Mathematics C1

1. (*a*) (i) Gradient of $AB(CD) = \frac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $AB = -2$, gradient of $CD = -2$,
(or equivalent, at least one correct) A1

Gradient of AB = gradient of $CD \Rightarrow AB$ and CD are parallel
(c.a.o.) A1

(ii) A correct method for finding the equation of AB using candidate's gradient for AB M1

Equation of AB: $y - 14 = -2\,[x - (-5)]$ (or equivalent)
(f.t. candidate's gradient for AB) A1

(iii) Use of gradient $L \times$ gradient $AB = -1$ M1

$\left\{\begin{array}{l}\text{A correct method for finding the equation of } L \text{ using} \\ \text{candidate's gradient for } L\end{array}\right\}$ (M1)

(to be awarded only if corresponding M1 is not awarded in part (ii))

Equation of L : $y - 8 = {}^{1}/_{2}(x - 3)$ (or equivalent)
(f.t. candidate's gradient for AB) A1

Equation of L : $x - 2y + 13 = 0$ (convincing) A1

Note: Total mark for part (*a*) is 8 marks

(*b*) (i) An attempt to solve equations of AB and L simultaneously M1

$x = -1,\ y = 6$ (c.a.o.) A1

(ii) A correct method for finding the coordinates of the mid-point of AB M1

Mid-point of AB has coordinates $(-2, 8)$ A1

A correct method for finding the length of EF M1

$EF = \sqrt{5}$ (f.t. the candidate's derived coordinates for E and F)
A1

2. (*a*) $\frac{9 + 4\sqrt{2}}{5 + 3\sqrt{2}} = \frac{(9 + 4\sqrt{2})(5 - 3\sqrt{2})}{(5 + 3\sqrt{2})(5 - 3\sqrt{2})}$ M1

Numerator: $45 - 27\sqrt{2} + 20\sqrt{2} - 24$ A1

Denominator: $25 - 18$ A1

$\frac{9 + 4\sqrt{2}}{5 + 3\sqrt{2}} = 3 - \sqrt{2}$ (c.a.o.) A1

Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $5 + 3\sqrt{2}$

(*b*) $\sqrt{8} \times \sqrt{10} = 4\sqrt{5}$ B1

$\frac{\sqrt{90}}{\sqrt{2}} = 3\sqrt{5}$ B1

$\frac{30}{\sqrt{5}} = 6\sqrt{5}$ B1

$(\sqrt{8} \times \sqrt{10}) + \frac{\sqrt{90}}{\sqrt{2}} - \frac{30}{\sqrt{5}} = \sqrt{5}$ (c.a.o.) B1

3. y-coordinate of $P = 7$ B1

$\frac{dy}{dx} = 4x - 8$

(an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 3$ in candidate's expression for $\frac{dy}{dx}$ m1

Value of $\frac{dy}{dx}$ at $P = 4$ (c.a.o.) A1

Gradient of normal = $\frac{-1}{\text{candidate's value for } \frac{dy}{dx}}$ m1

Equation of normal to C at P: $y - 7 = -{}^{1}/_{4}(x - 3)$ (or equivalent)

(f.t. candidate's value for $\frac{dy}{dx}$ and the candidate's derived y-value at $x = 3$

provided M1 and both m1's awarded) A1

4. (*a*) $\left(x + \frac{3}{x}\right)^4 = x^4 + 4x^3\left(\frac{3}{x}\right) + 6x^2\left(\frac{3}{x}\right)^2 + 4x\left(\frac{3}{x}\right)^3 + \left(\frac{3}{x}\right)^4$

(all terms correct B2)

(3 or 4 terms correct B1)

$\left(x + \frac{3}{x}\right)^4 = x^4 + 12x^2 + 54 + \frac{108}{x^2} + \frac{81}{x^4}$

(all terms correct B2)

(3 or 4 terms correct B1)

(– 1 for further incorrect simplification)

(*b*) ${}^{n}C_2 \times 2^k = 760$ ($k = 1, 2$) M1

Either $2n^2 - 2n - 760 = 0$ or $n^2 - n - 380 = 0$ or $n(n - 1) = 380$ A1

$n = 20$ (c.a.o.) A1

5. (*a*) $a = 3$ B1

$b = -1$ B1

$c = 2$ B1

(*b*) An attempt to substitute 1 for x in an appropriate quadratic expression

(f.t. candidate's value for b) M1

Maximum value = ${}^{1}/_{8}$ (c.a.o.) A1

6. An expression for $b^2 - 4ac$, with at least two of a, b, c correct M1

$b^2 - 4ac = 4^2 - 4 \times (k + 6) \times (k + 3)$ A1

Putting $b^2 - 4ac < 0$ m1

$k^2 + 9k + 14 > 0$ (convincing) A1

Finding critical values $k = -7$, $k = -2$ B1

A statement (mathematical or otherwise) to the effect that

$k < -7$ or $-2 < k$ (or equivalent)

(f.t. only critical values of ± 7 and ± 2) B2

Deduct 1 mark for each of the following errors:

the use of non-strict inequalities

the use of the word 'and' instead of the word 'or'

7. (*a*) $y + \delta y = 8(x + \delta x)^2 - 5(x + \delta x) - 6$ B1

Subtracting y from above to find δy M1

$\delta y = 16x\delta x + 8(\delta x)^2 - 5\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 16x - 5$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = a \times (-1) \times x^{-2} + 10 \times \frac{1}{2} \times x^{-1/2}$ B1, B1

Attempting to substitute $x = 4$ in candidate's expression for $\frac{dy}{dx}$ and putting expression equal to 3 M1

$a = -8$ (c.a.o.) A1

8. (*a*) Use of $f(3) = 35$ M1

$27a - 63 - 10 = 35 \Rightarrow a = 4$ (convincing) A1

(*b*) Attempting to find $f(r) = 0$ for some value of r M1

$f(-2) = 0 \Rightarrow x + 2$ is a factor A1

$f(x) = (x + 2)(4x^2 + ax + b)$ with one of a, b correct M1

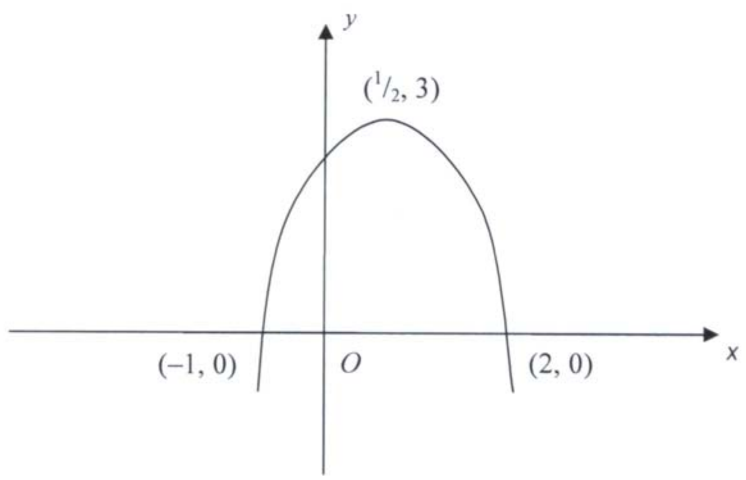
$f(x) = (x + 2)(4x^2 - 8x - 5)$ A1

$f(x) = (x + 2)(2x + 1)(2x - 5)$ (f.t. only $4x^2 + 8x - 5$ in above line) A1

Special case

Candidates who, after having found $x + 2$ as one factor, then find one of the remaining factors by using e.g. the factor theorem, are then awarded B1 instead of the final three marks

9. (*a*)

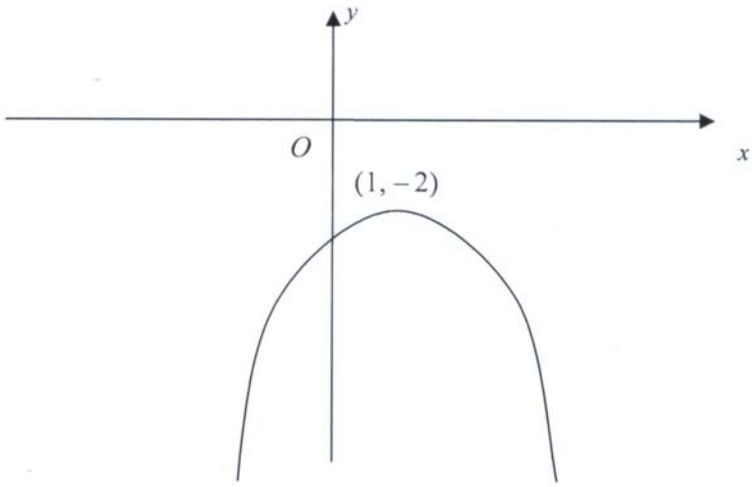


Concave down curve with y-coordinate of maximum = 3 B1

x-coordinate of maximum = $^1/_2$ B1

Both points of intersection with x-axis B1

(*b*)



(i) Concave down curve with x-coordinate of maximum = 1 B1
Graph below x-axis and y-coordinate of maximum = – 2 B1

(ii) No real roots (f.t. the number of times the candidate's curve cuts the x-axis) B1

10. (*a*) $\frac{dy}{dx} = 3x^2 - 12x + 12$ B1

Putting derived $\frac{dy}{dx} = 0$ M1

$3(x-2)^2 = 0 \Rightarrow x = 2$ A1

$x = 2 \Rightarrow y = -1$ (c.a.o) A1

(*b*) **Either:**

An attempt to consider value of $\frac{dy}{dx}$ at $x = 2^-$ and $x = 2^+$ M1

$\frac{dy}{dx}$ has same sign at $x = 2^-$ and $x = 2^+ \Rightarrow (2, -1)$ is a point of inflection A1

Or:

An attempt to find value of $\frac{d^2y}{dx^2}$ at $x = 2$, $x = 2^-$ and $x = 2^+$ M1

$\frac{d^2y}{dx^2} = 0$ at $x = 2$ and $\frac{d^2y}{dx^2}$ has different signs at $x = 2^-$ and $x = 2^+$

$\Rightarrow (2, -1)$ is a point of inflection A1

Or:

An attempt to find the value of y at $x = 2^-$ and $x = 2^+$ M1

Value of y at $x = 2^- < -1$ and value of y at $x = 2^+ > -1 \Rightarrow (2, -1)$ is a point of inflection A1

Or:

An attempt to find values of $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ at $x = 2$ M1

$\frac{d^2y}{dx^2} = 0$ and $\frac{d^3y}{dx^3} \neq 0$ at $x = 2 \Rightarrow (2, -1)$ is a point of inflection A1