Mathematics C1 January 2014

Solutions and Mark Scheme

Final Version

1. (*a*) (i) Gradient of $AB = \dfrac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $AB = -{}^{3}/_{2}$ (or equivalent) A1

(ii) Use of gradient $L_1 \times$ gradient $AB = -1$ M1

A correct method for finding the equation of L_1 using candidate's gradient for L_1 M1

Equation of L_1: $y - 1 = {}^{2}/_{3}(x - 4)$ (or equivalent)
(f.t. candidate's gradient for AB) A1

(*b*) (i) An attempt to solve equations of L_1 and L_2 simultaneously M1

$x = -2, y = -3$ (convincing) A1

(ii) A correct method for finding the coordinates of the mid-point of AC M1

Mid-point of AC has coordinates $(2, -2{\cdot}5)$ (c.a.o.) A1

(iii) A correct method for finding the length of $AB(BC)$ M1

$AB = \sqrt{13}$ A1

$BC = \sqrt{52}$ (or equivalent) A1

A correct method for finding the area of triangle ABC m1

Area of triangle $ABC = 13$ (c.a.o.) A1

2. $\dfrac{3\sqrt{3} - 2\sqrt{5}}{2\sqrt{3} + \sqrt{5}} = \dfrac{(3\sqrt{3} - 2\sqrt{5})(2\sqrt{3} - \sqrt{5})}{(2\sqrt{3} + \sqrt{5})(2\sqrt{3} - \sqrt{5})}$ M1

Numerator: $6 \times 3 - 3 \times \sqrt{3} \times \sqrt{5} - 4 \times \sqrt{5} \times \sqrt{3} + 10$ A1

Denominator: $12 - 5$ A1

$\dfrac{3\sqrt{3} - 2\sqrt{5}}{2\sqrt{3} + \sqrt{5}} = 4 - \sqrt{15}$ (c.a.o.) A1

Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $2\sqrt{3} + \sqrt{5}$

3. An attempt to differentiate, at least one non-zero term correct M1

$\frac{dy}{dx} = 20 \times -1 \times x^{-2} + 4x$ A1

An attempt to substitute $x = 2$ in candidate's derived expression for $\frac{dy}{dx}$ m1

Value of $\frac{dy}{dx}$ at $P = 3$ (c.a.o.) A1

Gradient of normal = $\frac{-1}{\text{candidate's derived value for } \frac{dy}{dx}}$ m1

Equation of normal to C at P: $y - 7 = -\frac{1}{3}(x - 2)$ (or equivalent)

(f.t. candidate's value for $\frac{dy}{dx}$ provided all three method marks are awarded) A1

4. Either $p = 0{\cdot}8$ or a sight of $(x + 0{\cdot}8)^2$ B1

A convincing argument to show that the value 25 is correct B1

$x^2 + 1{\cdot}6x - 24{\cdot}36 = 0 \Rightarrow (x + 0{\cdot}8)^2 = 25$ (f.t. candidate's value for p) M1

$x = 4{\cdot}2$ (f.t. candidate's value for p) A1

$x = -5{\cdot}8$ (f.t. candidate's value for p) A1

5. (*a*) $(1 + \sqrt{6})^5 = (1)^5 + 5(1)^4(\sqrt{6}) + 10(1)^3(\sqrt{6})^2 + 10(1)^2(\sqrt{6})^3$
$+ 5(1)(\sqrt{6})^4 + (\sqrt{6})^5$ (five or six terms correct) B2

(**If B2 not awarded**, award B1 for four correct terms)

$(1 + \sqrt{6})^5 = 1 + 5\sqrt{6} + 60 + 60\sqrt{6} + 180 + 36\sqrt{6}$
(six terms correct) B2

(**If B2 not awarded**, award B1 for four or five correct terms)

$(1 + \sqrt{6})^5 = 241 + 101\sqrt{6}$ (f.t. one error) B1

(*b*) ${}^nC_2 \times 3^k = 495$ (k = 1, **2**) M1

Either $9n^2 - 9n - 990 = 0$ or $n^2 - n - 110 = 0$ or $n(n - 1) = 110$ A1

$n = 11$ (c.a.o.) A1

6. An expression for $b^2 - 4ac$, with at least two of a, b, c correct M1

$b^2 - 4ac = 8^2 - 4 \times (2k - 3) \times (2k + 3)$ A1

Putting $b^2 - 4ac <(\leq) 0$ m1

$100 - 16k^2 < 0$ (o.e.) (c.a.o.) A1

Finding critical values $k = -5/2$, $k = 5/2$
(o.e.) (f.t. candidate's values for m, n) B1

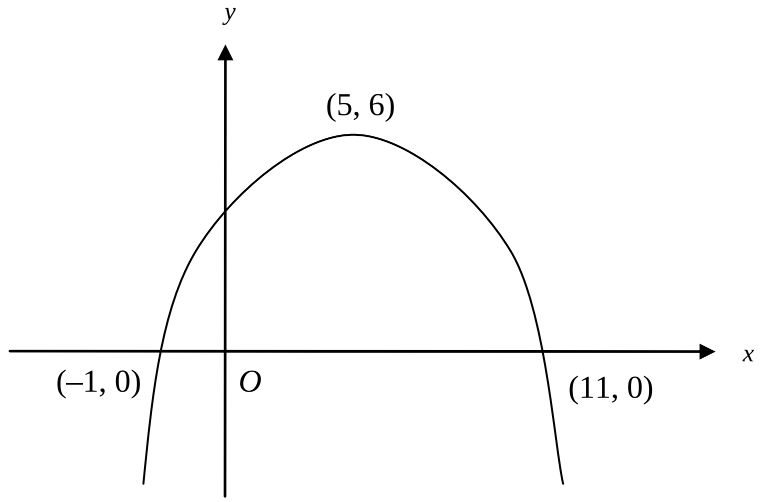
$k < -5/2$ or $5/2 < k$ (o.e.) (f.t. only critical values of $-a$ and a) B1

Each of the following errors earns a final B0

the use of non-strict inequalities

the use of the word 'and' instead of the word 'or'

7. (*a*)



Concave down curve and y-coordinate of maximum = 6 B1
x-coordinate of maximum = 5 B1
Both points of intersection with x-axis B1

(*b*) $y = f(-2x)$ B2
(**If B2 not awarded**, award B1 for either $y = f(-{}^{1}/_{2}\,x)$ or $y = f(2x)$)

8. (*a*) $y + \delta y = 7(x + \delta x)^2 - 6(x + \delta x) - 3$ B1
Subtracting y from above to find δy M1
$\delta y = 14x\delta x + 7(\delta x)^2 - 6\delta x$ A1
Dividing by δx and letting $\delta x \to 0$ M1
$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 14x - 6$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = a \times \frac{4}{3} \times x^{1/3} + 24 \times \frac{1}{2} \times x^{-1/2}$ B1, B1
Attempting to substitute $x = 64$ in candidate's expression for $\frac{dy}{dx}$ and
putting expression equal to $\frac{11}{2}$ M1
(The M1 is only awarded if at least one B1 has been awarded)
$a = \frac{3}{4}$ (c.a.o.) A1

9.	(*a*)	Use of $f(-3) = -39$	M1
		$-27a + 117 + 30 - 24 = -39 \Rightarrow a = 6$ (convincing)	A1
	(*b*)	Attempting to find $f(r) = 0$ for some value of r	M1
		$f(-2) = 0 \Rightarrow x + 2$ is a factor	A1
		$f(x) = (x + 2)(6x^2 + ax + b)$ with one of a, b correct	M1
		$f(x) = (x + 2)(6x^2 + x - 12)$	A1
		$f(x) = (x + 2)(2x + 3)(3x - 4)$ (f.t. only $6x^2 - x - 12$ in above line)	A1
		$x = -2, -{}^3/_2, {}^4/_3$ (f.t. for factors $2x \pm 3$, $3x \pm 4$)	A1

Special case

Candidates who, after having found $x + 2$ as one factor, then find just one of the remaining factors by using e.g. the factor theorem, are awarded B1 for the final 4 marks

10. (*a*) $\frac{dy}{dx} = -6x^2 + 24x - 18$ B1

Putting derived $\frac{dy}{dx} = 0$ M1

$x = 1, 3$ (both correct) (f.t. candidate's $\frac{dy}{dx}$) A1

Stationary points are $(1, -3)$ and $(3, 5)$ (both correct) (c.a.o) A1

A correct method for finding nature of stationary points yielding

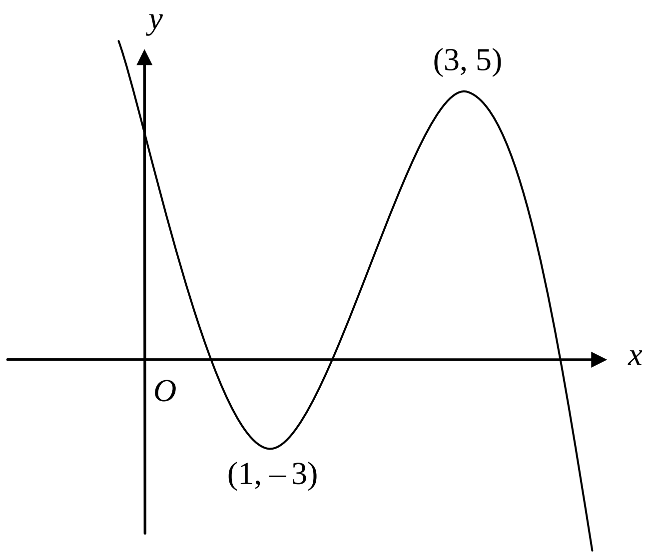
either $(1, -3)$ is a minimum point

or $(3, 5)$ is a maximum point (f.t. candidate's derived values) M1

Correct conclusion for other point

(f.t. candidate's derived values) A1

(*b*)



Graph in shape of a negative cubic with two turning points M1

Correct marking of both stationary points

(f.t. candidate's derived maximum and minimum points) A1

(*c*) Use of both $k = -3$, $k = 5$ to find the range of values for k

(f.t. candidate's y-values at stationary points) M1

$-3 < k < 5$ (f.t. candidate's y-values at stationary points) A1