C3

1. (*a*)

0	0
$\pi/9$	$-0{\cdot}062202456$
$2\pi/9$	$-0{\cdot}266515091$
$\pi/3$	$-0{\cdot}693147181$
$4\pi/9$	$-1{\cdot}750723994$

(5 values correct) B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = \pi/9$ M1

$$I \approx \frac{\pi/9}{3} \times \{0 + (-1{\cdot}750723994) + 4[(-0{\cdot}062202456) + (-0{\cdot}693147181)] + 2(-0{\cdot}266515091)\}$$

$I \approx -5{\cdot}305152724 \times (\pi/9) \div 3$

$I \approx -0{\cdot}617282549$

$I \approx -0{\cdot}6173$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

(*b*)

$\int_0^{4\pi/9} \ln(\sec x)\,dx \approx 0{\cdot}6173$ (f.t. candidate's answer to (*a*)) B1

2. (*a*) $7\operatorname{cosec}^2\theta - 4(\operatorname{cosec}^2\theta - 1) = 16 + 5\operatorname{cosec}\theta$

(correct use of $\cot^2\theta = \operatorname{cosec}^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in cosec θ, either by using the quadratic formula or by getting the expression into the form $(a\operatorname{cosec}\theta + b)(c\operatorname{cosec}\theta + d)$, with $a \times c$ = candidate's coefficient of $\operatorname{cosec}^2\theta$ and $b \times d$ = candidate's constant m1

$3\operatorname{cosec}^2\theta - 5\operatorname{cosec}\theta - 12 = 0 \Rightarrow (\operatorname{cosec}\theta - 3)(3\operatorname{cosec}\theta + 4) = 0$

$\Rightarrow \operatorname{cosec}\theta = 3$, $\operatorname{cosec}\theta = -\frac{4}{3}$

$\Rightarrow \sin\theta = \frac{1}{3}$, $\sin\theta = -\frac{3}{4}$ (c.a.o.) A1

$\theta = 19{\cdot}47°, 160{\cdot}53°$ B1

$\theta = 311{\cdot}41°, 228{\cdot}59°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\sin\theta = +, -$, f.t. for 3 marks, $\sin\theta = -, -$, f.t. for 2 marks

$\sin\theta = +, +$, f.t. for 1 mark

(*b*) $\sec\phi \ge 1$, $\operatorname{cosec}\phi \ge 1$ and thus $4\sec\phi + 3\operatorname{cosec}\phi \ge 7$ E1

3. (*a*) $\frac{d}{dx}(x^3) = 3x^2$ $\frac{d}{dx}(1) = 0$ $\frac{d}{dx}(\pi^2/4) = 0$ B1

$\frac{d}{dx}(2x\cos y) = 2x(-\sin y)\frac{dy}{dx} + 2\cos y$ B1

$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$ B1

$\frac{dy}{dx} = \frac{3}{2-\pi}$ (c.a.o.) B1

(*b*) $\frac{d^2y}{dx^2} = \frac{d}{dx}(x^2y) = x^2\frac{dy}{dx} + 2xy$ B1

Substituting x^2y for $\frac{dy}{dx}$ in candidate's derived expression for $\frac{d^2y}{dx^2}$ M1

$\frac{d^2y}{dx^2} = x^2(x^2y) + 2xy = x^4y + 2xy$ (o.e.) (c.a.o.) A1

4. (*a*) candidate's *x*-derivative = $\frac{1}{1+t^2}$ B1

candidate's *y*-derivative = $\frac{1}{t}$ B1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{1+t^2}{t}$ A1

(*b*) $\frac{d}{dt}\left(\frac{dy}{dx}\right) = -t^{-2} + 1$ (o.e.) B1

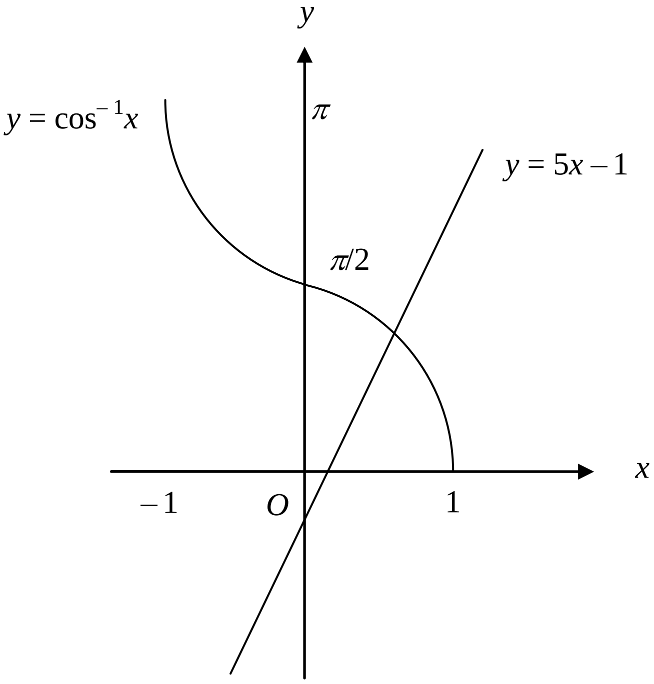
Use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div$ candidate's *x*-derivative M1

$\frac{d^2y}{dx^2} = (-t^{-2} + 1)(1 + t^2)$ (o.e.) (f.t. one slip) A1

$\frac{d^2y}{dx^2} = 0 \Rightarrow t = 1$ (c.a.o.) A1

$\frac{d^2y}{dx^2} = 0 \Rightarrow x = \frac{\pi}{4}$ (f.t. candidate's derived value for *t*) A1

5. (*a*)



Correct shape for $y = \cos^{-1}x$ — B1

A straight line with negative y-intercept and positive gradient intersecting once with $y = \cos^{-1}x$ in the first quadrant. — B1

(*b*)

$x_0 = 0{\cdot}4$

$x_1 = 0{\cdot}431855896$ (x_1 correct, at least 4 places after the point) — B1

$x_2 = 0{\cdot}424849379$

$x_3 = 0{\cdot}426400166$

$x_4 = 0{\cdot}426057413 = 0{\cdot}4261$ (x_4 correct to 4 decimal places) — B1

Let $h(x) = \cos^{-1}x - 5x + 1$

An attempt to check values or signs of $h(x)$ at $x = 0{\cdot}42605$, $x = 0{\cdot}42615$ — M1

$h(0{\cdot}42605) = 4{\cdot}24 \times 10^{-4} > 0$, $h(0{\cdot}42615) = -1{\cdot}86 \times 10^{-4} < 0$ — A1

Change of sign $\Rightarrow \alpha = 0{\cdot}4261$ correct to four decimal places — A1

6. (*a*) (i) $\dfrac{dy}{dx} = \dfrac{a + bx}{4x^2 - 3x - 5}$ (including $a = 1$, $b = 0$) M1

$\dfrac{dy}{dx} = \dfrac{8x - 3}{4x^2 - 3x - 5}$ A1

(ii) $\dfrac{dy}{dx} = e^{\sqrt{x}} \times f(x)$ ($f(x) \neq 1, 0$) M1

$\dfrac{dy}{dx} = e^{\sqrt{x}} \times \dfrac{1}{2} x^{-1/2}$ A1

(iii) $\dfrac{dy}{dx} = \dfrac{(a - b\sin x) \times m\cos x - (a + b\sin x) \times k\cos x}{(a - b\sin x)^2}$

($m = \pm b$, $k = \pm b$) M1

$\dfrac{dy}{dx} = \dfrac{(a - b\sin x) \times b\cos x - (a + b\sin x) \times (-b)\cos x}{(a - b\sin x)^2}$ A1

$\dfrac{dy}{dx} = \dfrac{2ab\cos x}{(a - b\sin x)^2}$ A1

(*b*) $\dfrac{d}{dx}(\cot x) = \dfrac{d}{dx}\left(\tan x\right)^{-1} = (-1) \times \left(\tan x\right)^{-2} \times f(x)$ ($f(x) \neq 1, 0$) M1

$\dfrac{d}{dx}\left(\tan x\right)^{-1} = (-1) \times \left(\tan x\right)^{-2} \times \sec^2 x$ A1

$\dfrac{d}{dx}\left(\tan x\right)^{-1} = -\mathrm{cosec}^2 x$ (convincing) A1

7. (*a*) (i) $\displaystyle\int \frac{(7x^2 - 2)}{x}\,dx = \int 7x\,dx - \int \frac{2}{x}\,dx$

Correctly rewriting as two terms and an attempt to integrate M1

$\displaystyle\int \frac{(7x^2 - 2)}{x}\,dx = \frac{7}{2}x^2 - 2\ln x + c$ A1 A1

(ii) $\int \sin({}^{2x}/_3 - \pi)\,dx = k \times \cos({}^{2x}/_3 - \pi) + c$

$(k = -1, -{}^{\mathbf{3}}/_{\mathbf{2}}, {}^3/_2, -{}^2/_3)$ M1

$\int \sin({}^{2x}/_3 - \pi)\,dx = -{}^3/_2 \times \cos({}^{2x}/_3 - \pi) + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\displaystyle\int (5x - 14)^{-1/4}\,dx = \frac{k \times (5x - 14)^{3/4}}{3/4}$ $(k = 1, 5, {}^{\mathbf{1}}/_{\mathbf{5}})$ M1

$\displaystyle\int (5x - 14)^{-1/4}\,dx = {}^1/_5 \times \frac{(5x - 14)^{3/4}}{3/4}$ A1

A correct method for substitution of the correct limits limits in an expression of the form $m \times (5x - 14)^{3/4}$ M1

$\displaystyle\int_3^6 (5x - 14)^{-1/4}\,dx = \frac{28}{15}$ (= 1·867)

(f.t. only for solutions of $\frac{28}{3}$ (= 9·333) and $\frac{140}{3}$ (= 46·667) from $k = 1$, $k = 5$ respectively) A1

Note: Answer only with no working shown earns 0 marks

8. (*a*) Trying to solve either $3x - 5 \le 1$ or $3x - 5 \ge -1$ M1

$3x - 5 \le 1 \Rightarrow x \le 2$

$3x - 5 \ge -1 \Rightarrow x \ge {}^4/_3$ (both inequalities) A1

Required range: ${}^4/_3 \le x \le 2$ (f.t. one slip) A1

Alternative mark scheme

$(3x - 5)^2 \le 1$

(squaring both sides, forming and trying to solve quadratic) M1

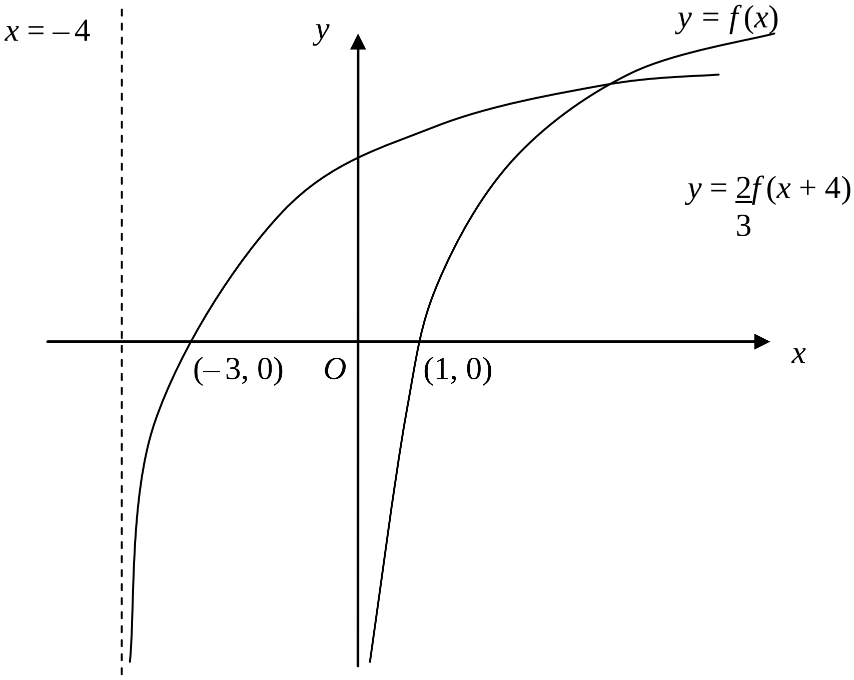
Critical values $x = {}^4/_3$ and $x = 2$ A1

Required range: ${}^4/_3 \le x \le 2$ (f.t. one slip in critical values) A1

(*b*) ${}^4/_3 \le 1/y \le 2$ (f.t. candidate's $a \le x \le b$, $a > 0$, $b > 0$) M1

${}^1/_2 \le y \le {}^3/_4$ (f.t. candidate's $a \le x \le b$, $a > 0$, $b > 0$) A1

9.



Correct shape, including the fact that the y-axis is an asymptote for $y = f(x)$ at $-\infty$ — B1

$y = f(x)$ cuts x-axis at (1, 0) — B1

Correct shape, including the fact that $x = -4$ is an asymptote for $y = \frac{2}{3}f(x + 4)$ at $-\infty$ — B1

$y = \frac{2}{3}f(x + 4)$ cuts x-axis at (– 3, 0) (f.t. candidate's x-intercept for $f(x)$) — B1

The diagram shows that the graph of $y = f(x)$ is steeper than the graph of $y = \frac{2}{3}f(x + 4)$ in the first quadrant — B1

10. (*a*) Choice of h, k such that $h(x) = k(x) + c$, $c \neq 0$ — M1

Convincing verification of the fact that $h'(x) = k'(x)$ — A1

(*b*) (i) $y - 3 = 2\ln(4x + 5)$ — B1

An attempt to express candidate's equation as an exponential equation — M1

$x = \frac{(e^{(y-3)/2} - 5)}{4}$ (c.a.o.) — A1

$f^{-1}(x) = \frac{(e^{(x-3)/2} - 5)}{4}$

(f.t. one slip in candidate's expression for x) — A1

(ii) $D(f^{-1}) = [10, 14]$ — B1 B1

(iii) $gf(x) = e^{2\ln(4x + 5) + 3}$ — B1

$e^{2\ln(4x + 5)} = (4x + 5)^2$ — B1

$gf(x) = e^3(4x + 5)^2$ (c.a.o.) — B1