Mathematics C2 January 2013

Solutions and Mark Scheme

Final Version

1.

0	3·16227766		
0·5	3·142451272		
1	3		
1·5	2·573907535		
2	1·414213562	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0·5$ M1

$$I \approx \frac{0·5}{2} \times \{3·16227766 + 1·414213562 + 2(3·142451272 + 3 + 2·573907535)\}$$

$I \approx 22·00920884 \times 0·5 \div 2$

$I \approx 5·50230221$

$I \approx 5·5023$ (f.t. one slip) A1

Special case for candidates who put $h = 0·4$

0	3·16227766		
0·4	3·152142129		
0·8	3·080259729		
1·2	2·876108482		
1·6	2·429814808		
2	1·414213562	(all values correct)	B1

Correct formula with $h = 0·4$ M1

$$I \approx \frac{0·4}{2} \times \{3·16227766 + 1·414213562 + 2(3·152142129 + 3·080259729 + 2·876108482 + 2·429814808)\}$$

$I \approx 27·65314152 \times 0·4 \div 2$

$I \approx 5·530628304$

$I \approx 5·5306$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

2. (*a*) $7\sin^2\theta - \sin\theta = 3(1 - \sin^2\theta)$

(correct use of $\cos^2\theta = 1 - \sin^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = candidate's coefficient of $\sin^2\theta$ and $b \times d$ = candidate's constant m1

$10\sin^2\theta - \sin\theta - 3 = 0 \Rightarrow (2\sin\theta + 1)(5\sin\theta - 3) = 0$

$\Rightarrow \sin\theta = -\frac{1}{2},\ \sin\theta = \frac{3}{5}$ (c.a.o.) A1

$\theta = 210°, 330°$ B1, B1

$\theta = 36{\cdot}87°, 143{\cdot}13°$ B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\sin\theta$ = +, –, f.t. for 3 marks, $\sin\theta$ = –, –, f.t. for 2 marks
$\sin\theta$ = +, +, f.t. for 1 mark

(*b*) $3x - 20° = 52°, 232°, 412°$ (one value) B1

$x = 24°, 84°, 144°$ B1, B1, B1

Note: Subtract (from final three marks) 1 mark for each additional root in range, ignore roots outside range.

3. (*a*) $x^2 = 10^2 + (x + 4)^2 - 2 \times 10 \times (x + 4) \times \frac{3}{5}$ (correct use of cos rule) M1

$x^2 = 100 + x^2 + 8x + 16 - 12x - 48$ A1

$x = 17$ (f.t. one slip) A1

(*b*) $\sin\alpha = \frac{4}{5}$ B1

Area of triangle $ABC = \frac{1}{2} \times 10 \times 21 \times \frac{4}{5}$

(substituting the correct values in the correct places in the area formula, f.t. candidate's values for x and $\sin\alpha$) M1

Area of triangle $ABC = 84\,\text{cm}^2$ (f.t. candidate's value for x) A1

4. (*a*) (i) nth term $= 1 + 4(n - 1) = 1 + 4n - 4 = 4n - 3$ (convincing) B1

(ii) $S_n = 1 + 5 + \ldots + (4n - 7) + (4n - 3)$

$S_n = (4n - 3) + (4n - 7) + \ldots + 5 + 1$

Reversing and adding M1

Either:

$2S_n = (4n - 2) + (4n - 2) + \ldots + (4n - 2) + (4n - 2)$

Or:

$2S_n = (4n - 2) + \ldots$ (n times) A1

$2S_n = n(4n - 2)$

$S_n = n(2n - 1)$ (convincing) A1

(*b*) $\frac{10}{2} \times [2a + 9d] = 55$ B1

Either: $(a + 3d) + (a + 6d) + (a + 8d) = 27$

Or: $(a + 4d) + (a + 7d) + (a + 9d) = 27$ M1

$3a + 17d = 27$ (seen or implied by later work) A1

An attempt to solve candidate's derived linear equations simultaneously by eliminating one unknown M1

$a = -8$, $d = 3$ (both values) (c.a.o.) A1

5. (*a*) $r = 1{\cdot}5$ B1

A correct method for finding $(p + 4)$ th term M1

$(p + 4)$ th term $= 81$ (c.a.o.) A1

(*b*) Either: $\frac{a(1 - r^3)}{1 - r} = 22{\cdot}8$

Or: $a + ar + ar^2 = 22{\cdot}8$ B1

$\frac{a}{1 - r} = 18{\cdot}75$ B1

An attempt to solve these equations simultaneously by eliminating a M1

$r^3 = -0{\cdot}216$ A1

$r = -0{\cdot}6$ (c.a.o.) A1

$a = 30$ (f.t. candidate's derived value for r) A1

6. (*a*) $5 \times \dfrac{x^{-3}}{-3} - 7 \times \dfrac{x^{5/3}}{5/3} + c$ B1, B1

(–1 if no constant term present)

(*b*) (i) $9 - a^2 = 0 \Rightarrow a = 3$ B1

(ii) $\dfrac{dy}{dx} = \pm 2x$ M1

Gradient of tangent = ± 6 (f.t. candidate's value for a) A1

$b = 18$ (convincing) A1

(iii) **Either:**

Area of triangle = 27 (f.t. candidate's value for a) B1

Use of integration to find the area under the curve M1

$\int (9 - x^2)\,dx = 9x - (1/3)x^3$ (correct integration) B1

Correct method of substitution of candidate's limits m1

$[9x - (1/3)x^3]_0^3 = (27 - 9) - 0 = 18$

Use of 0 and candidate's value for a as limits and trying to find total area by subtracting area under curve from area of triangle m1

Shaded area = 27 – 18 = 9 (c.a.o.) A1

Or:

Equation of tangent is $y = -6x + 18$

Use of integration to find an area M1

$\int (-6x + 18)\,dx = -3x^2 + 18x$ (correct integration)

(f.t. one slip in candidate's equation of tangent) B1

$\int (9 - x^2)\,dx = 9x - (1/3)x^3$ (correct integration) B1

Correct method of substitution of candidate's limits m1

$[-3x^2 + 18x]_0^3 = (-27 + 54) - 0 = 27$

(f.t. one slip in candidate's equation of tangent)

$[9x - (1/3)x^3]_0^3 = (27 - 9) - 0 = 18$

Use of 0 and candidate's value for a as limits and trying to find total area by subtracting area under curve from area under line m1

Shaded area = 27 – 18 = 9 (c.a.o.) A1

7. (*a*) Let $p = \log_a x$, $q = \log_a y$

Then $x = a^p$, $y = a^q$ (the relationship between log and power) B1

$\frac{x}{y} = \frac{a^p}{a^q} = a^{p-q}$ (the laws of indicies) B1

$\log_a x/y = p - q$ (the relationship between log and power)

$\log_a x/y = p - q = \log_a x - \log_a y$ (convincing) B1

(*b*) **Either:**

$(2x + 5)\log_{10} 6 = \log_{10} 7$

(taking logs on both sides and using the power law) M1

$x = \frac{\log_{10} 7 - 5\log_{10} 6}{2\log_{10} 6}$ (o.e.) A1

$x = -1{\cdot}957$ (f.t. one slip, see below) A1

Or:

$2x + 5 = \log_6 7$ (rewriting as a log equation) M1

$x = \frac{\log_6 7 - 5}{2}$ (o.e.) A1

$x = -1{\cdot}957$ (f.t. one slip, see below) A1

Note: an answer of $x = 1{\cdot}957$ from $x = \frac{5\log_{10} 6 - \log_{10} 7}{2\log_{10} 6}$

earns M1 A0 A1

an answer of $x = 3{\cdot}043$ from $x = \frac{\log_{10} 7 + 5\log_{10} 6}{2\log_{10} 6}$

earns M1 A0 A1

an answer of $x = -3{\cdot}914$ from $x = \frac{\log_{10} 7 - 5\log_{10} 6}{\log_{10} 6}$

earns M1 A0 A1

Note: Answer only with no working shown earns 0 marks

8. (*a*) (i) $A(-3, 5)$ B1

A correct method for finding radius M1

Radius = $\sqrt{20}$ A1

(ii) **Either:**

A correct method for finding AP^2 M1

$AP^2 = 18\ (< 20) \Rightarrow P$ is inside C

(f.t. candidate's coordinates for A) A1

Or:

An attempt to substitute $x = -6$, $y = 2$ in the equation of C M1

$(-6)^2 + 2^2 + 6 \times (-6) - 10 \times 2 + 14 = -2\ (< 0)$

$\Rightarrow P$ is inside C A1

(*b*) (i) An attempt to substitute $(2x + 1)$ for y in the equation of the circle M1

$5x^2 - 10x + 5 = 0$ A1

Either: Use of $b^2 - 4ac$ m1

Discriminant = 0 ($\Rightarrow y = 2x + 1$ is a tangent to the circle) A1

$x = 1, y = 3$ A1

Or: An attempt to factorise candidate's quadratic m1

Repeated (single) root ($\Rightarrow y = 2x + 1$ is a tangent to the circle) A1

$x = 1, y = 3$ A1

(ii) **Either:**

$RQ = \sqrt{45}$ or $RA = \sqrt{65}$

(f.t. candidate's coordinates for A and Q) B1

Correct substitution of candidate's values in an expression for sin R, cos R or tan R M1

$ARQ = 33{\cdot}69°$ (f.t. one numerical slip) A1

Or:

$RQ = \sqrt{45}$ or $RA = \sqrt{65}$

(f.t. candidate's coordinates for A and Q) B1

Correct substitution of candidate's values in the cos rule to find cos R M1

$ARQ = 33{\cdot}69°$ (f.t. one numerical slip) A1

9. (*a*) $\frac{1}{2} \times 11 \times 11 \times \theta = 43{\cdot}56$ M1

$\theta = 0{\cdot}72$ radians A1

(*b*) $BC = 11\phi$ B1

$CD = 11(\pi - \phi)$ B1

$11\phi = 11(\pi - \phi) \pm 13$ M1

$\phi = 0{\cdot}98$ radians (c.a.o.) A1