C3

1. (*a*)

1	1·945910149		
1·5	2·238046572		
2	2·63905733		
2·5	3·073850053		
3	3·496507561	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0·5$ M1

$I \approx \frac{0·5}{3} \times \{1·945910149 + 3·496507561 + 4(2·238046572 + 3·073850053) + 2(2·63905733)\}$

$I \approx 31·96811887 \times 0·5 \div 3$

$I \approx 5·328019812$

$I \approx 5·328$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

(*b*) $\int_1^3 \ln \sqrt{(x^3 + 6)}\,dx \approx 2·664$ (f.t. candidate's answer to (*a*)) B1

2. (*a*) $4(\text{cosec}^2\theta - 1) - 8 = 2\,\text{cosec}^2\theta - 5\,\text{cosec}\,\theta$

(correct use of $\cot^2\theta = \text{cosec}^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in cosec θ, either by using the quadratic formula or by getting the expression into the form $(a\,\text{cosec}\,\theta + b)(c\,\text{cosec}\,\theta + d)$, with $a \times c$ = coefficient of $\text{cosec}^2\theta$ and $b \times d$ = candidate's constant m1

$2\,\text{cosec}^2\theta + 5\,\text{cosec}\,\theta - 12 = 0 \Rightarrow (2\,\text{cosec}\,\theta - 3)(\text{cosec}\,\theta + 4) = 0$

$\Rightarrow \text{cosec}\,\theta = \frac{3}{2}, \text{cosec}\,\theta = -4$

$\Rightarrow \sin\theta = \frac{2}{3}, \sin\theta = -\frac{1}{4}$ (c.a.o.) A1

$\theta = 41·81°, 138·19°$ B1

$\theta = 194·48°, 345·52°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\sin\theta$ = +, –, f.t. for 3 marks, $\sin\theta$ = –, –, f.t. for 2 marks

$\sin\theta$ = +, +, f.t. for 1 mark

(*b*) Correct use of $\sec\phi = \frac{1}{\cos\phi}$ and $\tan\phi = \frac{\sin\phi}{\cos\phi}$ (o.e.) M1

$\sin\phi = -\frac{1}{2}$ A1

$\phi = 210°, 330°$ (f.t. for $\sin\phi = -a$) A1

3. (*a*) Use of product formula yielding $x^3 \times 2y \times \frac{dy}{dx} + 3x^2 \times y^2$ B1 B1

$\frac{dy}{dx} = -\frac{3x^2y^2}{2x^3y}$ (c.a.o.) B1

(*b*) (i) Putting candidate's expression for $\frac{dy}{dx} = 3$ and an attempt to simplify M1

$-\frac{3a^2b^2}{2a^3b} = 3 \Rightarrow b = -2a$ (convincing) A1

(ii) Substituting a for x and $-2a$ for y in the equation for C M1

$a = 2,\ b = -4$ A1

4. (*a*) Differentiating $\ln t$ and $5t^4$ with respect to t, at least one correct M1

candidate's x-derivative $= \frac{1}{t}$,

candidate's y-derivative $= 20t^3$ (both values) A1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = 20t^4$ (c.a.o.) A1

(*b*) $\frac{d}{dt}\left(\frac{dy}{dx}\right) = 80t^3$ (f.t. candidate's expression for $\frac{dy}{dx}$) B1

Use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div$ candidate's x-derivative M1

$\frac{d^2y}{dx^2} = 80t^4$ (f.t. one slip) A1

$\frac{d^2y}{dx^2} = 0{\cdot}648 \Rightarrow t = 0{\cdot}3$ (c.a.o.) A1

5. (*a*) $\frac{dy}{dx} = 5 \times (7 - 9x^2)^4 \times f(x)$, $(f(x) \neq 1)$ M1

$\frac{dy}{dx} = -90x \times (7 - 9x^2)^4$ A1

(*b*) $\frac{dy}{dx} = \boldsymbol{\frac{6}{1 + (6x)^2}}$ **or** $\frac{1}{1 + (6x)^2}$ **or** $\frac{6}{1 + 6x^2}$ M1

$\frac{dy}{dx} = \frac{6}{1 + 36x^2}$ A1

(*c*) $\frac{dy}{dx} = e^{4x} \times m \sec^2 2x + \tan 2x \times ke^{4x}$ ($m = 1$, **2**, $k = 1$, **4**) M1

$\frac{dy}{dx} = e^{4x} \times 2 \sec^2 2x + \tan 2x \times 4e^{4x}$ (at least one correct term) B1

$\frac{dy}{dx} = e^{4x} \times 2 \sec^2 2x + \tan 2x \times 4e^{4x}$ (c.a.o.) A1 (*d*) $\frac{dy}{dx} = \frac{(2 + \cos x) \times m\cos x - (3 + \sin x) \times k\sin x}{(2 + \cos x)^2}$ (m = **1**, – 1 k = 1, **– 1**) M1

$\frac{dy}{dx} = \frac{(2 + \cos x) \times (\cos x) - (3 + \sin x) \times (-\sin x)}{(2 + \cos x)^2}$ A1

$\frac{dy}{dx} = \frac{2\cos x + 3\sin x + 1}{(2 + \cos x)^2}$ A1

6. (*a*) (i) $\int \cos(3x + \pi/2)\,dx = k \times \sin(3x + \pi/2) + c$

(k = 1, 3, **$^1/_3$**, – $^1/_3$) M1

$\int \cos(3x + \pi/2)\,dx = {}^1/_3 \times \sin(3x + \pi/2) + c$ A1

(ii) $\int e^{3-4x}\,dx = k \times e^{3-4x} + c$ (k = 1, – 4, $^1/_4$, **– $^1/_4$**) M1

$\int e^{3-4x}\,dx = -{}^1/_4 \times e^{3-4x} + c$ A1

(iii) $\int \frac{7}{8x+5}\,dx = 7 \times k \times \ln|8x + 5| + c$ (k = 1, 8, **$^1/_8$**) M1

$\int \frac{7}{8x+5}\,dx = 7 \times {}^1/_8 \times \ln|8x + 5| + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int (2x - 1)^{-4}\,dx = k \times \frac{(2x-1)^{-3}}{-3}$ (k = 1, 2, **$^1/_2$**) M1

$\int_1^2 9 \times (2x - 1)^{-4}\,dx = \left[9 \times {}^1/_2 \times \frac{(2x-1)^{-3}}{-3}\right]_1^2$ A1

Correct method for substitution of limits in an expression of the form $m \times (2x - 1)^{-3}$ M1

$\int_1^2 9 \times (2x - 1)^{-4}\,dx = \frac{13}{9} = 1{\cdot}44$ (f.t. for k = 1, 2 only) A1

Note: Answer only with no working earns 0 marks

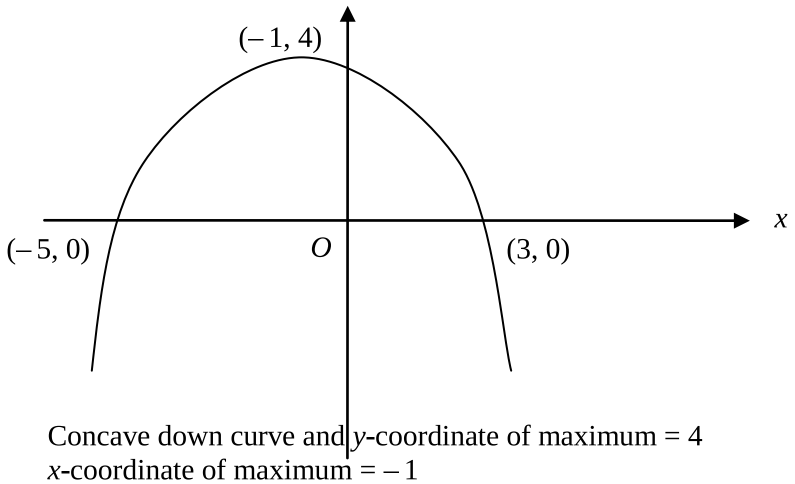
7. (*a*) Choice of $a \neq -1$ and $b = -a - 2$ M1
Correct verification that given equation is satisfied A1

(*b*) Trying to solve either $x^2 - 10 \leq 6$ or $x^2 - 10 \geq -6$ M1
$x^2 - 10 \leq 6 \Rightarrow x^2 \leq 16$
$x^2 - 10 \geq -6 \Rightarrow x^2 \geq 4$ (both inequalities) A1
At least one of: $2 \leq x \leq 4$, $-4 \leq x \leq -2$ (f.t. one slip) A1
Required range: $2 \leq x \leq 4$ or $-4 \leq x \leq -2$ (c.a.o.) A1

Alternative mark scheme
$(x^2 - 10)^2 \leq 36$ (forming and trying to solve quadratic in x^2)M1
Critical values $x^2 = 4$ and $x^2 = 16$ A1
At least one of: $2 \leq x \leq 4$, $-4 \leq x \leq -2$ (f.t. one slip) A1
Required range: $2 \leq x \leq 4$ or $-4 \leq x \leq -2$ (c.a.o.) A1

8. $x_0 = -1{\cdot}5$
$x_1 = -1{\cdot}666394263$ (x_1 correct, at least 5 places after the point) B1
$x_2 = -1{\cdot}676625462$
$x_3 = -1{\cdot}677198866$
$x_4 = -1{\cdot}677230823 = -1{\cdot}67723$ (x_4 correct to 5 decimal places) B1
Let $f(x) = x^2 + e^x - 3$
An attempt to check values or signs of $f(x)$ at $x = -1{\cdot}677225$, $x = -1{\cdot}677235$
M1
$f(-1{\cdot}677225) = -2{\cdot}44 \times 10^{-5} < 0$, $f(-1{\cdot}677235) = 7{\cdot}26 \times 10^{-6} > 0$ A1
Change of sign $\Rightarrow \alpha = -1{\cdot}67723$ correct to five decimal places A1

9.



Concave down curve and y-coordinate of maximum = 4 B1
x-coordinate of maximum = – 1 B1
Both points of intersection with x-axis B1

10. (*a*) $y - 6 = e^{5 - x/2}$. B1

An attempt to express equation as a logarithmic equation and to isolate x M1

$x = 2\,[5 - \ln\,(y - 6)]$ (c.a.o.) A1

$f^{-1}(x) = 2\,[5 - \ln\,(x - 6)]$

(f.t. one slip in candidate's expression for x) A1

(*b*) $D(f^{-1}) = [7, \infty)$ B1 B1

11. (*a*) (i) $D(fg) = (0, \pi/4]$ B1

(ii) $R(fg) = (-\infty, 0]$ B1 B1

(*b*) (i) $fg(x) = -0{\cdot}4 \Rightarrow \tan x = e^{-0{\cdot}4}$ M1

$x = 0{\cdot}59$ A1

(ii) Equation has solution only if $k \in R(fg)$.

$\therefore$ choose any $k \notin R(fg)$ (f.t. candidate's $R(fg)$) B1