

C3

1. (a) 0 2.197224577
 0.75 2.314217179
 1.5 2.524262696
 2.25 2.861499826
 3 3.335254744 (5 values correct) B2
 (If B2 not awarded, award B1 for either 3 or 4 values correct)
 Correct formula with $h = 0.75$ M1
 $I \approx \frac{0.75}{3} \times \{2.197224577 + 3.335254744$
 $+ 4(2.314217179 + 2.861499826) + 2(2.524262696)\}$
 $I \approx 31.28387273 \times 0.75 \div 3$
 $I \approx 7.820968183$
 $I \approx 7.82$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

- (b) $\int_0^3 \ln(16 + 2e^x) dx = \int_0^3 \ln(8 + e^x) dx + \int_0^3 \ln 2 dx$ M1
 $\int_0^3 \ln(16 + 2e^x) dx = 7.82 + 2.08 = 9.90$
 (f.t. candidate's answer to (a)) A1

Note: Answer only with no working shown earns 0 marks

2. $8(\sec^2 \theta - 1) - 5 \sec^2 \theta = 7 + 4 \sec \theta$. (correct use of $\tan^2 \theta = \sec^2 \theta - 1$) M1
 An attempt to collect terms, form and solve quadratic equation in $\sec \theta$, either by using the quadratic formula or by getting the expression into the form $(a \sec \theta + b)(c \sec \theta + d)$, with $a \times c =$ candidate's coefficient of $\sec^2 \theta$ and $b \times d =$ candidate's constant m1
 $3 \sec^2 \theta - 4 \sec \theta - 15 = 0 \Rightarrow (3 \sec \theta + 5)(\sec \theta - 3) = 0$
 $\Rightarrow \sec \theta = -\frac{5}{3}, \sec \theta = 3$
 $\Rightarrow \cos \theta = -\frac{3}{5}, \cos \theta = \frac{1}{3}$ (c.a.o.) A1
 $\theta = 126.87^\circ, 233.13^\circ$ B1 B1
 $\theta = 70.53^\circ, 289.47^\circ$ B1

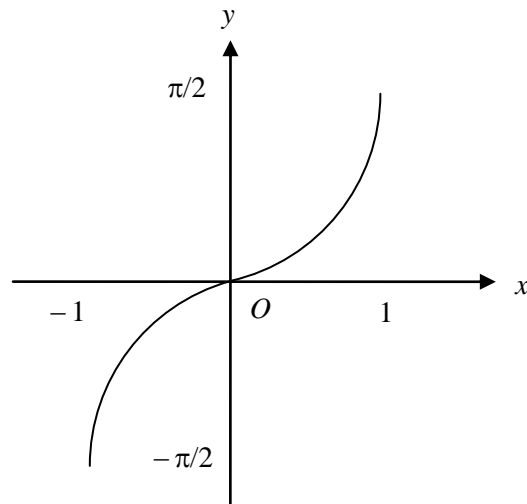
Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
 $\cos \theta = +, -, \text{ f.t. for 3 marks, } \cos \theta = -, -, \text{ f.t. for 2 marks}$
 $\cos \theta = +, +, \text{ f.t. for 1 mark}$

3. (a) $\frac{d}{dx}(y^4) = 4y^3 \frac{dy}{dx}$ B1
 $\frac{d}{dx}(8xy^2) = (8x)(2y)\frac{dy}{dx} + 8y^2$ B1
 $\frac{d}{dx}(2x^2) = 4x, \frac{d}{dx}(9) = 0$ B1
 $\frac{dy}{dx} = \frac{x - 2y^2}{y^3 + 4xy}$ (convincing) (c.a.o.) B1
- (b) $\frac{dy}{dx} = 0 \Rightarrow x = 2y^2$ B1
Substitute $2y^2$ for x in equation of C M1
 $9y^4 + 9 = 0$ (o.e.) (c.a.o.) A1
 $9y^4 + 9 > 0$ for any real y (o.e.) and thus no such point exists A1
4. candidate's x -derivative = $2e^t$ B1
candidate's y -derivative = $-8e^{-t} + 3e^t$ B1
 $\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1
 $\frac{dy}{dx} = \frac{-8e^{-t} + 3e^t}{2e^t}$ (o.e.) (c.a.o.) A1
Putting candidate's $\frac{dy}{dx} = -1$, rearranging and obtaining either an equation in
both e^t and e^{-t} , or an equation in e^{2t} , or an equation in e^{-2t} . M1
Either $e^{2t} = \frac{8}{5}$ or $e^{-2t} = \frac{5}{8}$
(f.t. one numerical slip in candidate's derived expression for $\frac{dy}{dx}$) A1
 $t = 0.235$ (c.a.o.) A1
5. (a) $\frac{d}{dx}[\ln(3x^2 - 2x - 1)] = \frac{ax + b}{3x^2 - 2x - 1}$ (including $a = 0, b = 1$) M1
 $\frac{d}{dx}[\ln(3x^2 - 2x - 1)] = \frac{6x - 2}{3x^2 - 2x - 1}$ A1
 $6x - 2 = 8x(3x^2 - 2x - 1)$ (o.e.) (f.t. candidate's a, b) A1
 $12x^3 - 8x^2 - 7x + 1 = 0$ (convincing) A1
- (b) $x_0 = -0.6$
 $x_1 = -0.578232165$ (x_1 correct, at least 4 places after the point) B1
 $x_2 = -0.582586354$
 $x_3 = -0.581770386$
 $x_4 = -0.581925366 = -0.5819$ (x_4 correct to 4 decimal places) B1
Let $g(x) = 12x^3 - 8x^2 - 7x + 1$
An attempt to check values or signs of $g(x)$ at $x = -0.58185$,
 $x = -0.58195$ M1
 $g(-0.58185) = 7.35 \times 10^{-4}, g(-0.58195) = -7.15 \times 10^{-4}$ A1
Change of sign $\Rightarrow \alpha = -0.5819$ correct to four decimal places A1

6. (a) (i) $\frac{dy}{dx} = -\frac{1}{4} \times (9 - 4x^5)^{-5/4} \times f(x) \quad (f(x) \neq 1) \quad \text{M1}$
 $\frac{dy}{dx} = -\frac{1}{4} \times (9 - 4x^5)^{-5/4} \times (-20x^4) \quad \text{A1}$
 $\frac{dy}{dx} = 5x^4 \times (9 - 4x^5)^{-5/4} \quad \text{A1}$

(ii) $\frac{dy}{dx} = \frac{(7 - x^3) \times f(x) - (3 + 2x^3) \times g(x)}{(7 - x^3)^2} \quad (f(x), g(x) \neq 1) \quad \text{M1}$
 $\frac{dy}{dx} = \frac{(7 - x^3) \times 6x^2 - (3 + 2x^3) \times (-3x^2)}{(7 - x^3)^2} \quad \text{A1}$
 $\frac{dy}{dx} = \frac{51x^2}{(7 - x^3)^2} \quad (\text{c.a.o.}) \quad \text{A1}$

(b) (i)



G1

(ii) $x = \sin y \Rightarrow \frac{dx}{dy} = \cos y \quad \text{B1}$
 $\frac{dx}{dy} = \pm\sqrt{1 - \sin^2 y} \quad \text{B1}$
 The +ive sign is chosen because the graph shows the gradient to be positive E1
 $\frac{dx}{dy} = \sqrt{1 - x^2} \quad \text{B1}$
 $\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}} \quad \text{B1}$

7. (a) (i) $\int \cos(2-5x) dx = k \times \sin(2-5x) + c$ (k = 1, 1/5, -5, -1/5) M1
 $\int \cos(2-5x) dx = -1/5 \times \sin(2-5x) + c$ A1
- (ii) $\int \frac{4}{e^{3x-2}} dx = k \times 4 \times e^{2-3x} + c$ (k = 1, -3, 1/3 - 1/3) M1
 $\int \frac{4}{e^{3x-2}} dx = -4/3 \times e^{2-3x} + c$ A1
- (iii) $\int \frac{5}{\sqrt[1]{6x-3}} dx = k \times 5 \times \ln|\sqrt[1]{6x-3}| + c$ (k = 1, 1/6, 6) M1
 $\int \frac{5}{\sqrt[1]{6x-3}} dx = 30 \times \ln|\sqrt[1]{6x-3}| + c$ A1

Note: The omission of the constant of integration is only penalised once.

- (b) $\int (4x+1)^{1/2} dx = k \times \frac{(4x+1)^{3/2}}{3/2}$ (k = 1, 4, 1/4) M1
 $\int_2^6 (4x+1)^{1/2} dx = \left[\frac{1}{4} \times \frac{(4x+1)^{3/2}}{3/2} \right]_2^6$ A1

A correct method for substitution of limits in an expression of the form $m \times (4x+1)^{3/2}$ M1

$$\int_2^6 (4x+1)^{1/2} dx = \frac{125}{6} - \frac{27}{6} = \frac{98}{6} = 16.33$$

(f.t. only for solutions of $\frac{392}{6}$ and $\frac{1568}{6}$ from k = 1, 4 respectively) A1

Note: Answer only with no working shown earns 0 marks

8. (a) Choice of a, b, with one positive and one negative and one side correctly evaluated M1
Both sides of identity evaluated correctly A1
- (b) Trying to solve $3x-2=7x$ M1
Trying to solve $3x-2=-7x$ M1
 $x=-0.5, x=0.2$ (both values) (c.a.o.) A1

Alternative mark scheme

$$(3x-2)^2 = 7^2 \times x^2 \quad \text{(squaring both sides) M1}$$

$$40x^2 + 12x - 4 = 0 \quad \text{(o.e.) (c.a.o.) A1}$$

$$x = -0.5, x = 0.2 \quad \text{(both values, f.t. one slip in quadratic) A1}$$

9. (a) $f(x) = (x - 4)^2 - 9$ B1
- (b) $y = (x - 4)^2 - 9$ and an attempt to isolate x
 (f.t. candidate's expression for $f(x)$ of form $(x + a)^2 + b$, with a, b derived) M1
- $x = (\pm)\sqrt{(y + 9)} + 4$
 (f.t. candidate's expression for $f(x)$ of form $(x + a)^2 + b$, with a, b derived) A1
- $x = -\sqrt{(y + 9)} + 4$ (o.e.) (c.a.o.) A1
- $f^{-1}(x) = -\sqrt{(x + 9)} + 4$ (o.e.)
- (f.t. only incorrect choice of sign in front of the $\sqrt{\quad}$ sign and candidate's expression for $f(x)$ of form $(x + a)^2 + b$, with a, b derived) A1
10. (a) $R(g) = [2k - 4, \infty)$ B1
- (b) (i) $2k - 4 \geq -2$ M1
 $k \geq 1$ (\Rightarrow least value of k is 1)
 (f.t. candidate's $R(g)$ provided it is of form $[a, \infty)$ A1
- (ii) $fg(x) = (kx - 4)^2 + k(kx - 4) - 8$ B1
- (iii) $(3k - 4)^2 + k(3k - 4) - 8 = 0$
 (substituting 3 for x in candidate's expression for $fg(x)$ and putting equal to 0) M1
- Either $12k^2 - 28k + 8 = 0$ or $6k^2 - 14k + 4 = 0$
 or $3k^2 - 7k + 2 = 0$ (c.a.o.) A1
- $k = \frac{1}{3}, 2$ (f.t. candidate's quadratic in k) A1
- $k = 2$ (c.a.o.) A1