Mathematics S1 January 2013

Solutions and Mark Scheme

Final Version

Ques	Solution	Mark	Notes
1(a)	Use of $P(A\cup B) + P(A\cap B) = P(A) + P(B)$	**M1**	
	Use of $P(A\cap B) = P(A)P(B)$	**m1**	
	$0.4 + 0.2P(B) = 0.2 + P(B)$	**A1**	
	$P(B) = 0.25$	**A1**	
(b)	EITHER		
	We require $P(A\cap B') + P(A'\cap B)$	**M1**	FT their P(B)
	$= 0.2 \times (1 - 0.25) + 0.25 \times (1 - 0.2)$	**A1**	
	$= 0.35$	**A1**	
	OR		
	We require $P(A\cup B) - P(A\cap B)$	**M1**	FT their P(B)
	$= 0.4 - 0.2\times0.25$	**A1**	
	$= 0.35$	**A1**	
2(a)	$E(X) = 3.2$, $Var(X) = 2.56$	**B1B1**	
	$E(Y) = 2 \times 3.2 + 5 = 11.4$ cao	**M1A1**	
	$Var(Y) = 4 \times 2.56 = 10.24$ cao	**M1A1**	
(b)	$Y = 11 \Rightarrow X = 3$	**B1**	
	$P(X = 3) = \binom{16}{3} \times 0.2^3 \times 0.8^{13} = 0.246$	**M1A1**	FT their derived value of X M0 if no working
3(a)	$P(2\text{ red}) = \frac{6}{11}\times\frac{5}{10}\times\frac{5}{9}\times3$ or $\binom{6}{2}\binom{5}{1}\div\binom{11}{3}$	**M1A1**	
	$= \frac{5}{11}$ (0.455)	**A1**	
(b)	$P(2\text{ green}) = \frac{4}{11}\times\frac{3}{10}\times\frac{7}{9}\times3$ or $\binom{4}{2}\binom{7}{1}\div\binom{11}{3}$	**M1A1**	
	$= \frac{14}{55}$ (0.255)	**A1**	
	$P(2\text{ the same}) = \frac{5}{11} + \frac{14}{55}$	**M1**	
	$= \frac{39}{55}$ (0.709)	**A1**	FT on their probs

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4(a)(i)	Poisson mean = 6	**B1**	
	P(4 arrivals) = $e^{-6} \times \frac{6^4}{4!} = 0.134$ cao	**M1A1**	Accept 0.2851 – 0.1512 or 0.8488 – 0.7149 M0 if no working
(ii)	EITHER		
	P(between 2 and 8) = 0.8472 – 0.0174	**B1B1**	
	or 0.9826 – 0.1528		
	= 0.8298 cao	**B1**	M0 if no working
	OR		
	P(between 2 and 8) = $\sum_{x=2}^{8} e^{-6} \times \frac{6^x}{x!}$	**M1**	
	= 0.0446 + 0.0892 + 0.1339 + 0.1606 + 0.1606 + 0.1377 + 0.1033	**A1**	M0 if no working
	= 0.83 cao	**A1**	
(b)	E(X) = 12	**B1**	
	$E(X^2) = \mathrm{E}(X) + [E(X)]^2 = 156$	**M1A1**	M1 requires Var(X) = E(X) FT their mean
5(a)(i)	Let X denote the number of seeds producing red flowers so that X is B(20,0.7) si	**B1**	
	P(X = 15) = $\binom{20}{15} \times 0.7^{15} \times 0.3^5$	**M1**	M0 if no working
	= 0.179	**A1**	Accept 0.4164 – 0.2375 or 0.7625 – 0.5836
(ii)	The number of seeds not producing red flowers, X', is B(20,0.3)	**M1**	
	We require P(X > 12) = P(X' < 8)	**m1**	
(b)	= 0.7723	**A1**	
	Number of seeds producing white flowers Y is B(150,0.09) ≈ Poi(13.5) si	**B1**	
	P(Y = 10) = $e^{-13.5} \times \frac{13.5^{10}}{10!}$	**M1**	Do not accept use of interpolation in tables
	= 0.076	**A1**	M0 if no working

Ques	Solution	Mark	Notes
6(a)	$k(2 + 3 + 4 + 5) = 1$ $14k = 1$	**M1**	
	$k = 1/14$	**A1**	Must be convincing
(b)	$\mathrm{E}(X) = \frac{2}{14}\times1+\frac{3}{14}\times2+\frac{4}{14}\times3+\frac{5}{14}\times4$	**M1**	
	$= \frac{20}{7}$ (2.86)	**A1**	Accept $40k$
	$E(X^2) = \frac{2}{14}\times1+\frac{3}{14}\times4+\frac{4}{14}\times9+\frac{5}{14}\times16$ (65/7)	**B1**	Accept in terms of k
	$\mathrm{Var}(X) = 65/7-(20/7)^2$	**M1**	
	= 1.12 (55/49)	**A1**	Numerical value required
(c)	The possibilities are $(x_1, x_2) = (1,2),(2,3),(3,4)$ si	**B1**	
	$\text{Prob} = \frac{2}{14}\times\frac{3}{14}+\frac{3}{14}\times\frac{4}{14}+\frac{4}{14}\times\frac{5}{14}$	**M1A1**	
	= 0.194 (19/98)	**A1**	Numerical value required
7(a)	$\mathrm{P}(+) = 0.02\times0.96+0.98\times0.01$ = 0.029	**M1A1** **A1**	M1 Use of Law of Total Prob (Accept tree diagram)
(b)(i)	$\mathrm{P}(\text{Disease}\mid+) = \frac{0.02\times0.96}{0.029}$	**B1B1**	FT denominator from (a) B1 num, B1 denom
(ii)	= 0.662 (96/145) cao	**B1**	
	EITHER $\mathrm{P}(+) = 0.662 \times 0.96 + 0.338 \times 0.01$ = 0.639	**M1A1** **A1**	M1 Use of Law of Total Prob (Accept tree diagram) FT from (b)(i)
	OR $\mathrm{P}(+) = \frac{0.02\times0.96^2+0.98\times0.01^2}{0.029}$	**M1A1**	M1 valid attempt to use conditional probability
	= 0.639	**A1**	

Ques	Solution	Mark	Notes
8(a)(i)	$\mathrm{P}(0.25 \le X \le 0.75) = F(0.75) - F(0.25)$ $= 0.6875 \quad (11/16)$	**M1** **A1**	
(ii)	The median satisfies $2m^2 - m^4 = 0.5$ $2m^4 - 4m^2 + 1 = 0$	**B1**	
(iii)	$(\text{Root}) = \dfrac{4 \pm \sqrt{16-8}}{4} \quad (= 0.29289..)$ $m = \sqrt{0.29289..} = 0.541$	**M1A1** **M1A1**	Condone the omission of the redundant root
(b)(i)	$f(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}(2x^2 - x^4)$ $= 4x - 4x^3$	**M1** **A1**	
(ii)	$E\left(\sqrt{X}\right) = \int_0^1 \sqrt{x}(4x - 4x^3)\mathrm{d}x$ $= \left[4x^{5/2} \times \dfrac{2}{5} - 4x^{9/2} \times \dfrac{2}{9}\right]_0^1$ $= \dfrac{32}{45} \quad (0.711)$	**M1A1** **A1** **A1**	M1 for the integral of $\sqrt{x}\,f(x)$ A1 for completely correct although limits may be left until 2nd line. FT their $f(x)$ from (b)(i) if M1 awarded there.