4 - \ln(x - 8)]$

(f.t. one slip in candidate's expression for x) A1

(*b*) $D(f^{-1}) = [9, \infty)$ B1 B1

10. (*a*)

$$hh(x) = \frac{4 \times \dfrac{4x + 3}{5x - 4} + 3}{5 \times \dfrac{4x + 3}{5x - 4} - 4}$$ M1

$$hh(x) = \frac{16x + 12 + 15x - 12}{20x + 15 - 20x + 16}$$ A1

$hh(x) = x$ (convincing) A1

(*b*) $h^{-1}(x) = h(x)$ B1

$h^{-1}(-1) = h(-1) = \dfrac{1}{9}$ (awarded only if first B1 awarded) B1