WJEC CBAC

GCE AS/A level

983/01

MATHEMATICS S1

Statistics

P.M. WEDNESDAY, 26 January 2011

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Use black ink or ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Jean has two fair dice, each in the shape of a regular tetrahedron. The four faces of each dice are numbered 1, 2, 3, 4 respectively. She throws the two dice simultaneously and the score on each dice is defined as the number on the face in contact with the table.

(a) Write down the sample space. [2]

(b) Calculate the probability that

(i) the sum of the scores on the two dice is 6,

(ii) the scores on the two dice are consecutive integers. [4]

2. The events A and B are such that

$$P(A) = P(B) = p \text{ and } P(A \cup B) = 0{\cdot}64.$$

(a) Given that A and B are mutually exclusive, find the value of p. [2]

(b) Given, instead, that A and B are independent, show that

$$25p^2 - 50p + k = 0,$$

where k is a constant whose value should be found.
Hence find the value of p. [5]

3. David owns a small collection of 12 CDs which he classifies into three types, classical, pop and jazz. He has 6 classical CDs, 4 pop CDs and 2 jazz CDs. One evening, he chooses 3 CDs at random from his collection to play. Find the probability that he chooses

(a) one of each type, [3]

(b) all classical, [2]

(c) all of the same type. [3]

4. The random variable X has the binomial distribution $\mathrm{B}(n, 0{\cdot}2)$. Given that the mean of X is twice its standard deviation, find the value of n. [5]

5. The number of cars, X, using a small all-day car park on a weekday may be assumed to follow a Poisson distribution with mean 15.

(a) Determine

(i) $P(X = 8)$,

(ii) $P(10 \leqslant X \leqslant 20)$. [5]

(b) The charge for all-day parking is £8 and the attendant who collects the money is paid £50 per day. The daily profit from the car park is denoted by £Y.

(i) Find an expression for Y in terms of X.

(ii) Find the mean and variance of Y. [5]

6. In a certain country, 80% of the defendants being tried in the Law Courts actually committed the crime. For those who committed the crime, the probability of being found guilty is 0·9. For those who did not commit the crime, the probability of being found guilty is 0·05.

(a) Find the probability that a randomly chosen defendant is found guilty. [3]

(b) Given that a randomly chosen defendant is found guilty, find the probability that this defendant committed the crime. [3]

7. The probability distribution of the discrete random variable X is given in the following table.

x	1	2	3
$P(X = x)$	$0{\cdot}4 - \alpha$	2α	$0{\cdot}6 - \alpha$

(a) (i) State the range of possible values of the constant α.

(ii) Show that $E(X)$ is independent of α.

(iii) Given that $\mathrm{Var}(X) = 0{\cdot}66$, find the value of α. [7]

(b) Assume now that $\alpha = 0{\cdot}25$. Given that X_1, X_2 are two independent values of X, determine the value of $P(X_1 = X_2)$. [4]

8. Wine glasses are packed in boxes, each containing 20 glasses. Each glass has a probability of 0·05 of being broken in transit, independently of all other glasses.

(a) Let X denote the number of glasses in a box broken in transit.

(i) State the distribution of X.

(ii) **Without** the use of tables, calculate $P(X = 1)$.

(iii) **Using tables**, determine the value of $P(X \geqslant 3)$. [5]

(b) A retailer buys 10 of these boxes. Use a Poisson approximation to find the probability that less than 5 of the 200 glasses are broken in transit. [3]

9. The continuous random variable X has probability density function f given by

$$f(x) = \frac{1}{6}(x + 1) \quad \text{for } 1 \leqslant x \leqslant 3,$$

$$f(x) = 0 \quad \text{otherwise.}$$

(a) Calculate $E(X)$. [4]

(b) (i) Find an expression for $F(x)$, valid for $1 \leqslant x \leqslant 3$, where F denotes the cumulative distribution function of X.

(ii) State the value of $F(4)$.

(iii) Evaluate $P(1{\cdot}5 \leqslant X \leqslant 2)$.

(iv) Find the median of X. [10]