Mathematics C1 January 2013

Solutions and Mark Scheme

Final Version

1.	(*a*)	Gradient of $AB = \frac{\text{increase in } y}{\text{increase in } x}$	M1
		Gradient of $AB = {}^4/_2$ (or equivalent)	A1
		A correct method for finding the equation of AB using the candidate's value for the gradient of AB.	M1
		Equation of AB : $y - 1 = 2(x - 4)$ (or equivalent) (f.t. the candidate's value for the gradient of AB)	A1
		Equation of AB : $2x - y - 7 = 0$ (f.t. one error if both M1's are awarded)	A1
	(*b*)	Gradient of $L = -{}^1/_2$ (or equivalent)	B1
		An attempt to use the fact that the product of perpendicular lines = –1 (or equivalent)	M1
		Gradient $AB \times$ Gradient $L = -1 \Rightarrow AB, L$ perpendicular (convincing)	A1
	(*c*)	An attempt to solve equations of AB and L simultaneously	M1
		$x = 5, y = 3$ (convincing)	A1
	(*d*)	A correct method for finding the length of $AB(AC)$	M1
		$AB = \sqrt{20}$	A1
		$AC = \sqrt{45}$	A1
		$k = {}^2/_3$ (c.a.o.)	A1
2.	(*a*)	$\frac{6\sqrt{7} - 11\sqrt{2}}{\sqrt{7} - \sqrt{2}} = \frac{(6\sqrt{7} - 11\sqrt{2})(\sqrt{7} + \sqrt{2})}{(\sqrt{7} - \sqrt{2})(\sqrt{7} + \sqrt{2})}$	M1
		Numerator: $6 \times 7 + 6 \times \sqrt{7} \times \sqrt{2} - 11 \times \sqrt{7} \times \sqrt{2} - 11 \times 2$	A1
		Denominator: $7 - 2$	A1
		$\frac{6\sqrt{7} - 11\sqrt{2}}{\sqrt{7} - \sqrt{2}} = 4 - \sqrt{14}$ (c.a.o.)	A1
		Special case If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $\sqrt{7} - \sqrt{2}$	
	(*b*)	$\frac{3}{2\sqrt{6}} = p\sqrt{6}$, where p is a fraction equivalent to ${}^1/_4$	B1
		$\left(\frac{\sqrt{6}}{2}\right)^3 = q\sqrt{6}$, where q is a fraction equivalent to ${}^3/_4$	B1
		$\frac{3}{2\sqrt{6}} + \left(\frac{\sqrt{6}}{2}\right)^3 = \sqrt{6}$ (c.a.o.)	B1

3. y-coordinate at $P = -2$ B1

$\frac{dy}{dx} = 6x - 14$ (an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 3$ in candidate's expression for $\frac{dy}{dx}$ m1

Use of candidate's numerical value for $\frac{dy}{dx}$ as gradient of tangent at P m1

Equation of tangent at P: $y - (-2) = 4(x - 3)$ (or equivalent)
(f.t. only candidate's derived value for y-coordinate at P) A1

4. (*a*) (i) $a = 4$ B1
$b = -11$ B1

(ii) least value -33 (f.t. candidate's value for b) B1
corresponding x-value $= -4$
(f.t. candidate's value for a) B1

(*b*) $x^2 - x - 9 = 2x - 5$ M1

An attempt to collect terms, form and use a correct method to solve their quadratic equation m1

$x^2 - 3x - 4 = 0 \Rightarrow (x - 4)(x + 1) = 0 \Rightarrow x = 4, x = -1$
(both values, c.a.o.) A1

When $x = 4$, $y = 3$, when $x = -1$, $y = -7$
(both values, f.t. one slip) A1

The line [$y = 2x - 5$] intersects the curve [$y = x^2 - x - 9$] at two points $(-1, -7)$ and $(4, 3)$ (f.t. candidate's x and y-values) E1

5. (*a*) An expression for $b^2 - 4ac$, with at least two of a, b or c correct M1

$b^2 - 4ac = 6^2 - 4 \times 5 \times (-3k)$ A1

$b^2 - 4ac > 0$ m1

$k > -{}^3/_5$ (o.e.)
[f.t. only for $k < {}^3/_5$ from $b^2 - 4ac = 6^2 - 4 \times 5 \times (3k)$] A1

(*b*) Finding critical values $x = 2{\cdot}5$, $x = 3$ B1

$2{\cdot}5 \le x \le 3$ **or** $3 \ge x \ge 2{\cdot}5$ **or** $[2{\cdot}5, 3]$ **or** $2{\cdot}5 \le x$ and $x \le 3$ **or** a correctly worded statement to the effect that x lies between $2{\cdot}5$ and 3 (both values inclusive) (f.t. candidate's derived critical values) B2

Note:
$2{\cdot}5 < x < 3$
$2{\cdot}5 \le x, x \le 3$
$2{\cdot}5 \le x \quad x \le 3$
$2{\cdot}5 \le x$ or $x \le 3$
all earn B1

6. (*a*) $y + \delta y = -(x + \delta x)^2 + 4(x + \delta x) - 6$ B1

Subtracting y from above to find δy M1

$\delta y = -2x\delta x - (\delta x)^2 + 4\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = -2x + 4$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = 5 \times \frac{4}{3} \times x^{1/3} - 9 \times -\frac{1}{2} \times x^{-3/2}$ B1, B1

7. Coefficient of $x = {}^6C_1 \times a^5 \times 4(x)$ B1

Coefficient of $x^2 = {}^6C_2 \times a^4 \times 4^2(x^2)$ B1

$15 \times a^4 \times m = k \times 6 \times a^5 \times 4$ (m = **16** or 4 or 8, k = **2** or $^1/_2$) M1

$a = 5$ (c.a.o.) A1

8. (*a*) Use of $f(-2) = 0$ M1

$-8p + 72 + 8 - 8 = 0 \Rightarrow p = 9$ (convincing) A1

Special case

Candidates who assume $p = 9$ and show $f(-2) = 0$ are awarded B1

(*b*) $f(x) = (x + 2)(9x^2 + ax + b)$ with one of a, b correct M1

$f(x) = (x + 2)(9x^2 + 0x - 4)$ A1

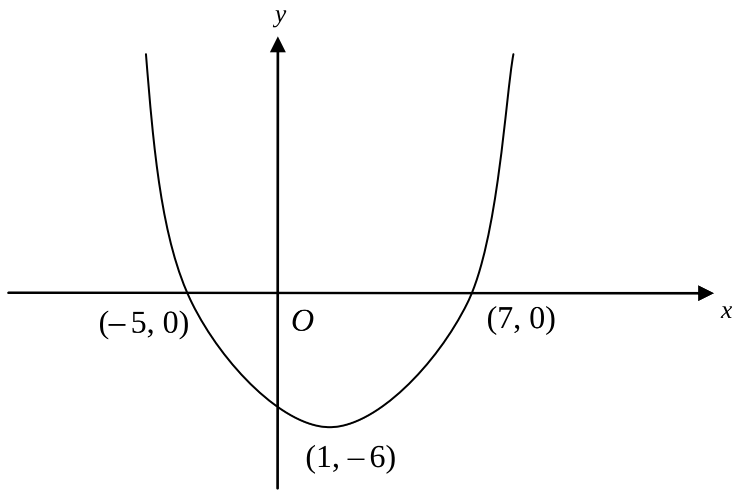
$f(x) = (x + 2)(3x - 2)(3x + 2)$ A1

Roots are $x = -2, 2/3, -2/3$ A1

Special case

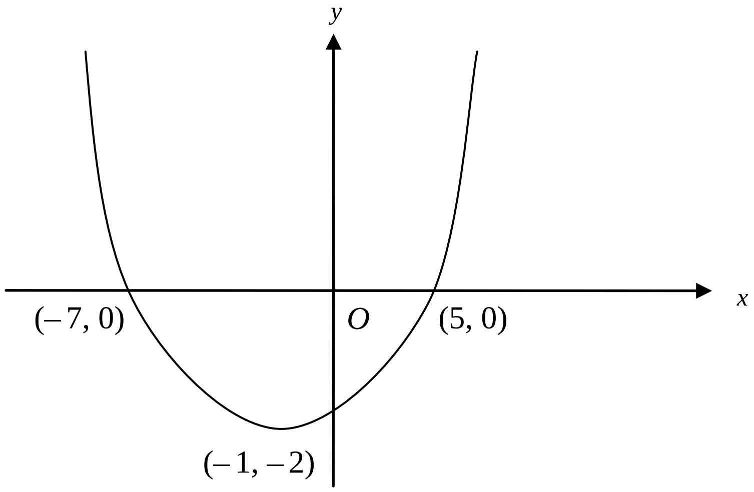
Candidates who find one of the remaining factors, $(3x - 2)$ or $(3x + 2)$, using e.g. factor theorem, are awarded B1

9. (*a*)



Concave up curve and x-coordinate of minimum = 1	B1
y-coordinate of minimum = – 6	B1
Both points of intersection with x-axis	B1

(*b*)



Concave up curve and y-coordinate of minimum = – 2	B1
x-coordinate of minimum = – 1	B1
Both points of intersection with x-axis	B1

Note: A candidate who draws a curve with no changes to the original graph is awarded 0 marks (both parts)

10. (*a*) $\frac{dy}{dx} = 3x^2 - 6x - 9$ B1

Putting candidate's derived $\frac{dy}{dx} = 0$ M1

$x = -1, 3$ (both correct) (f.t. candidate's $\frac{dy}{dx}$) A1

Stationary points are $(-1, 19)$ and $(3, -13)$ (both correct) (c.a.o) A1

A correct method for finding nature of stationary points yielding

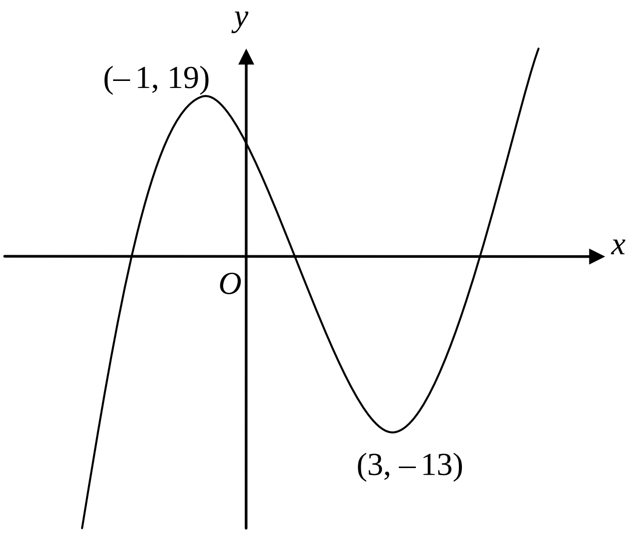
either $(-1, 19)$ is a maximum point

or $(3, -13)$ is a minimum point (f.t. candidate's derived values) M1

Correct conclusion for other point

(f.t. candidate's derived values) A1

(*b*)



Graph in shape of a positive cubic with two turning points M1

Correct marking of both stationary points

(f.t. candidate's derived maximum and minimum points) A1

(*c*) $k < -13$ B1

$19 < k$ B1

(f.t. candidate's *y*-values at stationary points)