Mathematics C3

1. (*a*)

0	1		
$\pi/12$	0·933012701		
$\pi/6$	0·75		
$\pi/4$	0·5	(5 values correct)	B2
$\pi/3$	0·25	(3 or 4 values correct)	B1

Correct formula with $h = \pi/12$ M1

$I \approx \dfrac{\pi/12}{3} \times \{1 + 0{\cdot}25 + 4(0{\cdot}933012701 + 0{\cdot}5) + 2(0{\cdot}75)\}$

$I \approx 8{\cdot}482050804 \times (\pi/12) \div 3$

$I \approx 0{\cdot}740198569$

$I \approx 0{\cdot}7402$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

(*b*) $\displaystyle\int_0^{\pi/3} \sin^2 x\,dx = \int_0^{\pi/3} 1\,dx - \int_0^{\pi/3} \cos^2 x\,dx$ M1

$\displaystyle\int_0^{\pi/3} \sin^2 x\,dx = 0{\cdot}3070$ (f.t. candidate's answer to (*a*)) A1

Note: Answer only with no working shown earns 0 marks

2. (*a*) e.g. $\theta = \pi/2$, $\phi = \pi$

$\sin(\theta + \phi) = -1$ (choice of θ, ϕ and one correct evaluation) B1

$\sin\theta + \sin\phi = 1$ (both evaluations correct but different) B1

(*b*) $\sec^2\theta + 8 = 4(\sec^2\theta - 1) + 5\sec\theta.$

(correct use of $\tan^2\theta = \sec^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in $\sec\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sec\theta + b)(c\sec\theta + d)$, with $a \times c$ = candidate's coefficient of $\sec^2\theta$ and $b \times d$ = candidate's constant m1

$3\sec^2\theta + 5\sec\theta - 12 = 0 \Rightarrow (3\sec\theta - 4)(\sec\theta + 3) = 0$

$\Rightarrow \sec\theta = \dfrac{4}{3}$, $\sec\theta = -3$

$\Rightarrow \cos\theta = \dfrac{3}{4}$, $\cos\theta = -\dfrac{1}{3}$ (c.a.o.) A1

$\theta = 41{\cdot}41°, 318{\cdot}59°$ B1

$\theta = 109{\cdot}47°, 250{\cdot}53°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\cos\theta$ = +, –, f.t. for 3 marks, $\cos\theta$ = –, –, f.t. for 2 marks
$\cos\theta$ = +, +, f.t. for 1 mark

3. (*a*) (i) candidate's x-derivative = $6t$,
candidate's y-derivative = $6t^5 - 12t^2$
(at least two of the three terms correct) B1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{6t^5 - 12t^2}{6t}$ (c.a.o.) A1

(ii) $\frac{6t^5 - 12t^2}{6t} = \frac{7}{2}$ (f.t. candidate's expression from (i)) M1

$2t^4 - 4t - 7 = 0$

(convincing) A1

(*b*) $f(t) = 2t^4 - 4t - 7$
An attempt to check values or signs of $f(t)$ at $t = 1$, $t = 2$ M1
$f(1) = -9 < 0$, $f(2) = 17 > 0$
Change of sign $\Rightarrow f(t) = 0$ has root in (1, 2) A1
$t_0 = 1{\cdot}6$
$t_1 = 1{\cdot}608861654$ (t_1 correct, at least 5 places after the point) B1
$t_2 = 1{\cdot}609924568$
$t_3 = 1{\cdot}610051919$
$t_4 = 1{\cdot}610067175 = 1{\cdot}61007$ (t_4 correct to 5 decimal places) B1
An attempt to check values or signs of $f(t)$ at $t = 1{\cdot}610065$,
$t = 1{\cdot}610075$ M1
$f(1{\cdot}610065) = -1{\cdot}25 \times 10^{-4} < 0$, $f(1{\cdot}610075) = 1{\cdot}69 \times 10^{-4} > 0$ A1
Change of sign $\Rightarrow \alpha = 1{\cdot}61007$ correct to five decimal places A1

Note: 'Change of sign' must appear at least once.

4. $\frac{d}{dx}(x^2y^2) = x^2 \times 2y\frac{dy}{dx} + 2x \times y^2$ B1

$\frac{d}{dx}(2y^3) = 6y^2 \times \frac{dy}{dx}$ B1

$\frac{d}{dx}(x^4 - 2x + 6) = 4x^3 - 2$ B1

$x = 2, y = 3 \Rightarrow \frac{dy}{dx} = \frac{66}{30} = \frac{11}{5}$ (o.e.) (c.a.o.) B1

5. (*a*) $\frac{dy}{dx} = \frac{4}{1+(4x)^2}$ **or** $\frac{1}{1+(4x)^2}$ **or** $\frac{4}{1+4x^2}$ M1

$\frac{dy}{dx} = \frac{4}{1+16x^2}$ A1

(*b*) $\frac{dy}{dx} = e^{x^3} \times f(x)$ $(f(x) \neq 1)$ M1

$\frac{dy}{dx} = 3x^2 \times e^{x^3}$ A1

(*c*) $\frac{dy}{dx} = x^5 \times f(x) + \ln x \times g(x)$ $(f(x), g(x) \neq 1)$ M1

$\frac{dy}{dx} = x^5 \times f(x) + \ln x \times g(x)$ (either $f(x) = 1/x$ or $g(x) = 5x^4$) A1

$\frac{dy}{dx} = x^4 + 5x^4 \times \ln x$ (c.a.o.) A1

(*d*) $\frac{dy}{dx} = \frac{(5-4x^2) \times f(x) - (3-2x^2) \times g(x)}{(5-4x^2)^2}$ $(f(x), g(x) \neq 1)$ M1

$\frac{dy}{dx} = \frac{(5-4x^2) \times f(x) - (3-2x^2) \times g(x)}{(5-4x^2)^2}$

(either $f(x) = -4x$ or $g(x) = -8x$) A1

$\frac{dy}{dx} = \frac{4x}{(5-4x^2)^2}$ (c.a.o.) A1

6. (*a*) (i) $\int \sin(x/4)\,dx = k \times \cos(x/4) + c$ $(k = -1, 4, \mathbf{-4}, -\frac{1}{4})$ M1

$\int \sin(x/4)\,dx = -4 \times \cos(x/4) + c$ A1

(ii) $\int e^{2x/3}dx = k \times e^{2x/3} + c$ $(k = 1, \frac{2}{3}, \mathbf{\frac{3}{2}})$ M1

$\int e^{2x/3}dx = \frac{3}{2} \times e^{2x/3} + c$ A1

(iii) $\int \frac{7}{8x-2}\,dx = k \times 7 \times \ln|8x-2| + c$ $(k = 1, 8, \mathbf{\frac{1}{8}})$ M1

$\int \frac{7}{8x-2}\,dx = \frac{1}{8} \times 7 \times \ln|8x-2| + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int (5x + 4)^{-1/2}\,dx = k \times \frac{(5x+4)^{1/2}}{1/2}$ ($k = 1, 5, \frac{1}{5}$) M1

$\int_1^9 3 \times (5x + 4)^{-1/2}\,dx = \left[3 \times \frac{1}{5} \times \frac{(5x+4)^{1/2}}{1/2}\right]_1^9$ A1

A correct method for substitution of limits in an expression of the form $m \times (5x + 4)^{1/2}$ M1

$\int_1^9 3 \times (5x + 4)^{-1/2}\,dx = \frac{42}{5} - \frac{18}{5} = \frac{24}{5} = 4{\cdot}8$

(f.t. only for solutions of 24 and 120 from $k = 1, 5$ respectively) A1

Note: Answer only with no working shown earns 0 marks

7. (*a*) Trying to solve either $4x - 5 \geq 3$ or $4x - 5 \leq -3$ M1

$4x - 5 \geq 3 \Rightarrow x \geq 2$

$4x - 5 \leq -3 \Rightarrow x \leq \frac{1}{2}$ (solving both inequalities correctly) A1

Required range: $x \leq \frac{1}{2}$ or $x \geq 2$ (f.t. one slip) A1

Alternative mark scheme

$(4x - 5)^2 \geq 9$ (forming and trying to solve quadratic) M1

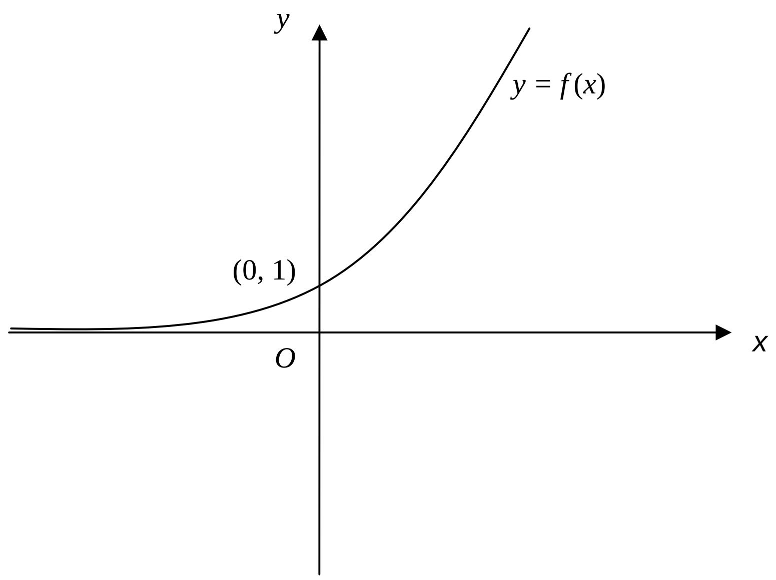
Critical values $x = \frac{1}{2}$ and $x = 2$ A1

Required range: $x \leq \frac{1}{2}$ or $x \geq 2$ (f.t. one slip) A1

(*b*) $(3|x| + 1)^{1/3} = 4 \Rightarrow 3|x| + 1 = 4^3$ M1

$x = \pm 21$ A1

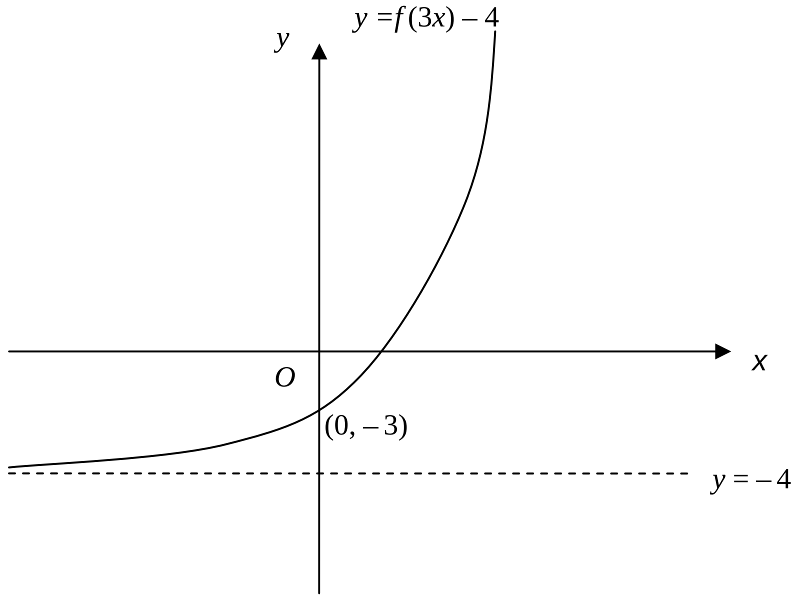
8. (*a*)



Correct shape, including the fact that the *x*-axis is an asymptote for $y = f(x)$ at $-\infty$ B1

$y = f(x)$ cuts *y*-axis at (0, 1) B1

(*b*) (i)



Correct shape, including the fact that $y = -4$ is an asymptote for $y = f(3x) - 4$ at $-\infty$ B1

(ii) $y = f(3x) - 4$ at cuts y-axis at $(0, -3)$ B1

(iii) $e^{3x} = 4 \Rightarrow 3x = \ln 4$ M1
$x = 0{\cdot}462$ A1

Note: Answer only with no working shown earns M0 A0

9. (*a*) $y = 3 - \dfrac{1}{\sqrt{x-2}} \Rightarrow 3 \pm y = \pm\dfrac{1}{\sqrt{x-2}}$ (separating variables) M1

$x - 2 = \dfrac{1}{(3 \pm y)^2}$ or $\dfrac{1}{(y \pm 3)^2}$ m1

$x = 2 + \dfrac{1}{(3 - y)^2}$ (c.a.o.) A1

$f^{-1}(x) = 2 + \dfrac{1}{(3 - x)^2}$ (f.t. one slip) A1

(*b*) $D(f^{-1}) = [2{\cdot}5, 3)$
$[2{\cdot}5$ B1
$3)$ B1

10. (*a*) $R(f) = [3 + k, \infty)$ B1

(*b*) $3 + k \geq -2$ M1

$k \geq -5$ ($\Rightarrow$ least value of k is -5)

(f.t. candidate's $R(f)$ provided it is of form $[a, \infty)$) A1

(*c*) (i) $gf(x) = (3x + k)^2 - 6$ B1

(ii) $(3 \times 2 + k)^2 - 6 = 3$

(substituting 2 for x in candidate's expression for $gf(x)$ and putting equal to 3) M1

Either $k^2 + 12k + 27 = 0$ or $(6 + k)^2 = 9$ (c.a.o.) A1

$k = -3, -9$ (f.t. candidate's quadratic in k) A1

$k = -3$ (c.a.o.) A1