

# Mathematics C3 January 2013

## Solutions and Mark Scheme

### Final Version

- 1.
- |      |             |                       |
|------|-------------|-----------------------|
| 1    | 0.211941557 |                       |
| 1.25 | 0.182137984 |                       |
| 1.5  | 0.154280773 |                       |
| 1.75 | 0.128955672 |                       |
| 2    | 0.106506978 | (5 values correct) B2 |
- (If B2 not awarded, award B1 for either 3 or 4 values correct)

Correct formula with  $h = 0.25$  M1

$$I \approx \frac{0.25}{3} \times \{0.211941557 + 0.106506978 + 4(0.182137984 + 0.128955672) + 2(0.154280773)\}$$

$$I \approx 1.871384705 \times 0.25 \div 3$$

$$I \approx 0.155948725$$

$$I \approx 0.156 \quad (\text{f.t. one slip}) \quad \text{A1}$$

**Note: Answer only with no working earns 0 marks**

2. (a) (i) e.g.  $\theta = 20^\circ$
- |                            |   |    |
|----------------------------|---|----|
| $\cos^3 \theta = 0.83$     | (choice of $\theta$ and one correct evaluation) | B1 |
| $1 - \sin^3 \theta = 0.96$ | (both evaluations correct but different)        | B1 |
- (ii)  $\theta = 0^\circ$  or  $\theta = 90^\circ$  B1
- (b)  $4(1 + \cot^2 \theta) = 9 - 8 \cot \theta$ . (correct use of  $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$ ) M1
- An attempt to collect terms, form and solve quadratic equation in  $\cot \theta$ , either by using the quadratic formula or by getting the expression into the form  $(a \cot \theta + b)(c \cot \theta + d)$ , with  $a \times c =$  candidate's coefficient of  $\cot^2 \theta$  and  $b \times d =$  candidate's constant m1
- $$4 \cot^2 \theta + 8 \cot \theta - 5 = 0 \Rightarrow (2 \cot \theta - 1)(2 \cot \theta + 5) = 0$$
- $$\Rightarrow \cot \theta = \frac{1}{2}, \cot \theta = -\frac{5}{2}$$
- $$\Rightarrow \tan \theta = 2, \tan \theta = -\frac{2}{5} \quad (\text{c.a.o.}) \quad \text{A1}$$
- $\theta = 63.43^\circ, 243.43^\circ$  B1
- $\theta = 158.2^\circ, 338.2^\circ$  B1, B1

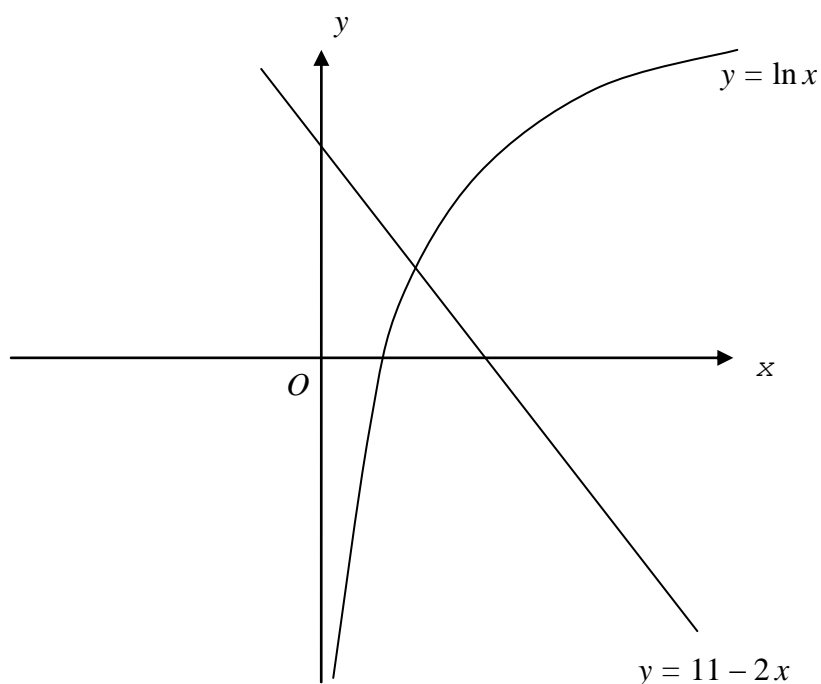
Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\tan \theta = +, -$ , f.t. for 3 marks,  $\tan \theta = -, -$ , f.t. for 2 marks

$\tan \theta = +, +$ , f.t. for 1 mark

3. (a)  $\frac{d(2y^3)}{dx} = 6y^2 \frac{dy}{dx}$  B1  
 $\frac{d(5x^4y)}{dx} = 5x^4 \frac{dy}{dx} + 20x^3y$  B1  
 $\frac{d(x^3)}{dx} = 3x^2, \frac{d(7)}{dx} = 0$  B1  
 $\frac{dy}{dx} = \frac{20x^3y + 3x^2}{6y^2 - 5x^4}$  (o.e.) (c.a.o.) B1
- (b) (i) candidate's  $x$ -derivative =  $3t^2$  B1  
candidate's  $y$ -derivative =  $4t^3 + 35t^4$  B1  
 $\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$  M1  
 $\frac{dy}{dx} = \frac{4t^3 + 35t^4}{3t^2}$  (c.a.o.) A1
- (ii)  $\frac{d\left(\frac{dy}{dx}\right)}{dt} = \frac{4 + 70t}{3}$  (o.e.) B1
- Use of  $\frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dt} \div \frac{dx}{dt}$   
(f.t. candidate's expression for  $\frac{dy}{dx}$ ) M1  
 $\frac{d^2y}{dx^2} = \frac{4 + 70t}{9t^2}$  (o.e.) A1
- (iii) An attempt to solve  $t^3 - 5 = 3$  and substitution of answer in candidate's expression for  $\frac{d^2y}{dx^2}$  M1  
 $\frac{d^2y}{dx^2} = 4$  (c.a.o.) A1

4. (a)



Correct shape for  $y = \ln x$ , including the fact that the  $y$ -axis is an asymptote at  $-\infty$  B1  
 A straight line with positive intercept and negative gradient intersecting once with  $y = \ln x$  in the first quadrant. B1  
 Equation has one root (c.a.o.) B1

(b)  $x_0 = 4.7$   
 $x_1 = 4.726218746$  ( $x_1$  correct, at least 5 places after the point) B1  
 $x_2 = 4.723437268$   
 $x_3 = 4.723731615$   
 $x_4 = 4.723700458 = 4.72370$  ( $x_4$  correct to 5 decimal places) B1  
 Let  $h(x) = \ln x + 2x - 11$   
 An attempt to check values or signs of  $h(x)$  at  $x = 4.723695$ ,  
 $x = 4.723705$  M1  
 $h(4.723695) = -1.87 \times 10^{-5} < 0$ ,  $h(4.723705) = 3.45 \times 10^{-6} > 0$  A1  
 Change of sign  $\Rightarrow \alpha = 4.72370$  correct to five decimal places A1

5. (a) (i)  $\frac{dy}{dx} = \frac{1}{2} \times (5x^2 - 3x)^{-1/2} \times f(x)$  ( $f(x) \neq 1$ ) M1  
 $\frac{dy}{dx} = \frac{1}{2} \times (5x^2 - 3x)^{-1/2} \times (10x - 3)$  A1
- (ii)  $\frac{dy}{dx} = \frac{\pm 7}{\sqrt{1 - (7x)^2}}$  **or**  $\frac{1}{\sqrt{1 - (7x)^2}}$  **or**  $\frac{7}{\sqrt{1 - 7x^2}}$  M1  
 $\frac{dy}{dx} = \frac{7}{\sqrt{1 - 49x^2}}$  A1
- (iii)  $\frac{dy}{dx} = e^{3x} \times f(x) + \ln x \times g(x)$  M1  
 $\frac{dy}{dx} = e^{3x} \times f(x) + \ln x \times g(x)$   
(either  $f(x) = 1/x$  or  $g(x) = 3e^{3x}$ ) A1  
 $\frac{dy}{dx} = \frac{e^{3x}}{x} + 3e^{3x} \ln x$  (all correct) A1
- (b)  $\frac{d}{dx}(\cot x) = \frac{\sin x \times m \sin x - \cos x \times k \cos x}{\sin^2 x}$  ( $m = 1, -1, k = 1, -1$ ) M1  
 $\frac{d}{dx}(\cot x) = \frac{\sin x \times (-\sin x) - \cos x \times (\cos x)}{\sin^2 x}$  A1  
 $\frac{d}{dx}(\cot x) = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$   
 $\frac{d}{dx}(\cot x) = \frac{-1}{\sin^2 x} = -\operatorname{cosec}^2 x$  (convincing) A1

6. (a) (i)  $\int \frac{\cos\left[\frac{4x+5}{3}\right]}{\left[\frac{4x+5}{3}\right]} dx = k \times \sin\left[\frac{4x+5}{3}\right] + c$  ( $k = 1, \frac{4}{3}, \frac{3}{4}, -\frac{3}{4}, \frac{1}{4}$ ) M1

$$\int \frac{\cos\left[\frac{4x+5}{3}\right]}{\left[\frac{4x+5}{3}\right]} dx = \frac{3}{4} \times \sin\left[\frac{4x+5}{3}\right] + c \quad \text{A1}$$

(ii)  $\int e^{2x+9} dx = k \times e^{2x+9} + c$  ( $k = 1, 2, \frac{1}{2}$ ) M1

$$\int e^{2x+9} dx = \frac{1}{2} \times e^{2x+9} + c \quad \text{A1}$$

(iii)  $\int \frac{3}{(7-2x)^6} dx = \frac{3}{-5k} \times (7-2x)^{-5} + c$  ( $k = 1, 2, -2, -\frac{1}{2}$ ) M1

$$\int \frac{3}{(7-2x)^6} dx = \frac{3}{-5 \times -2} \times (7-2x)^{-5} + c \quad \text{A1}$$

**Note: The omission of the constant of integration is only penalised once.**

(b)  $\int \frac{1}{3x-4} dx = k \times \ln|3x-4|$  ( $k = 1, 3, \frac{1}{3}$ ) M1

$$\int \frac{1}{3x-4} dx = \left[ \frac{1}{3} \times \ln|3x-4| \right] \quad \text{A1}$$

A correct method for substitution of limits 2, 44, in an expression of the form  $k \times \ln|3x-4|$  ( $k = 1, 3, \frac{1}{3}$ ) m1

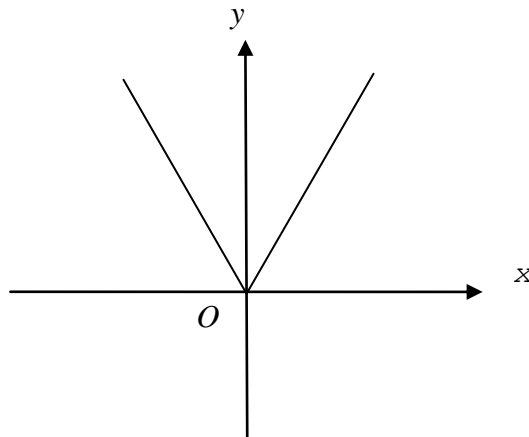
$$\int_2^{44} \frac{1}{3x-4} dx = \ln 4 \quad \text{(c.a.o.)} \quad \text{A1}$$

7. (a) Trying to solve either  $3x - 4 > 5$  or  $3x - 4 < -5$  M1  
 $3x - 4 > 5 \Rightarrow x > 3$   
 $3x - 4 < -5 \Rightarrow x < -\frac{1}{3}$  (both inequalities) A1  
Required range:  $x < -\frac{1}{3}$  or  $x > 3$  (f.t. one slip) A1

**Alternative mark scheme**

- $(3x - 4)^2 > 25$   
(squaring both sides, forming and trying to solve quadratic) M1  
Critical values  $x = -\frac{1}{3}$  and  $x = 3$  A1  
Required range:  $x < -\frac{1}{3}$  or  $x > 3$  (f.t. one slip in critical values) A1

- (b) (i)



G1

- (ii)  $a = -2$  B1  
 $b = -4$  B1

8. (a)  $y + 2 = \ln(4x + 5)$  B1  
An attempt to express candidate's equation as an exponential equation M1  
 $x = \frac{(e^{y+2} - 5)}{4}$  (f.t. one slip) A1  
 $f^{-1}(x) = \frac{(e^{x+2} - 5)}{4}$  (f.t. one slip) A1
- (b)  $D(f^{-1}) = [-2, \infty)$  B1

9. (a) (i)  $D(fg) = (0, \infty)$  B1  
(ii)  $R(fg) = [a, b]$  with B1  
 $a = -25$  B1  
 $b = \infty$  B1  
(iii)  $fg(x) = (2x - 3)^2 - 25$  B1  
(iv) Putting candidate's expression for  $fg(x)$  equal to 0 and using a correct method to try and solve the resulting quadratic in  $x$  M1  
 $x = 4, x = -1,$  (c.a.o.) A1  
 $x = 4$  (c.a.o.) A1
- (b) (i)  $hh(x) = \frac{2 \times \frac{2x+7}{5x-2} + 7}{5 \times \frac{2x+7}{5x-2} - 2}$  M1  
 $hh(x) = \frac{4x+14+35x-14}{10x+35-10x+4}$   
 $hh(x) = x$  (convincing) A1  
(ii)  $h^{-1}(x) = h(x)$  B1