

C3

1. (a)
- | | | | | | |
|--|---|-------------|-------------------------|--|----|
| | 0 | 1 | | | |
| | 0.25 | 1.064494459 | | | |
| | 0.5 | 1.284025417 | | | |
| | 0.75 | 1.755054657 | (5 values correct) | | B2 |
| | 1 | 2.718281828 | (3 or 4 values correct) | | B1 |
| | Correct formula with $h = 0.25$ | | | | M1 |
| | $I \approx \frac{0.25}{3} \times \{1 + 2 \cdot 718281828 + 4(1 \cdot 064494459 + 1 \cdot 755054657) + 2(1 \cdot 284025417)\}$ | | | | |
| | $I \approx 17.56452913 \times 0.25 \div 3$ | | | | |
| | $I \approx 1.463710761$ | | | | |
| | $I \approx 1.4637$ | | | | |
| | | | (f.t. one slip) | | A1 |

Note: Answer only with no working shown earns 0 marks

- (b)
- | | | | | |
|--|--|----------------------------------|--|----|
| | $\int_0^1 e^{x^2+3} dx = e^3 \times \int_0^1 e^{x^2} dx$ | | | |
| | | | | M1 |
| | $\int_0^1 e^{x^2+3} dx = 29.399$ | (f.t. candidate's answer to (a)) | | A1 |

Note: Answer only with no working shown earns 0 marks

2. (a) $\phi = 360^\circ - \theta$ or $\phi = -\theta$ and noting that $\cos \theta = \cos \phi$ B1
 $\sin \theta \neq \sin \phi$ (including correct evaluations) B1
- (b) $13 \tan^2 \theta = 5(1 + \tan^2 \theta) + 6 \tan \theta$.
 (correct use of $\sec^2 \theta = 1 + \tan^2 \theta$) M1
 An attempt to collect terms, form and solve quadratic equation
 in $\tan \theta$, either by using the quadratic formula or by getting the
 expression into the form $(a \tan \theta + b)(c \tan \theta + d)$,
 with $a \times c =$ candidate's coefficient of $\tan^2 \theta$ and
 $b \times d =$ candidate's constant m1
 $8 \tan^2 \theta - 6 \tan \theta - 5 = 0 \Rightarrow (4 \tan \theta - 5)(2 \tan \theta + 1) = 0$
 $\Rightarrow \tan \theta = \frac{5}{4}, \tan \theta = -\frac{1}{2}$ (c.a.o.) A1
- $\theta = 51.34^\circ, 231.34^\circ$ B1
 $\theta = 153.43^\circ, 333.43^\circ$ B1 B1
 Note: Subtract 1 mark for each additional root in range for each
 branch, ignore roots outside range.
 $\tan \theta = +, -, \text{f.t.}$ for 3 marks, $\tan \theta = -, -, \text{f.t.}$ for 2 marks
 $\tan \theta = +, +, \text{f.t.}$ for 1 mark

3. (a) $\frac{d(x^3)}{dx} = 3x^2$ $\frac{d(-3x-2)}{dx} = -3$ B1
 $\frac{d(-4x^2y)}{dx} = -4x^2 \frac{dy}{dx} - 8xy$ B1
 $\frac{d(2y^3)}{dx} = 6y^2 \frac{dy}{dx}$ B1
 $x = 3, y = 1 \Rightarrow \frac{dy}{dx} = \frac{6}{42} = \frac{1}{7}$ (c.a.o.) B1
- (b) (i) Differentiating $\sin at$ and $\cos at$ with respect to t , at least one
 correct M1
 candidate's x -derivative $= a \cos at$,
 candidate's y -derivative $= -a \sin at$ (both values) A1
 $\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1
 $\frac{dy}{dx} = -\tan at$ (c.a.o.) A1
- (ii) $\frac{d}{dt} \left[\frac{dy}{dx} \right] = -a \sec^2 at$ (f.t. candidate's expression for $\frac{dy}{dx}$) B1
 Use of $\frac{d^2y}{dx^2} = \frac{d}{dt} \left[\frac{dy}{dx} \right] \div \text{candidate's } x\text{-derivative}$ M1
 $\frac{d^2y}{dx^2} = -\sec^3 at$ (c.a.o.) A1

4. $f(x) = \cos x - 5x + 2$
 An attempt to check values or signs of $f(x)$ at $x = 0, x = \pi/4$ M1
 $f(0) = 3 > 0, f(\pi/4) = -1.22 < 0$
 Change of sign $\Rightarrow f(x) = 0$ has root in $(0, \pi/4)$ A1
 $x_0 = 0.6$
 $x_1 = 0.565067123$ B1
 $x_2 = 0.568910532$
 $x_3 = 0.568497677$
 $x_4 = 0.568542145 = 0.56854$ (x_4 correct to 5 decimal places) B1
 An attempt to check values or signs of $f(x)$ at $x = 0.568535, x = 0.568545$ M1
 $f(0.568535) = 1.563 \times 10^{-5} > 0, f(0.568545) = -3.975 \times 10^{-5} < 0$ A1
 Change of sign $\Rightarrow \alpha = 0.56854$ correct to five decimal places A1
Note: 'change of sign' must appear at least once

5. (a) $\frac{dy}{dx} = \frac{a + bx}{7 + 2x - 3x^2}$ (including $a = 1, b = 0$) M1
 $\frac{dy}{dx} = \frac{2 - 6x}{7 + 2x - 3x^2}$ A1
- (b) $\frac{dy}{dx} = e^{\tan x} \times f(x)$ ($f(x) \neq 1, 0$) M1
 $\frac{dy}{dx} = e^{\tan x} \times \sec^2 x$ A1
- (c) $\frac{dy}{dx} = 5x^2 \times f(x) + \sin^{-1} x \times g(x)$ ($f(x), g(x) \neq 1, 0$) M1
 $\frac{dy}{dx} = 5x^2 \times f(x) + \sin^{-1} x \times g(x)$
 (either $f(x) = \frac{1}{\sqrt{1-x^2}}$ or $g(x) = 10x$) A1
 $\frac{dy}{dx} = 5x^2 \times \frac{1}{\sqrt{1-x^2}} + 10x \times \sin^{-1} x$ A1

6. (a) (i) $\int 3e^{2-x/4} dx = k \times 3e^{2-x/4} + c$ ($k = 1, -1/4, 4, -4$) M1
 $\int 3e^{2-x/4} dx = -4 \times 3e^{2-x/4} + c$ A1
- (ii) $\int \frac{9}{(2x-3)^6} dx = \frac{k \times 9 \times (2x-3)^{-5}}{-5} + c$ ($k = 1, 2, 1/2$) M1
 $\int \frac{9}{(2x-3)^6} dx = \frac{9 \times (2x-3)^{-5}}{-5 \times 2} + c$ A1
- (iii) $\int \frac{7}{3x+1} dx = k \times 7 \times \ln|3x+1| + c$ ($k = 1, 3, 1/3$) M1
 $\int \frac{7}{3x+1} dx = 7/3 \times \ln|3x+1| + c$ A1

Note: The omission of the constant of integration is only penalised once.

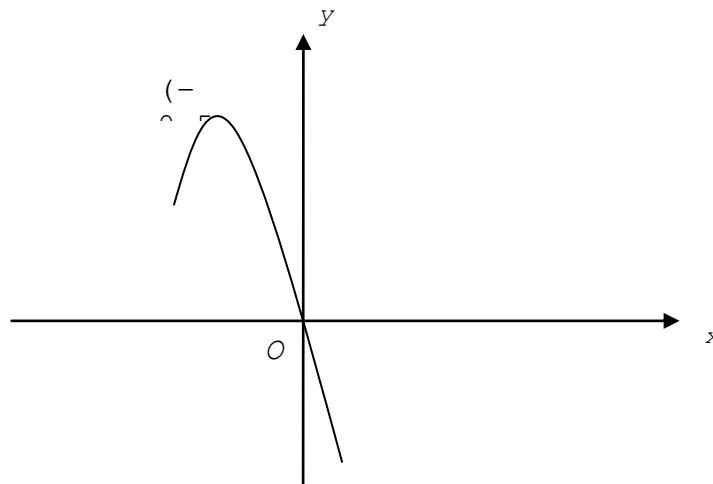
- (b) $\int \sin 2x dx = k \times \cos 2x$ ($k = -1, -2, 1/2, -1/2$) M1
 $\int \sin 2x dx = -\frac{1}{2} \times \cos 2x$ A1
 $k \times (\cos 2a - \cos 0) = 1/4$
(f.t. candidate's value for k) M1
 $\cos 2a = 1/2$ (c.a.o.) A1
 $a = \pi/6$ (f.t. $\cos 2a = b$ provided both M's are awarded) A1

7. (a) $9|x-3| = 6$ B1
 $x-3 = \pm 2/3$ (f.t. candidate's $a|x-3| = b$,
with at least one of a, b correct) B1
 $x = 11/3, 7/3$ (f.t. candidate's $a|x-3| = b$,
with at least one of a, b correct) B1
- (b) Trying to solve either $5x-2 \leq 3$ or $5x-2 \geq -3$ M1
 $5x-2 \leq 3 \Rightarrow x \leq 1$
 $5x-2 \geq -3 \Rightarrow x \geq -1/5$ (both inequalities) A1
Required range: $-1/5 \leq x \leq 1$ (f.t. one slip) A1

Alternative mark scheme

- $(5x-2)^2 \leq 9$ (forming and trying to solve quadratic) M1
Critical points $x = -1/5$ and $x = 1$ A1
Required range: $-1/5 \leq x \leq 1$ (f.t. one slip) A1

8.



Concave down curve passing through the origin with maximum point in the second quadrant B1
 x -coordinate of stationary point = -0.5 B1
 y -coordinate of stationary point = 8 B1

9. (a) (i) $f'(x) = \frac{(x^2 + 5) \times f(x) - (x^2 + 3) \times g(x)}{(x^2 + 5)^2}$ ($f(x), g(x) \neq 1$) M1
 $f'(x) = \frac{(x^2 + 5) \times 2x - (x^2 + 3) \times 2x}{(x^2 + 5)^2}$ A1
 $f'(x) = \frac{4x}{(x^2 + 5)^2}$ (c.a.o.) A1
 $f'(x) < 0$ since numerator is negative and denominator is positive B1
- (ii) $R(f) = (\sqrt[3]{5}, 1)$ B1 B1
- (b) (i) $x^2 = \frac{3-5y}{y-1}$ (o.e.) (condone any incorrect signs) M1
 $x = (\pm) \sqrt{\frac{3-5y}{y-1}}$ (f.t. at most one incorrect sign) A1
 $x = - \sqrt{\frac{3-5y}{y-1}}$ (f.t. at most one incorrect sign) A1
 $f^{-1}(x) = - \sqrt{\frac{3-5x}{x-1}}$ (c.a.o.) A1
- (ii) $R(f^{-1}) = (-\infty, 0), D(f^{-1}) = (\sqrt[3]{5}, 1),$
 (both intervals, f.t. candidate's $R(f)$) B1

10. $gg(x) = (3(g(x))^2 + 7)^{1/2}$ or $gg(x) = g((3x^2 + 7)^{1/2})$ M1
 $gg(x) = (3(3x^2 + 7) + 7)^{1/2}$ A1
 An attempt to solve the equation by squaring both sides M1
 $gg(x) = 8 \Rightarrow 9x^2 = 36$ (o.e.) (c.a.o.) A1
 $x = \pm 2$ (c.a.o.) A1