Mathematics C2 January 2014

Solutions and Mark Scheme

Final Version

1.

2	2		
2·5	1·843908891		
3	1·732050808		
3·5	1·647508942		
4	1·58113883	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0{\cdot}5$ M1

$I \approx \frac{0{\cdot}5}{2} \times \{2 + 1{\cdot}58113883 + 2(1{\cdot}843908891 + 1{\cdot}732050808 + 1{\cdot}647508942)\}$

$I \approx 14{\cdot}02807611 \times 0{\cdot}5 \div 2$

$I \approx 3{\cdot}507019028$

$I \approx 3{\cdot}507$ (f.t. one slip) A1

Special case for candidates who put $h = 0{\cdot}4$

2	2		
2·4	1·870828693		
2·8	1·772810521		
3·2	1·695582496		
3·6	1·632993162		
4	1·58113883	(all values correct)	B1

Correct formula with $h = 0{\cdot}4$ M1

$I \approx \frac{0{\cdot}4}{2} \times \{2 + 1{\cdot}58113883 + 2(1{\cdot}870828693 + 1{\cdot}772810521 + 1{\cdot}695582496 + 1{\cdot}632993162)\}$

$I \approx 17{\cdot}52556857 \times 0{\cdot}4 \div 2$

$I \approx 3{\cdot}505113715$

$I \approx 3{\cdot}505$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

2. (*a*) $8\cos^2\theta - 7(1 - \cos^2\theta) = 4\cos\theta - 3$

(correct use of $\sin^2\theta = 1 - \cos^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = candidate's coefficient of $\cos^2\theta$ and $b \times d$ = candidate's constant m1

$15\cos^2\theta - 4\cos\theta - 4 = 0 \Rightarrow (5\cos\theta + 2)(3\cos\theta - 2) = 0$

$\Rightarrow \cos\theta = \frac{2}{3}$, $\cos\theta = -\frac{2}{5}$ (c.a.o.) A1

$\theta = 48{\cdot}19°, 311{\cdot}81°$ B1

$\theta = 113{\cdot}58°, 246{\cdot}42°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\cos\theta$ = +, –, f.t. for 3 marks, $\cos\theta$ = –, –, f.t. for 2 marks
$\cos\theta$ = +, +, f.t. for 1 mark

(*b*) $X = 114°$ B1

$Y - Z = 20°$ B1

$114° + Y + Z = 180°$ (f.t. only for an obtuse value for X) M1

$Y = 43°, Z = 23°$ (f.t. one error) A1

3. (*a*) $a + 2d + a + 7d = 0$ B1

$a + 4d + a + 6d + a + 9d = 22$ B1

An attempt to solve the candidate's linear equations simultaneously by eliminating one unknown M1

$a = -18, d = 4$ (both values) (c.a.o.) A1

(*b*) $S_n = \frac{n}{2}[2 \times 9 + (n - 1) \times 2]$ B1

$S_{2n} = \frac{2n}{2}[2 \times 9 + (2n - 1) \times 2]$ B1

$\frac{2n}{2}[2 \times 9 + (2n - 1) \times 2] = k \times \frac{n}{2}[2 \times 9 + (n - 1) \times 2]$ (k = **3**, $^1/_3$)

(f.t. candidate's quadratic expressions for S_{2n}, S_n provided at least one of the first two B marks is awarded) M1

An attempt to solve this equation including dividing both sides by n to reach a linear equation in n. m1

$n = 8$ (c.a.o.) A1

4. (*a*) $S_n = a + ar + \ldots + ar^{n-1}$ (at least 3 terms, one at each end) B1

$rS_n = \quad ar + \ldots + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$ (multiply first line by r and subtract) M1

$(1 - r)S_n = a(1 - r^n)$

$S_n = \dfrac{a(1 - r^n)}{1 - r}$ (convincing) A1

(*b*) (i) $ar^3 = -108$ and $ar^6 = 4$ B1

$r^3 = \dfrac{4}{-108}$ (o.e.) M1

$r = -\,^1/_3$ (c.a.o.) A1

(ii) $a \times (-\,^1/_3)^3 = -108 \Rightarrow a = 2916$

(f.t. candidate's derived value for r) B1

$S_\infty = \dfrac{2916}{1 - (-\,^1/_3)}$ (use of formula for sum to infinity)

(f.t. candidate's derived values for r and a) M1

$S_\infty = 2187$ (c.a.o.) A1

5. (*a*) (i) **Either**: $5^2 = 3^2 + x^2 - 2 \times 3 \times x \times \cos ADB$ (o.e.)

Or: $6^2 = 1^2 + x^2 - 2 \times 1 \times x \times \cos ADC$ (o.e.)

(at least one correct use of cos rule) M1

$\cos ADB = \dfrac{x^2 - 16}{6x}$ (convincing) A1

$\cos ADC = \dfrac{x^2 - 35}{2x}$ A1

(ii) $\dfrac{x^2 - 16}{6x} + \dfrac{x^2 - 35}{2x} = 0$ (o.e.)

(f.t. candidate's derived expression for $\cos ADC$) M1

$4x^2 = 121$ (f.t. candidate's derived expression for $\cos ADC$ providing it is of similar form) A1

$x = 5{\cdot}5$ (convincing) (c.a.o.) A1

(*b*) $ADB = 64{\cdot}42°$ B1

Area of triangle $ADB = \dfrac{5{\cdot}5 \times 3 \times \sin 64{\cdot}42°}{2}$

(f.t. candidate's derived value for angle ADB) M1

Area of triangle $ADB = 7{\cdot}44 \text{ cm}^2$ (c.a.o.) A1

6. (*a*) $5 \times \frac{x^{-2}}{-2} - 2 \times \frac{x^{4/3}}{4/3} - 4x + c$ B1, B1, B1

(–1 if no constant term present)

(*b*) Area = $\int_2^6 \left(3x^2 - \frac{1}{4}x^3\right) \mathrm{d}x$ (use of integration) M1

$\frac{3x^3}{3} - \frac{1}{4 \times 4}x^4$ (correct integration) B1

Area = (216 – 81) – (8 – 1)

(correct method for substituting limits) m1

Area = 128 (c.a.o.) A1

7. (*a*) Let $p = \log_a x$

Then $x = a^p$ (relationship between log and power) B1

$x^n = a^{pn}$ (the laws of indices) B1

$\therefore \log_a x^n = pn$ (relationship between log and power)

$\therefore \log_a x^n = pn = n \log_a x$ (convincing) B1

(*b*) **Either:**

$(5 - 4x) \log_{10} 7 = \log_{10} 11$

(taking logs on both sides and using the power law) M1

$x = \frac{5 \log_{10} 7 - \log_{10} 11}{4 \log_{10} 7}$ A1

$x = 0{\cdot}942$ (f.t. one slip, see below) A1

Or:

$5 - 4x = \log_7 11$ (rewriting as a log equation) M1

$x = \frac{5 - \log_7 11}{4}$ A1

$x = 0{\cdot}942$ (f.t. one slip, see below) A1

Note: an answer of $x = -0{\cdot}942$ from $x = \frac{\log_{10} 11 - 5 \log_{10} 7}{4 \log_{10} 7}$

earns M1 A0 A1

an answer of $x = 1{\cdot}558$ from $x = \frac{\log_{10} 11 + 5 \log_{10} 7}{4 \log_{10} 7}$

earns M1 A0 A1

Note: Answer only with no working shown earns 0 marks

(*c*) $\log_8 x = -\frac{1}{3} \Rightarrow x = 8^{-1/3}$

(rewriting log equation as power equation) M1

$x = 8^{-1/3} \Rightarrow x = \frac{1}{2}$ A1

8.	(*a*)	(i)	$A(2, -4)$	B1
		(ii)	Gradient $AP = \frac{\text{inc in } y}{\text{inc in } x}$	M1
			Gradient $AP = \frac{(-7) - (-4)}{6 - 2} = -\frac{3}{4}$	
			(f.t. candidate's coordinates for A)	A1
			Use of $m_{\tan} \times m_{\text{rad}} = -1$	M1
			Equation of tangent is:	
			$y - (-7) = \frac{4}{3}(x - 6)$ (f.t. candidate's gradient for AP)	A1
	(*b*)		An attempt to substitute $(x + 3)$ for y in the equation of the circle and form quadratic in x	M1
			$x^2 + (x + 3)^2 - 4x + 8(x + 3) - 5 = 0 \Rightarrow 2x^2 + 10x + 28 = 0$	A1
			An attempt to calculate value of discriminant	m1
			Discriminant = $100 - 224 < 0 \Rightarrow$ no points of intersection (f.t. one slip)	A1

9.	Denoting $A\hat{O}B$ by θ,	
	Area of sector $AOB = \frac{1}{2} \times 7^2 \times \theta$	
	Area of sector $COD = \frac{1}{2} \times 4^2 \times \theta$ (at least one correct)	M1
	$\frac{1}{2} \times 7^2 \times \theta - \frac{1}{2} \times 4^2 \times \theta = 23{\cdot}1$ (f.t candidate's expressions for the areas of the sectors)	m1
	$\theta = 1{\cdot}4$ (c.a.o.)	
	A1	
	$CD = 5{\cdot}6$ cm, $AB = 9{\cdot}8$ cm (both values, f.t candidate's value for θ)	B1
	Use of perimeter of $ACDB = AC + CD + DB + BA$	M1
	Perimeter of $ACDB = 21{\cdot}4$ cm (c.a.o.)	A1

10.	(*a*)	$t_2 = {}^3/_4$	B1
		$t_3 = -{}^1/_3$, $t_4 = 4$	B1
	(*b*)	The sequence repeats itself every third term	B1
		$t_{50} = {}^3/_4$	B1