# GCE AS/A level

983/01

# MATHEMATICS S1

## Statistics

A.M. TUESDAY, 15 June 2010

1½ hours

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

**INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

**1.** The independent events $A$ and $B$ are such that

$$P(A) = 0{\cdot}6, \quad P(B) = 0{\cdot}3.$$

Find the value of

*(a)* $P(A \cup B)$, [3]

*(b)* $P(A \cup B')$. [3]

**2.** The random variable $X$ has mean 4 and variance 2. The random variable $Y$ is given by

$$Y = 3X - 1.$$

*(a)* Find the mean and variance of $Y$. [4]

*(b)* Hence find the value of $E(Y^2)$. [2]

**3.** The number of customers arriving at a village shop during an interval of length $t$ minutes may be assumed to have a Poisson distribution with mean $0{\cdot}1t$.

*(a)* Find the probability that the number of customers arriving between 10 a.m. and 11 a.m. is

(i) exactly 3,

(ii) less than 5. [5]

*(b)* Given that the probability of no customers arriving during an interval of $t$ minutes is equal to $0{\cdot}25$, find the value of $t$ correct to two decimal places. [4]

**4.** Alan and Bill play a game with darts in which they throw a dart at the 'bull' on the dartboard alternately, starting with Alan, and the winner is the first to hit the 'bull'. Each time they throw a dart at the 'bull', Alan hits it with probability 0·2 and Bill hits it with probability 0·3.
Find the probability that

*(a)* Bill wins the game with his first throw, [2]

*(b)* Bill wins the game with his second throw, [2]

*(c)* Bill wins the game. [4]

**5.** Jack is taking part in a quiz programme. For each question in the quiz, four alternative answers are given, only one of which is correct. Jack has probability 0·6 of knowing the correct answer to a question, and when he does not know the correct answer he chooses one of the four answers at random.

*(a)* Calculate the probability that Jack gives the correct answer to a question. [3]

*(b)* Given that Jack gave the correct answer to a question, find the probability that he knew the correct answer. [3]

**6.** The probability distribution of the discrete random variable $X$ is given by

$$P(X = x) = kx \qquad \text{for } x = 1, 3, 5, 7,$$
$$P(X = x) = 0 \qquad \text{otherwise.}$$

*(a)* Show that $k = \frac{1}{16}$. [2]

*(b)* Determine

(i) $E(X)$,

(ii) $E\left(\frac{1}{X}\right)$. [5]

*(c)* Given that $X_1, X_2$ are two independent values of $X$, determine

(i) $P(X_1 + X_2 = 6)$,

(ii) $P(X_1 = X_2)$. [7]

**7.** Sheila buys two biased dice in a shop. Each time either of the dice is thrown, the probability of obtaining a six is 0·2.

*(a)* She throws one of the dice 50 times. Determine the probability that she obtains

(i) exactly 12 sixes,

(ii) at least 10 sixes. [5]

*(b)* She now throws the two dice simultaneously 200 times. Use a Poisson approximation to find the probability that between 5 and 10 (both inclusive) double sixes are obtained. [5]

**8.** The continuous random variable $X$ has probability density function $f$ given by

$$f(x) = kx(1 - x^2) \qquad \text{for } 0 \leqslant x \leqslant 1,$$
$$f(x) = 0 \qquad \text{otherwise,}$$

where $k$ is a constant.

*(a)* Show that $k = 4$. [3]

*(b)* Calculate $E(X)$. [4]

*(c)* (i) Find an expression for $F(x)$, valid for $0 \leqslant x \leqslant 1$, where $F$ denotes the cumulative distribution function of $X$.

(ii) Evaluate $P(0{\cdot}25 \leqslant X \leqslant 0{\cdot}75)$.

(iii) Find the median of $X$. [9]