Mathematics C2

1.

1	0·5		
1·5	0·674234614		
2	0·828427124		
2·5	0·968564716	(5 values correct)	B2
3	1·098076211	(3 or 4 values correct)	B1

Correct formula with h = 0·5 M1

$I \approx \frac{0 \cdot 5}{2} \times \{0 \cdot 5 + 1 \cdot 098076211 + 2(0 \cdot 674234614 + 0 \cdot 828427124 + 0 \cdot 968564716)\}$

$I \approx 6 \cdot 540529119 \div 4$

$I \approx 1 \cdot 63513228$

$I \approx 1 \cdot 635$ (f.t. one slip) A1

Special case for candidates who put h = 0·4

1	0·5		
1·4	0·641255848		
1·8	0·768691769		
2·2	0·885939445		
2·6	0·995233768		
3	1·098076211	(all values correct)	B1

Correct formula with h = 0·2 M1

$I \approx \frac{0 \cdot 4}{2} \times \{0 \cdot 5 + 1 \cdot 098076211 + 2(0 \cdot 641255848 + 0 \cdot 768691769 + 0 \cdot 885939445 + 0 \cdot 995233768)\}$

$I \approx 8 \cdot 180317871 \div 5$

$I \approx 1 \cdot 636063574$

$I \approx 1 \cdot 636$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

2. (a) $10(1 - \cos^2\theta) + 7\cos\theta = 5\cos^2\theta + 8$

(correct use of $\sin^2\theta = 1 - \cos^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = candidate's coefficient of $\cos^2\theta$ and $b \times d$ = candidate's constant m1

$15\cos^2\theta - 7\cos\theta - 2 = 0 \Rightarrow (3\cos\theta - 2)(5\cos\theta + 1) = 0$

$\Rightarrow \cos\theta = \frac{2}{3}, \quad \cos\theta = -\frac{1}{5}$ (c.a.o.) A1

$\theta = 48 \cdot 19°, 311 \cdot 81°$ B1

$\theta = 101 \cdot 54°, 258 \cdot 46°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\cos\theta$ = +, –, f.t. for 3 marks, $\cos\theta$ = –, –, f.t. for 2 marks

$\cos\theta$ = +, +, f.t. for 1 mark

(*b*) $x - 50 = -43°, 223°, 317°$ (at least one value) B1
$x = 7°, 273°$, (both values) B1 B1

Note: Subtract from final 2 marks 1 mark for each additional root in range, ignore roots outside range.

(*c*) $\sin \phi \le 1, \cos \phi \le 1$ and thus $\sin \phi + \cos \phi \le 2$ E1

3. (*a*) $\frac{1}{2} \times x \times (2x - 3) \times \sin 150° = 6\cdot75$
(substituting the correct values and expressions in the correct places in the area formula) M1
$2x^2 - 3x - 27 = 0$ (convincing) A1
An attempt to solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(ax + b)(cx + d)$, with $a \times c = 2$ and $b \times d = -27$ M1
$(x + 3)(2x - 9) = 0 \Rightarrow x = 4\cdot5$ (c.a.o.) A1

(*b*) $BC^2 = 4\cdot5^2 + 6^2 - 2 \times 4\cdot5 \times 6 \times \cos 150°$
(correct use of cos rule, f.t. candidate's derived value for x) M1
$BC = 10\cdot15$ cm (f.t. candidate's derived value for x) A1

(*c*) $\frac{1}{2} \times 10\cdot15 \times AD = 6\cdot75$ (f.t. candidate's derived value for BC) M1
$AD = 1\cdot33$ cm (c.a.o.) A1

4. (*a*) $a + 14d = k \times (a + 4d)$ ($k = 7, \frac{1}{7}$) M1
$a + 14d = 7 \times (a + 4d)$ A1
$3a + 7d = 0$
$\frac{11}{2} \times (2a + 10d) = 88$ B1
$a + 5d = 8$
An attempt to solve the candidate's two derived linear equations simultaneously by eliminating one unknown M1
$d = 3$ (c.a.o.) A1
$a = -7$ (f.t. candidate's value for d) A1

(*b*) $-7 + (n - 1) \times 3 = 143$ (f.t. candidate's values for a and d) M1
$n = 51$ (c.a.o.) A1

5. (*a*) $S_n = a + ar + ... + ar^{n-1}$ (at least 3 terms, one at each end) B1

$rS_n = \quad ar + ... + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$ (multiply first line by r and subtract) M1

$(1 - r)S_n = a(1 - r^n)$

$S_n = \frac{a(1 - r^n)}{1 - r}$ (convincing) A1

(*b*) $a + ar = 25{\cdot}2$ or $\frac{a(1 - r^2)}{(1 - r)} = 25{\cdot}2$ B1

$\frac{a}{1 - r} = 30$ B1

An attempt to solve the candidate's derived equations simultaneously by eliminating a M1

$30(1 - r) + 30(1 - r)r = 25{\cdot}2$ (a correct quadratic in r) A1

$r = 0{\cdot}4$ (c.a.o.) A1

$a = 18$ (f.t. candidate's value for r provided $r > 0$) A1

6. (*a*) $4 \times \frac{x^{-2}}{-2} - 3 \times \frac{x^{5/4}}{5/4} + c$ (Deduct 1 mark if no c present) B1,B1

(*b*) (i) $4 - x^2 = 0$ M1

$x = -2,\ x = 2$ (both values, c.a.o.) A1

(ii) Use of integration to find an area M1

$\int 4\,dx = 4x, \quad \int x^2\,dx = \frac{x^3}{3}$ B1, B1

$$\text{Total area} = \int_{-2}^{2} (4 - x^2)\,dx - \int_{2}^{3} (4 - x^2)\,dx$$

(subtraction of integrals with correct use of candidate's x_A, x_B and 3 as limits) m1

$$\text{Total area} = \left[4x - (1/3)x^3\right]_{-2}^{2} - \left[4x - (1/3)x^3\right]_{2}^{3}$$

$$= \{[8 - (8/3)] - [(-8) - (-8/3)]\} - \{[12 - 9] - [8 - (8/3)]\}$$

Correct method of substitution of candidate's limits in at least one integral m1

Total area = 13 (c.a.o.) A1

Note: Answer only with no working shown earns 0 marks

7. (*a*) Let $p = \log_a x$, $q = \log_a y$

Then $x = a^p$, $y = a^q$ (relationship between log and power) B1

$xy = a^p \times a^q = a^{p+q}$ (the laws of indices) B1

$\log_a xy = p + q$ (relationship between log and power)

$\log_a xy = p + q = \log_a x + \log_a y$ (convincing) B1

(*b*) **Either:**

$(3 - 5x)\log_{10} 2 = \log_{10} 12$

(taking logs on both sides and using the power law) M1

$x = \dfrac{3\log_{10} 2 - \log_{10} 12}{5\log_{10} 2}$ A1

$x = -0{\cdot}117$ (f.t. one slip, see below) A1

Or:

$3 - 5x = \log_2 12$ (rewriting as a log equation) M1

$x = \dfrac{3 - \log_2 12}{5}$ A1

$x = -0{\cdot}117$ (f.t. one slip, see below) A1

Note: an answer of $x = 0{\cdot}117$ from $x = \dfrac{\log_{10} 12 - 3\log_{10} 2}{5\log_{10} 2}$

earns M1 A0 A1

an answer of $x = 1{\cdot}317$ from $x = \dfrac{\log_{10} 12 + 3\log_{10} 2}{5\log_{10} 2}$

earns M1 A0 A1

an answer of $x = -0{\cdot}585$ from $x = \dfrac{3\log_{10} 2 - \log_{10} 12}{\log_{10} 2}$

earns M1 A0 A1

Note: Answer only with no working shown earns 0 marks

(*c*) (i) $2\log_9 (x + 1) = \log_9 (x + 1)^2$ (power law) B1

$\log_9 (3x - 1) + \log_9 (x + 4) - \log_9 (x + 1)^2$

$= \log_9 \dfrac{(3x - 1)(x + 4)}{(x + 1)^2}$

(addition law) B1

(subtraction law) B1

(ii) $\log_9 \dfrac{(3x - 1)(x + 4)}{(x + 1)^2} = 1/2 \Rightarrow \dfrac{(3x - 1)(x + 4)}{(x + 1)^2} = 3$

(f.t. candidate's log expression) M1

$x = 1{\cdot}4$ (c.a.o.) A1

8. (*a*) (i) $A(4, -1)$ B1

(ii) A correct method for finding radius M1
Radius = $\sqrt{50}$ (convincing) A1

(iii) Equation of C: $(x-4)^2 + (y-[-1])^2 = 50$
(f.t. candidate's coordinates for A) B1

(*b*) **Either:**
An attempt to substitute the coordinates of R in the candidate's equation for C M1
Verification that L.H.S. of equation of $C = 50 \Rightarrow R$ lies on C A1
Or:
An attempt to find AR^2 M1
$AR^2 = 50 \Rightarrow R$ lies on C A1

(*c*) **Either:**
$RQ = \sqrt{160}$ ($RP = \sqrt{40}$) B1
$\cos PQR = \dfrac{\sqrt{160}}{2\sqrt{50}}$ $\left(\sin PQR = \dfrac{\sqrt{40}}{2\sqrt{50}}\right)$ M1
$PQR = 26{\cdot}565°$ (f.t. one numerical slip) A1
Or:
$RQ = \sqrt{160}$ and $RP = \sqrt{40}$ (both values) B1
$(\sqrt{40})^2 = (\sqrt{160})^2 + (2\sqrt{50})^2 - 2 \times (\sqrt{160}) \times (2\sqrt{50}) \times \cos PQR$
(correct use of cos rule) M1
$PQR = 26{\cdot}565°$ (f.t. one numerical slip) A1

9. (*a*) $\frac{1}{2} \times 5^2 \times \theta + \frac{1}{2} \times 5^2 \times \phi = 22{\cdot}5$ M1
$\theta + \phi = 1{\cdot}8$ (convincing) A1

(*b*) $5 \times \theta - 5 \times \phi = 3{\cdot}5$ or $5 \times \phi - 5 \times \theta = 3{\cdot}5$ M1
$5 \times \theta - 5 \times \phi = 3{\cdot}5$ (o.e.) A1
An attempt to solve the candidate's two linear equations simultaneously by eliminating one unknown M1
$\theta = 1{\cdot}25$, $\phi = 0{\cdot}55$ (both values) A1
(f.t. only for $\theta = 0{\cdot}55$, $\phi = 1{\cdot}25$ from $5 \times \phi - 5 \times \theta = 3{\cdot}5$)