GCE AS/A level

0983/01

MATHEMATICS S1
Statistics

A.M. THURSDAY, 31 May 2012

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

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1. The events A and B are such that

$$P(A) = 0{\cdot}5,\ P(B) = 0{\cdot}3.$$

(a) Evaluate $P(A \cup B)$ when

(i) A, B are mutually exclusive,
(ii) A, B are independent. [5]

(b) Given that $P(A \cup B) = 0{\cdot}7$, find the value of $P(B|A)$. [3]

2. The random variable X has mean 8 and variance 2.

(a) Find the value of $E(X^2)$. [2]

(b) If $Y = 3X + 4$, find the mean and variance of Y. [4]

3. A bag contains 9 balls, of which 1 is red, 3 are blue and 5 are white. Ann selects 3 balls at random from the bag without replacement. Calculate the probability that

(a) no white balls are selected, [2]

(b) exactly 2 white balls are selected, [2]

(c) the selection contains 2 balls of the same colour and 1 ball of a different colour. [3]

4. Charlie and Dave regularly play chess against each other. When they play each other, Charlie wins with probability 0·75 and successive games are independent.

(a) One weekend they play 10 games against each other. Determine the probability that Charlie wins

(i) exactly 4 games,

(ii) more than 5 games. [5]

(b) The probability that a game lasts for less than one hour is 0·08.
They play 45 games against each other over a holiday period. Use a Poisson approximation to determine the probability that more than 6 of these games last for less than one hour. [3]

5. In a certain population, 60% are male and 40% are female. It is known that 8% of males are colour-blind and 3% of females are colour-blind. A member of the population is selected at random.

(a) Find the probability that this person is colour-blind. [3]

(b) Given that this person is colour-blind, find the probability that the person is female. [3]

6. Sue and Tim play the following game. They throw a fair dice alternately, starting with Sue, and the winner is the first to obtain a 6.

(a) Write down the probability that Sue wins with her first throw. [1]

(b) Find the probability that Sue wins with her second throw. [2]

(c) Write down the first three terms of the infinite geometric series for the probability that Sue wins the game. [2]

(d) Hence find the probability that Sue wins the game. [2]

7. Jim sells jars of honey at a Saturday market. The demand each Saturday for his jars can be modelled by a Poisson distribution with mean 12.

(a) Find the probability that the demand on a randomly chosen Saturday is

(i) exactly 10 jars,

(ii) more than 10 jars. [4]

(b) Jim wants the probability of being able to satisfy the demand for his honey to be at least 0·95. Find the minimum number of jars that he needs to take to the market. [2]

8. The probability distribution of the discrete random variable X is given by

x	2	3	4
$P(X = x)$	$0{\cdot}3 - \theta$	2θ	$0{\cdot}7 - \theta$

(a) State the range of possible values of the constant θ. [2]

(b) Show that $E(X)$ is independent of θ. [2]

(c) You are now given that the standard deviation of X is 0·8.

(i) Find the value of θ.

(ii) Two independent observations X_1, X_2 are taken from the distribution of X. Calculate $P(X_1 + X_2 = 6)$. [8]

TURN OVER.

9. The continuous random variable X has probability density function f given by

$$f(x) = \frac{1}{10}(2x + 3x^2) \qquad \text{for } 1 \leqslant x \leqslant 2,$$
$$f(x) = 0 \qquad \text{otherwise.}$$

(a) (i) Determine $E(X)$.

(ii) Show that

$$E(X^2) = 2{\cdot}61$$

and hence calculate the variance of X correct to 2 decimal places. [8]

(b) (i) Find an expression for $F(x)$, valid for $1 \leqslant x \leqslant 2$, where F denotes the cumulative distribution function of X.

(ii) Determine $P(X \leqslant 1{\cdot}4)$.

(iii) Hence, giving a reason, state whether the lower quartile of X is less than or greater than 1·4. [7]

