GCE AS/A level

973/01

MATHEMATICS C1

Pure Mathematics

A.M. WEDNESDAY, 18 May 2011

$1\frac{1}{2}$ hours

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ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Calculators are **not** allowed for this paper.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The points A and B have coordinates (3, 11) and (9, –1) respectively.
The line L_1 passes through the point B and is **perpendicular** to AB.

(a) Find the gradient of AB. [2]

(b) Find the equation of L_1 and simplify your answer. [4]

The line L_2 has equation $6x + 7y + 10 = 0$.
The lines L_1 and L_2 intersect at the point C.

(c) (i) Show that C has coordinates (3, –4).

(ii) Find the length of BC.

(iii) Find the coordinates of the mid-point of BC.

(iv) Write down the equation of the line AC. [7]

2. Simplify

(a) $\dfrac{9}{\sqrt{3}-1} + \dfrac{7}{\sqrt{3}+1}$, [4]

(b) $\dfrac{90}{\sqrt{3}} - \sqrt{6} \times \sqrt{8} - \left(2\sqrt{3}\right)^3$. [4]

3. The curve C has equation $y = 3x^2 - 9x + 1$. The point P, whose x-coordinate is 2, lies on the curve C. Find the equation of the tangent to C at P. [5]

4. Express $-x^2 + 6x - 7$ in the form $-(x + a)^2 + b$, where the values of the constants a and b are to be found.
Hence sketch the graph of $y = -x^2 + 6x - 7$, indicating the coordinates of its stationary point. [4]

5. The curve C has equation

$$y = x^2 + (4k + 3)x + 7,$$

and the line L has equation

$$y = x + k,$$

where k is a constant.
Given that L and C intersect at two distinct points,

(a) show that $4k^2 + 5k - 6 > 0$, [6]

(b) find the range of values of k satisfying this inequality. [3]

6. *(a)* Given that $y = 7x^2 - 5x + 2$, find $\frac{dy}{dx}$ from first principles. [5]

(b) Differentiate $4x^{\frac{2}{5}} - \frac{9}{x} - 6$ with respect to x. [2]

7. *(a)* Use the binomial theorem to expand $(3 + 2x)^4$, simplifying each term of the expansion. [4]

(b) In the binomial expansion of $\left(1 + \frac{x}{4}\right)^n$, the coefficient of x^2 is five times the coefficient of x.
Given that n is a positive integer, find the value of n. [4]

8. The polynomial $px^3 - x^2 - 31x + q$ has $x + 2$ as a factor. When the polynomial is divided by $x - 1$, the remainder is –36.

(a) Show that $p = 6$ and $q = -10$. [6]

(b) Factorise $6x^3 - x^2 - 31x - 10$. [3]

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9. Figure 1 shows a sketch of the graph of $y = f(x)$. The graph has a minimum point at $(-3, -4)$ and intersects the x-axis at the points $(-8, 0)$ and $(2, 0)$.

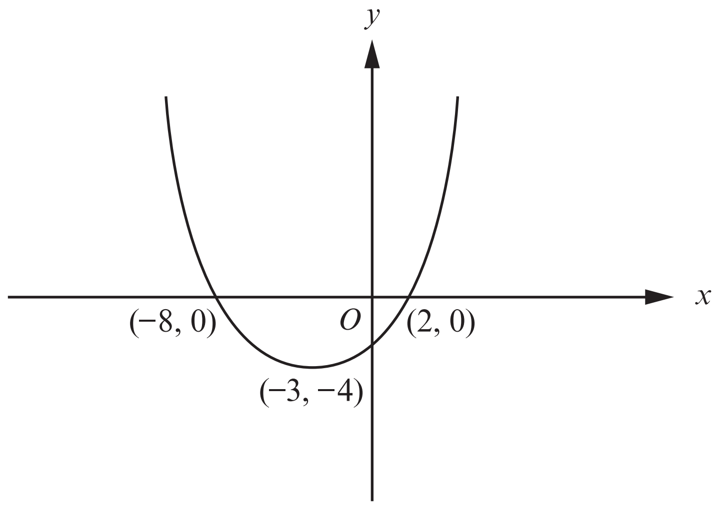


Figure 1

(a) Sketch the graph of $y = f(x + 3)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. [3]

(b) Figure 2 shows a sketch of the graph having **one** of the following equations with an appropriate value of either p, q or r.

$y = f(px)$, where p is a constant
$y = f(x) + q$, where q is a constant
$y = rf(x)$, where r is a constant.

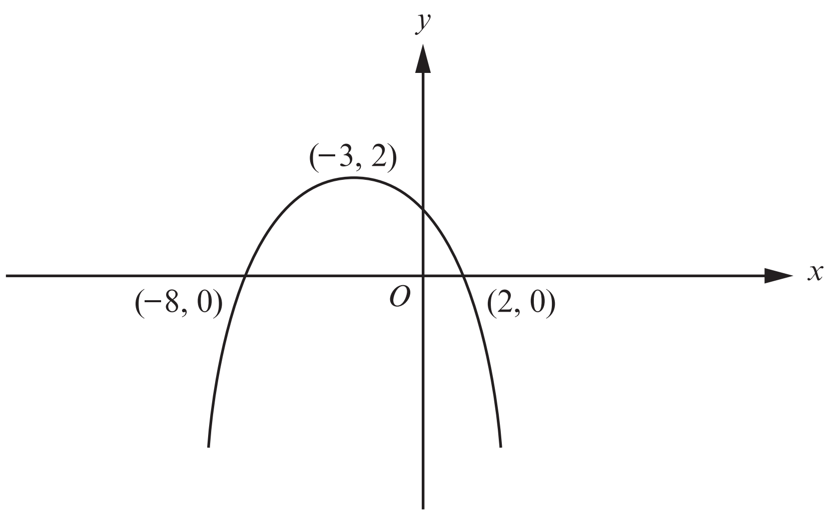
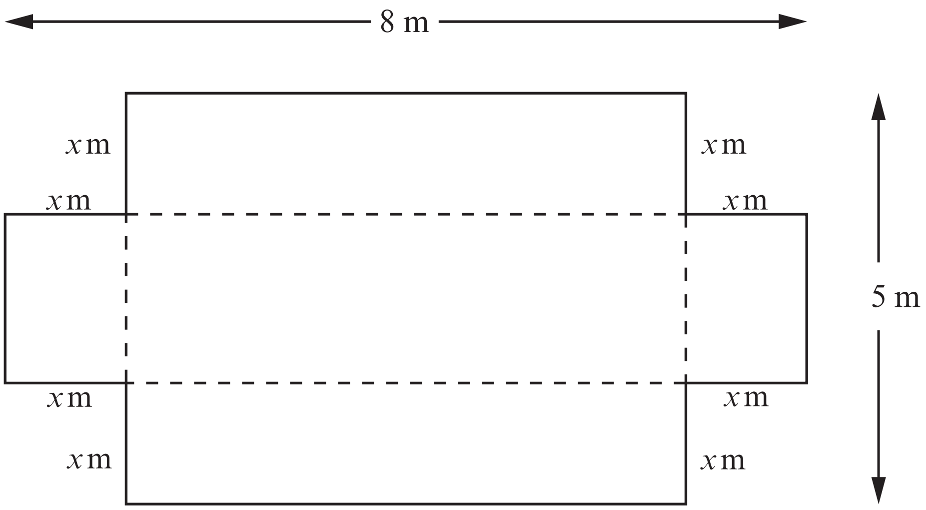


Figure 2

Write down the equation of the graph sketched in Figure 2, together with the value of the corresponding constant. [2]

10. A rectangular sheet of metal has length 8 m and width 5 m. Four squares, each of side x m, where $x < 2{\cdot}5$, have been cut away from the corners of the rectangular sheet, as shown in the diagram below. The rest of the metal sheet is now bent along the dotted lines to form an open tank in the form of a cuboid.



(a) Show that the volume V m^3 of this tank is given by

$$V = 4x^3 - 26x^2 + 40x\ .$$

[2]

(b) Find the maximum value of V, showing that the value you have found is a maximum value. [5]