GCE AS/A level

0973/01

MATHEMATICS – C1
Pure Mathematics

A.M. MONDAY, 13 January 2014

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Calculators are **not** allowed for this paper.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

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1. The points A and B have coordinates $(6, -2)$ and $(4, 1)$, respectively.
The line L_1 passes through the point B and is perpendicular to AB.

(a) (i) Find the gradient of AB.

(ii) Find the equation of L_1. [5]

(b) The line L_2 passes through A and has equation $x - 8y - 22 = 0$.
The lines L_1 and L_2 intersect at the point C.

(i) Show that C has coordinates $(-2, -3)$.

(ii) Find the coordinates of the mid-point of AC.

(iii) Find the area of triangle ABC, simplifying your answer. [9]

2. Simplify $\dfrac{3\sqrt{3} - 2\sqrt{5}}{2\sqrt{3} + \sqrt{5}}$. [4]

3. The curve C has equation $y = \dfrac{20}{x} + 2x^2 - 11$. The point P has coordinates $(2, 7)$ and lies on C.

Find the equation of the **normal** to C at P. [6]

4. Show that $x^2 + 1{\cdot}6x - 24{\cdot}36$ may be expressed in the form $(x + p)^2 - 25$, where p is a constant whose value is to be found.
Hence solve the quadratic equation $x^2 + 1{\cdot}6x - 24{\cdot}36 = 0$. [5]

5. *(a)* **Use the binomial theorem** to express $(1 + \sqrt{6})^5$ in the form $a + b\sqrt{6}$, where a, b are integers whose values are to be found. [5]

(b) The coefficient of x^2 in the expansion of $(1 + 3x)^n$ is 495. Given that n is a positive integer, find the value of n. [3]

6. Given that the quadratic equation

$$(2k - 3)x^2 + 8x + (2k + 3) = 0$$

has no real roots, show that k satisfies an inequality of the form

$$m - nk^2 < 0,$$

where m, n are integers whose values are to be found.

Hence find the range of values of k such that the quadratic equation

$$(2k - 3)x^2 + 8x + (2k + 3) = 0$$

has no real roots. [6]

7. **Figure 1** shows a sketch of the graph of $y = f(x)$. The graph has a maximum point at (2, 6) and intersects the x-axis at the points (–4, 0) and (8, 0).

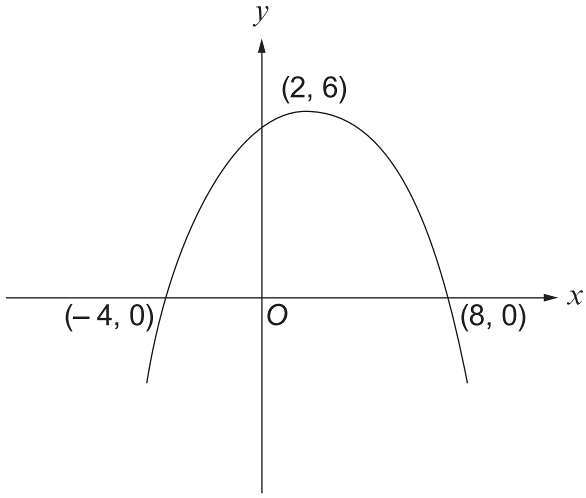

Figure 1

(a) Sketch the graph of $y = f(x - 3)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. [3]

(b) **Figure 2** shows a sketch of the graph having **one** of the following equations with an appropriate value of p, q or r.

$y = f(x) + p$, where p is a constant
$y = f(qx)$, where q is a constant
$y = rf(x)$, where r is a constant

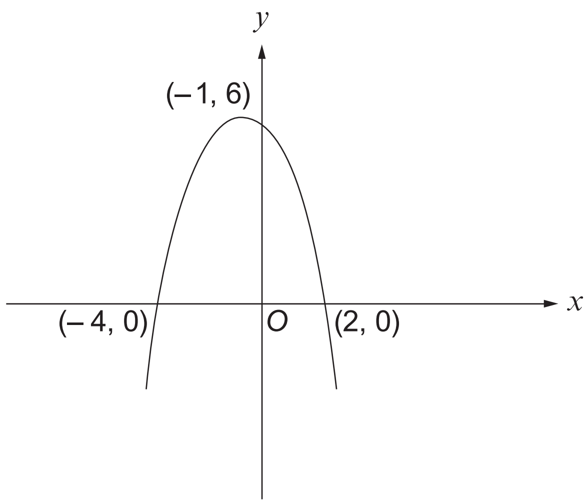

Figure 2

Write down the equation of the graph sketched in **Figure 2**, together with the value of the corresponding constant. [2]

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8. *(a)* Given that $y = 7x^2 - 6x - 3$, find $\frac{dy}{dx}$ from first principles. [5]

(b) Given that $y = ax^{\frac{4}{3}} + 24x^{\frac{1}{2}}$ and that $\frac{dy}{dx} = \frac{11}{2}$ when $x = 64$,

find the value of the constant a. [4]

9. *(a)* When $ax^3 + 13x^2 - 10x - 24$ is divided by $x + 3$, the remainder is -39.

Write down an equation satisfied by a and hence show that $a = 6$. [2]

(b) Solve the equation $6x^3 + 13x^2 - 10x - 24 = 0$. [6]

10. The curve *C* has equation

$$y = -2x^3 + 12x^2 - 18x + 5.$$

(a) Find the coordinates and the nature of each of the stationary points of *C*. [6]

(b) Sketch *C*, indicating the coordinates of each of the stationary points. [2]

(c) Given that the equation

$$-2x^3 + 12x^2 - 18x + 5 = k$$

has three distinct real roots, find the range of possible values for k. [2]