Mathematics S1

1. (a) $P(A \cap B) = 0.6 \times 0.3$ B1

$P(A \cup B) = 0.6 + 0.3 - 0.6 \times 0.3$ M1

$= 0.72$ A1

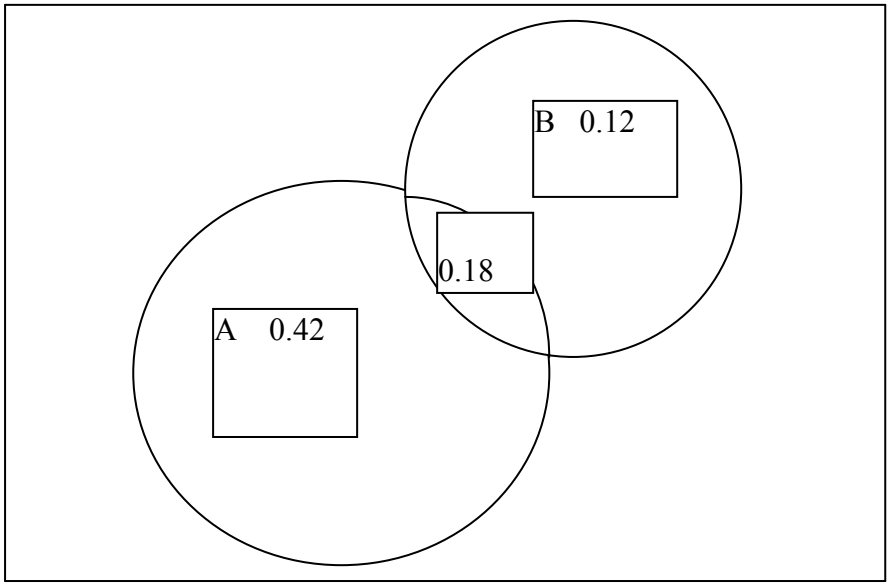
(b) EITHER

$P(B') = 1 - P(B) = 0.7$ B1

$P(A \cup B') = P(A) + P(B') - P(A)P(B')$ M1

$= 0.6 + 0.7 - 0.6 \times 0.7 = 0.88$ A1

OR



Correct Venn diagram B1

Required prob $= 1 - 0.12 = 0.88$ M1A1

2. (a) $E(Y) = 3 \times 4 - 1 = 11$ M1A1

$Var(Y) = 9 \times 2 = 18$ M1A1

(b) $E(Y^2) = Var(Y) + \{E(Y)\}^2$ M1

$= 18 + 121 = 139$ A1

[FT 1 arithmetic slip in (a)]

3. (a) Mean = 6 si B1

(i) Prob $= e^{-6} \times \frac{6^3}{3!} = 0.0892$ M1A1

(ii) Prob $= 1 - 0.7149 = 0.2851$ M1A1

[FT on mean]

(b) Prob of no arrivals $= e^{-0.1t}$ B1

Attempting to solve $e^{-0.1t} = 0.25$ M1

$-0.1t \log e = \log 0.25$ A1

$$t = -\frac{\log 0.25}{0.1 \log e} = 13.86$$ A1

[Award 2 marks for 14 using tables]

4. (a) Prob wins on 1st throw = $0.8 \times 0.3 = 0.24$ M1A1

(b) Prob wins on 2nd throw = $0.8 \times 0.7 \times 0.8 \times 0.3 = 0.1344$ M1A1
[FT from (a) if M1 awarded in (a)]

(c) Prob wins = $0.24 + 0.24 \times 0.56 + 0.24 \times 0.56^2 + ...$ M1A1

$= \dfrac{0.24}{1 - 0.56} = 6/11 \ \ (0.55)$ M1A1

[For candidates who solve for Bill first, award M0A0 for (a), M1A1 for 0.168 in (b), M1A1 for $0.3 + 0.3 \times 0.56 + ...$ and M1A1 for $0.3/(1 - 0.56) = 15/22$ (0.68) in (c)]

5. (a) P(correct ans) = $0.6 \times 1 + 0.4 \times 0.25$ M1A1

$= 0.7$ A1

[Award M1 if 1 and 0.25 reversed]

(b) Reqd prob = $\dfrac{0.6}{0.7}$ [FT denominator from (a) if answer < 1] B1B1

$= \dfrac{6}{7}$ cao B1

6. (a) $\sum p_x = 16k = 1$ so $k = 1/16$ M1A1

(b) (i) $E(X) = \dfrac{1}{16}(1 \times 1 + 3 \times 3 + 5 \times 5 + 7 \times 7)$ M1

$= 5.25$ [Accept $84k$] A1

(ii) $\mathrm{E}\left(\dfrac{1}{X}\right) = \dfrac{1}{16}\left(1 \times \dfrac{1}{1} + 3 \times \dfrac{1}{3} + 5 \times \dfrac{1}{5} + 7 \times \dfrac{1}{7}\right)$ M1A1

$= 0.25$ [Accept $4k$] A1

(c) (i) Possibilities are 1,5 and 3,3 si B1

Prob = $\dfrac{1}{256}(1 \times 5 + 5 \times 1 + 3 \times 3)$ M1A1

[Award M1 for 2 or 3 terms]

$= \dfrac{19}{256}$ (0.074) [Accept $19/k^2$] A1

(ii) Possibilities are 1,1 ; 3,3 ; 5,5 ; 7,7 si B1

Prob = $\dfrac{1}{256}(1^2 + 3^2 + 5^2 + 7^2)$ M1

[Award M1 for 3 or 4 terms]

$= 0.328$ (21/64) [Accept $84/k^2$] A1

7. (a) Number of 6s obtained, X, is B(50,0.2) B1

(i) Prob $= \binom{50}{12} \times 0.2^{12} \times 0.8^{38} = 0.1033$

or $0.8139 - 0.7107$ or $0.2893 - 0.1861 = 0.1032$ M1A1

(ii) P(at least 10) = 0.5563 or $1 - 0.4437 = 0.5563$ M1A1

(b) Prob of 2 6s $= 0.2^2 = 0.04$ B1

X is now B(200, 0.04) which is approx P(8) B1

[FT p from previous line]

$P(5 \le X \le 10) = 0.8159 - 0.0996$ or $0.9004 - 0.1841$ B1B1

$= 0.7163$ cao B1

8. (a) $\int_0^1 kx(1-x^2)\mathrm{d}x = 1$ M1

Integral $= k\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$ B1

[Limits must appear somewhere for M1]

$= \frac{k}{4}$ A1

so $k = 4$

(b) $E(X) = \int_0^1 x.4x(1-x^2)\mathrm{d}x$ M1A1

$= \left[\frac{4x^3}{3} - \frac{4x^5}{5}\right]_0^1$ A1

[Limits not required until 2nd line]

$= \frac{8}{15}$ A1

(c) (i) $F(x) = \int_0^x 4t(1-t^2)\mathrm{d}t$ M1

[Limits not required for M1]

$= \left[2t^2 - t^4\right]_0^x$ A1

[Limits must appear here]

$= 2x^2 - x^4$ A1

(ii) Prob = $F(0.75) - F(0.25)$

$= 2\times 0.75^2 - 0.75^4 - (2\times 0.25^2 - 0.25^4)$ M1

$= 0.6875$ A1

[FT if M1 awarded in (c)(i) and answer sensible]

(iii) The median m satisfies

$2m^2 - m^4 = 0.5$ M1

[FT from (c)(i) if M1 awarded there]

$2m^4 - 4m^2 + 1 = 0$ A1

$m^2 = \dfrac{4 \pm \sqrt{8}}{4}$ m1

$m = 0.541$ cao A1