WJEC: A2 Unit 3 Pure Mathematics B

5th June 2019 2 hour 30 minutes

PAPER WITH TEACHER'S SOLUTIONS

Total marks available: 120

Reminder: *Sufficient working must be shown to demonstrate the **mathematical** method employed.*

0 1 a) Express $\frac{9}{(x-1)(x+2)^2}$ in terms of partial fractions. [4]

b) Find $\int \frac{9}{(x-1)(x+2)^2}\,dx$. [3]

a) $\frac{9}{(x-1)(x+2)^2} \equiv \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$ $\times (x-1)(x+2)^2$

$9 \equiv A(x+2)^2 + B(x-1)(x+2) + C(x-1)$

Choose values of x to make terms on the right zero.

$x = -2$ $\quad 9 = -3C$

$C = -3$

$x = 1$ $\quad 9 = 9A$

$A = 1$

Substitute in $x = 0$ to find B.

$x = 0$ $\quad 9 = 4A - 2B - C$

$9 = 4(1) - 2B + 3$

$2 = -2B$

$B = -1$

$$= \frac{1}{x-1} - \frac{1}{x+2} - \frac{3}{(x+2)^2}$$

b) $$\int \frac{9}{(x-1)(x+2)^2}\,dx \equiv \int \frac{1}{x-1}\,dx - \int \frac{1}{x+2}\,dx - 3\int (x+2)^{-2}\,dx$$

$$= \ln|x-1| - \ln|x+2| - \frac{3(x+2)^{-1}}{-1} + C$$

$$= \ln|x-1| - \ln|x+2| + \frac{3}{(x+2)} + C$$

0 2 Expand $\frac{4-x}{\sqrt{1+2x}}$ in ascending powers of x up to and including the term in x^3. State the range of values of x for which the expansion is valid. [6]

$= (4-x)(1+2x)^{-1/2}$

Consider binomial expansion of $(1+2x)^{-1/2}$

formula booklet (page 2) gives

$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \cdots + \frac{n(n-1)\cdots(n-r+1)}{1.2.\cdots r}x^r + \cdots$

valid $|x|<1$ $n \in \mathbb{R}$

substitute into this $n = -1/2$ and $x = '2x'$

$(1+2x)^{-1/2} = 1 + (-\frac{1}{2})(2x) + \frac{(-\frac{1}{2})(-\frac{3}{2})(2x)^2}{2} + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})(2x)^3}{6} + \cdots$

(only need upto x^3 term)

$= 1 - x + \frac{3}{2}x^2 - \frac{15}{6}x^3$

$\therefore (4-x)(1+2x)^{-1/2} = (4-x)(1 - x + \frac{3}{2}x^2 - \frac{15}{6}x^3)$

$= 4 - 4x + 6x^2 - 10x^3$
$\quad - x + x^2 - \frac{3}{2}x^3$ } +

$= 4 - 5x + 7x^2 - \frac{23}{2}x^3 + \cdots$

Expansion valid $|x|<1$

but $x = '2x'$ $|2x|<1$

$|x| < \frac{1}{2}$

0 3 The nth term of a number sequence is denoted by x_n. The $(n + 1)$th term is defined by $x_{n+1} = 4x_n - 3$ and $x_3 = 113$.

a) Find the values of x_2 and x_1. [2]

b) Determine whether the sequence is an arithmetic sequence, a geometric sequence or neither. Give reasons for your answer. [2]

a) $x_{n+1} = 4x_n - 3$

$x_3 = 113$

$\therefore \quad 113 = 4x_2 - 3$

$116 = 4x_2$

$\frac{116}{4} = x_2$

$x_2 = 29$

$29 = 4x_1 - 3$

$32 = 4x_1$

$\frac{32}{4} = x_1$

$x_1 = 8$

b) If the sequence is arithmetic, it is of the form

$a, a+d, a+2d$

so we would get a common difference

8 , 29 , 113

+21 +84

There isn't a common difference, $21 \neq 84$

If the sequence is geometric, it is of the form

a, ar, ar^2

so we would get a common ratio

8 , 29 , 113

×3·625 ×3·897

There isn't a common ratio, $3{\cdot}625 \neq 3{\cdot}897$

The sequence is neither a geometric nor an arithmetic sequence.

0	4

a) Express $5\sin x - 12\cos x$ in the form $R\sin(x-\alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

b) Find the minimum value of $\dfrac{4}{5\sin x - 12\cos x + 15}$. [2]

c) Solve the equation

$$5\sin x - 12\cos x + 3 = 0$$

for values of x between 0° and 360°. [4]

a) Formula booklet gives

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\therefore$ $\sin(x-\alpha) = \sin x\cos\alpha - \cos x\sin\alpha$

So, expressing $5\sin x - 12\cos x$ in form $R\sin(x-\alpha)$ gives

$5\sin x - 12\cos x = R\sin x\cos\alpha - R\cos x\sin\alpha$

Equating $\sin x$ terms gives $5 = R\cos\alpha$ ①

Equating $\cos x$ terms gives $-12 = -R\sin\alpha$

$12 = R\sin\alpha$ ②

squaring ① gives $25 = R^2\cos^2\alpha$

squaring ② gives $144 = R^2\sin^2\alpha$

adding gives $169 = R^2(\sin^2\alpha + \cos^2\alpha)$

and $\sin^2\alpha + \cos^2\alpha = 1$

$\therefore$ $169 = R^2$, $R = 13$

② ÷ ① gives $\dfrac{12}{5} = \dfrac{R\sin\alpha}{R\cos\alpha}$, $\tan\alpha = 12/5$

$\alpha = 67.38°$

$= 13\sin(x - 67.38°)$

b) $\dfrac{4}{5\sin x - 12\cos x + 15} = \dfrac{4}{13\sin(x - 67.38^\circ) + 15}$

Require MINIMUM of the fraction. This means we want the denominator to be as big as possible

$\therefore$ choose $\sin(x - 67.38^\circ) = 1$

$\underline{\text{MINIMUM}} = \dfrac{4}{13+15} = \dfrac{4}{28} = \underline{\dfrac{1}{7}}$

c) $5\sin x - 12\cos x + 3 = 0$

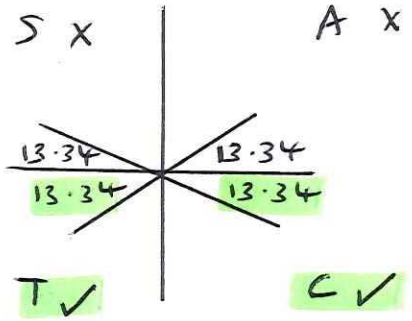
$13\sin(x - 67.38^\circ) + 3 = 0$

$\sin(x - 67.38) = \dfrac{-3}{13}$

$x - 67.38^\circ = -13.34^\circ$

$x = -13.34^\circ + 67.38^\circ$

$\underline{x = 54.04^\circ}$



second solution from CAST diagram

$x - 67.38^\circ = 193.34^\circ$

$x = 193.34 + 67.38^\circ$

$\underline{x = 260.72^\circ}$

0 5 **a)** Find the range of values of x for which $|1 - 3x| > 7$. [3]

b) Sketch the graph of $y = |1 - 3x| - 7$. Clearly label the minimum point and the points where the graph crosses the x-axis. [4]

a)

Either :-

$1 - 3x > 7$

$1 > 7 + 3x$

$-6 > 3x$

$-2 > x$

$x < -2$

or

$1 - 3x < -7$

$1 < -7 + 3x$

$8 < 3x$

$\frac{8}{3} < x$

$x > \frac{8}{3}$

Or :-

$(1 - 3x)^2 > 7^2$

$(1 - 3x)(1 - 3x) > 49$

$1 - 6x + 9x^2 > 49$

$9x^2 - 6x - 48 > 0 \quad \div 3$

$3x^2 - 2x - 16 > 0$

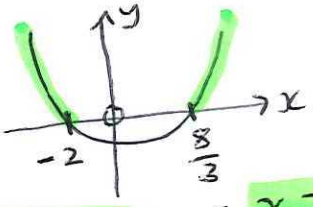
$\times ac$	$+b$
-48	-2

$6, -8$

$3x^2 + 6x - 8x - 16 > 0$

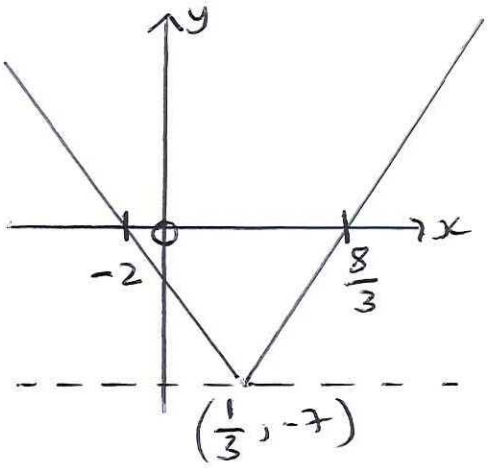
$3x(x + 2) - 8(x + 2) > 0$

$(3x - 8)(x + 2) > 0$



$x < -2$ or $x > \frac{8}{3}$

b) $y = |1 - 3x| - 7$



0 6 A curve C has parametric equations $x = \sin\theta$, $y = \cos 2\theta$.

a) The equation of the tangent to the curve C at the point P where $\theta = \frac{\pi}{4}$ is $y = mx + c$. Find the exact values of m and c. [6]

b) Find the coordinates of the points of intersection of the curve C and the straight line $x + y = 1$. [5]

a) $x = \sin\theta$ $\quad \frac{dx}{d\theta} = \cos\theta$ $\quad y = \cos 2\theta$ $\quad \frac{dy}{d\theta} = -2\sin 2\theta$

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-2\sin 2\theta}{\cos\theta}$ $\quad (\sin 2\theta = 2\sin\theta\cos\theta)$

$= \frac{-2\times(2\sin\theta\cos\theta)}{\cos\theta}$

$= -4\sin\theta$

Point $P = (\sin\frac{\pi}{4}, \cos\frac{\pi}{2})$ because $\theta = \pi/4$, $P = (\frac{1}{\sqrt{2}}, 0)$

gradient of tangent at $P = -4\sin\pi/4 = -2\sqrt{2} = m$

$y = mx + c$ gives, $0 = -2\sqrt{2}(\frac{1}{\sqrt{2}}) + c$

$0 = -2 + c$, $c = 2$

equation of tangent $y = -2\sqrt{2}x + 2$

b) $x + y = 1$ and $x = \sin\theta$ and $y = \cos 2\theta$

$\therefore \sin\theta + \cos 2\theta = 1$ (use $\cos 2\theta \equiv 1 - 2\sin^2\theta$)

$\sin\theta + 1 - 2\sin^2\theta - 1 = 0$

$-2\sin^2\theta + \sin\theta = 0$

$2\sin^2\theta - \sin\theta = 0$

$\sin\theta(2\sin\theta - 1) = 0$

$\sin\theta = 0$ $\quad 2\sin\theta = 1$, $\sin\theta = \frac{1}{2}$

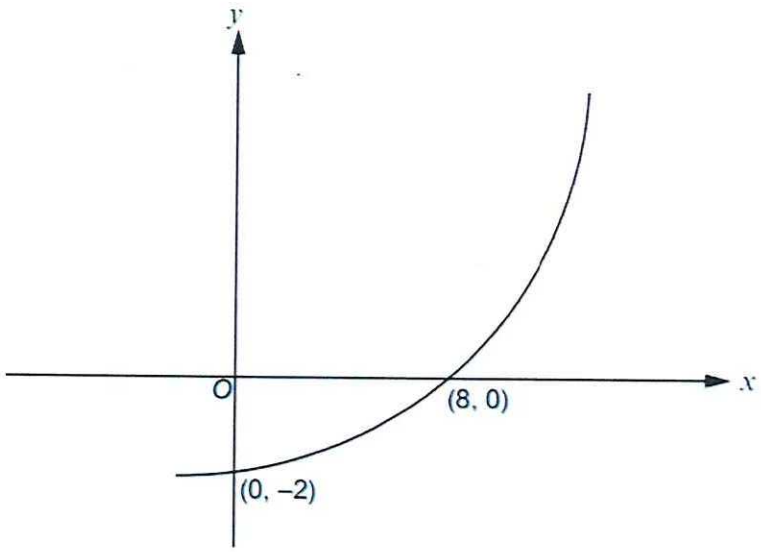
$x = \sin\theta$ $\therefore$ $x = 0$, $x = \frac{1}{2}$

$y = \cos 2\theta$, $y = 1 - 2\sin^2\theta$ $\therefore$ $y = 1$, $y = \frac{1}{2}$

Points of intersection are:-

$(0, 1)$ and $(\frac{1}{2}, \frac{1}{2})$

0 7 The diagram below shows a sketch of the graph of $y = f(x)$. The graph crosses the y-axis at the point (0, –2), and the x-axis at the point (8, 0).



a) Sketch the graph of $y = -4f(x + 3)$. Indicate the coordinates of the point where the graph crosses the x-axis and the y-coordinate of the point where $x = -3$. [3]

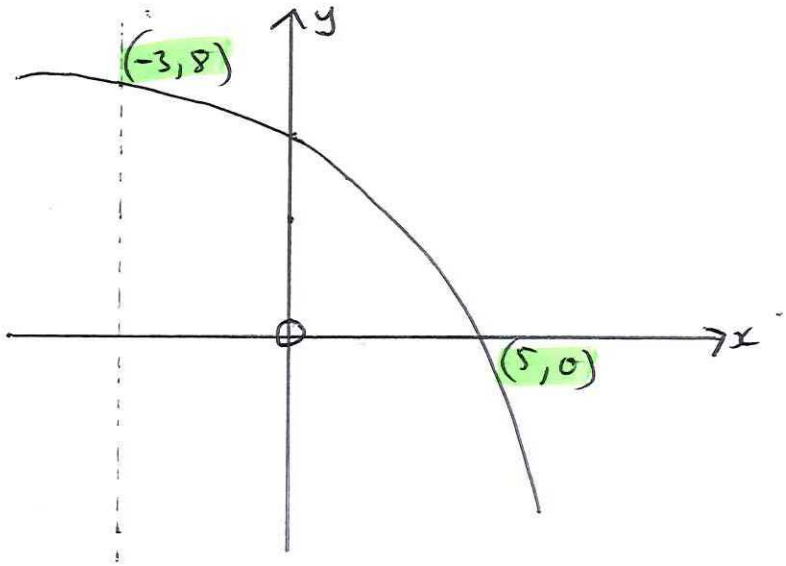
b) Sketch the graph of $y = 3 + f(2x)$. Indicate the y-coordinate of the point where $x = 4$. [2]

a) Consider $y = -4f(x+3)$

translation, -3, x direction

stretch x4 y direction

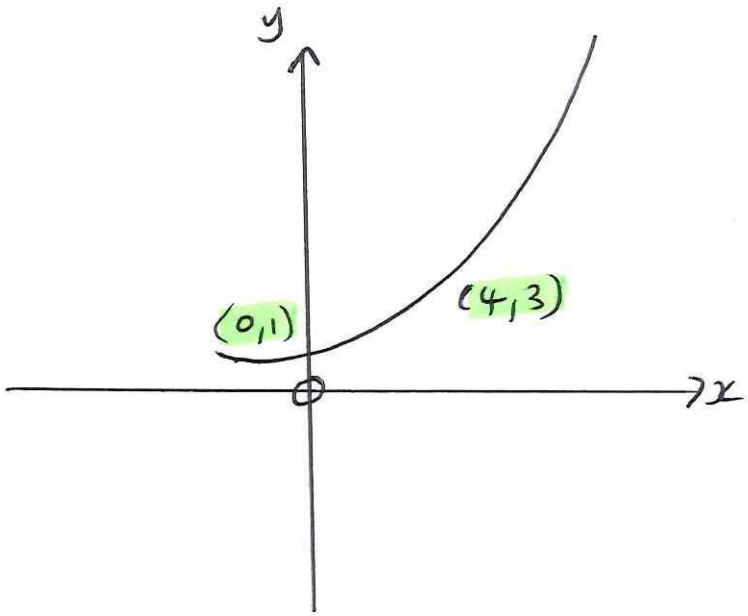
reflected across x axis, 'y' coordinates change



b) Consider $y = 3 + f(2x)$

stretch, $\times \frac{1}{2}$ x direction

translation, +3, y direction



0 8 **a)** The 3rd, 19th and 67th terms of an arithmetic sequence form a geometric sequence. Given that the arithmetic sequence is increasing and that the first term is 3, find the common difference of the arithmetic sequence. [5]

b) A firm has 100 employees on a particular Monday. The next day it adds 12 employees onto its staff and continues to do so on every successive working day, from Monday to Friday.

i) Find the number of employees at the end of the 8th week.

ii) Each employee is paid £55 per working day. Determine the total wage bill for the 8 week period. [6]

a) Let the common difference of the arithmetic sequence be 'd'

1st term = 3
3rd term = $3+2d$
19th term = $3+18d$
67th term = $3+66d$

Now, $3+2d$, $3+18d$, $3+66d$ form a geometric sequence with common ratio 'r'.

$\therefore \quad r = \frac{3+18d}{3+2d}$ and $r = \frac{3+66d}{3+18d}$

so, $\frac{3+66d}{3+18d} = \frac{3+18d}{3+2d}$

$(3+66d)(3+2d) = (3+18d)(3+18d)$

$9+198d+6d+132d^2 = 9+54d+54d+324d^2$

$0 = 192d^2 - 96d \quad (\div 96)$

$0 = 2d^2 - d$

$0 = d(2d-1)$

$d = 0$ and $d = \frac{1}{2}$

$d = 0$ discard not possible

common difference $= \frac{1}{2}$

b) i) 8 weeks, 40 working days (Mon → Fri)

$\therefore a = 100$

$d = 12$

$n = 40$

40th term = $a + 39d$

$= 100 + 39 \times 12$

$= 568$

568 employees at the end of 8th week

ii) We need to work out the total number of staff days.

$a, a+d, a+2d, \ldots, a+39d$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{40} = \frac{40}{2}[2(100) + 39(12)]$

$= 13360$

Each of these earn €55

$55 \times 13360 = €734,800$

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0 9 **a)** Given that α and β are two angles such that $\tan\alpha = 2\cot\beta$, show that

$$\tan(\alpha + \beta) = -(\tan\alpha + \tan\beta).$$ [2]

b) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying the equation

$$4\tan\theta = 3\sec^2\theta - 7.$$ [6]

a) $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$ (formula booklet)

given $\tan\alpha = 2\cot\beta$

$\therefore = \dfrac{\tan\alpha + \tan\beta}{1 - 2\cot\beta\tan\beta}$

$= \dfrac{\tan\alpha + \tan\beta}{-1} = -(\tan\alpha + \tan\beta)$

b) $4\tan\theta = 3\sec^2\theta - 7$ using $\sec^2\theta \equiv 1 + \tan^2\theta$

$4\tan\theta = 3 + 3\tan^2\theta - 7$

$3\tan^2\theta - 4\tan\theta - 4 = 0$

$\times a z$	$+b$
-12	-4

$-6, 2$

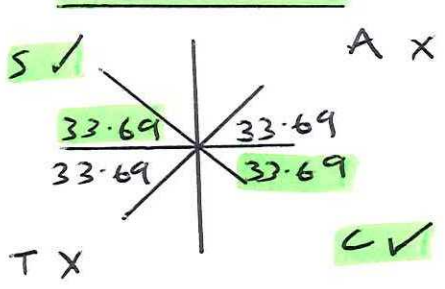
$3\tan^2\theta - 6\tan\theta + 2\tan\theta - 4 = 0$

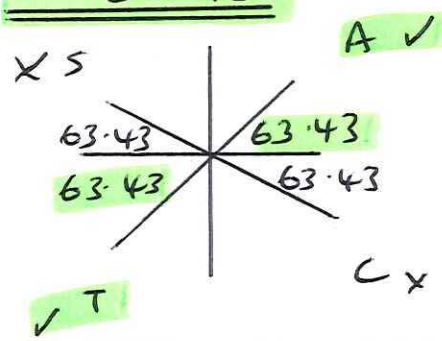
$3\tan\theta(\tan\theta - 2) + 2(\tan\theta - 2) = 0$

$(3\tan\theta + 2)(\tan\theta - 2) = 0$

$\tan\theta = -\frac{2}{3}$ $\qquad$ $\tan\theta = 2$

$\theta = -33.69°$ $\qquad$ $\theta = 63.43°$





$\theta = 146.31°, 326.31°$ $\qquad$ $\theta = 63.43°, 243.43°$

10 a) Differentiate each of the following functions with respect to x.

i) $x^5 \ln x$

ii) $\dfrac{e^{3x}}{x^3 - 1}$

iii) $(\tan x + 7x)^{\frac{1}{5}}$ [8]

b) A function is defined implicitly by

$$3y + 4xy^2 - 5x^3 = 8.$$

Find $\dfrac{dy}{dx}$ in terms of x and y. [3]

ai) $y = x^5 \ln x$ $\quad u = x^5$ $\quad v = \ln x$

$\dfrac{du}{dx} = 5x^4$ $\quad \dfrac{dv}{dx} = \dfrac{1}{x}$

$\dfrac{dy}{dx} = v\dfrac{du}{dx} + u\dfrac{dv}{dx}$ (product rule)

$\dfrac{dy}{dx} = \ln x\,(5x^4) + x^5\left(\dfrac{1}{x}\right)$

$\dfrac{dy}{dx} = 5x^4 \ln x + x^4$

ii) $y = \dfrac{e^{3x}}{x^3 - 1} = \dfrac{u}{v}$ $\quad u = e^{3x}$ $\quad v = x^3 - 1$

$\dfrac{du}{dx} = 3e^{3x}$ $\quad \dfrac{dv}{dx} = 3x^2$

$\dfrac{dy}{dx} = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ (quotient rule)

$\dfrac{dy}{dx} = \dfrac{(x^3 - 1)(3e^{3x}) - (e^{3x})(3x^2)}{(x^3 - 1)^2}$ ← (doesn't ask to 'simplify' so this gets all the marks)

iii) $y = (\tan x + 7x)^{1/2}$ Let $u = \tan x + 7x$

$y = u^{1/2}$ $\quad \frac{dy}{du} = \frac{1}{2}u^{-1/2}$ $\quad \frac{du}{dx} = \sec^2 x + 7$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ (chain rule)

$\frac{dy}{dx} = \frac{1}{2}u^{-1/2}(\sec^2 x + 7)$

$\frac{dy}{dx} = \frac{1}{2}(\tan x + 7x)^{-1/2}(\sec^2 x + 7)$

b) $3y + 4xy^2 - 5x^3 = 8$ Be careful here!

$3\frac{dy}{dx} + 4y^2 + 4x(2y\frac{dy}{dx}) - 15x^2 = 0$

$3\frac{dy}{dx} + 4y^2 + 8xy\frac{dy}{dx} - 15x^2 = 0$

$\frac{dy}{dx}(3 + 8xy) = 15x^2 - 4y^2$

$\frac{dy}{dx} = \frac{15x^2 - 4y^2}{3 + 8xy}$

1 1 The function $f(x)$ is defined by

$$f(x) = \frac{\sqrt{x^2-1}}{x}$$

with domain $x \geqslant 1$.

a) Find an expression for $f^{-1}(x)$. State the domain for f^{-1} and sketch both $f(x)$ and $f^{-1}(x)$ on the same diagram. [6]

b) Explain why the function $ff(x)$ cannot be formed. [1]

a) Let $y = \frac{\sqrt{x^2-1}}{x}$

$xy = \sqrt{x^2-1}$, squaring both sides gives

$x^2y^2 = x^2 - 1$

$1 = x^2 - x^2y^2$

$1 = x^2(1-y^2)$

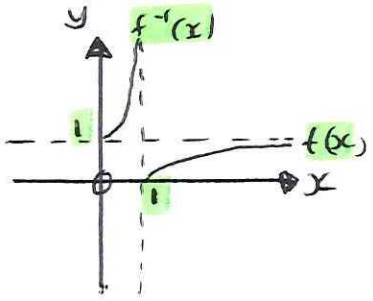
$\frac{1}{1-y^2} = x^2$, square root both sides

$$x = \pm\sqrt{\frac{1}{1-y^2}}$$

This gives $f^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$ because domain of $x \geqslant 1$

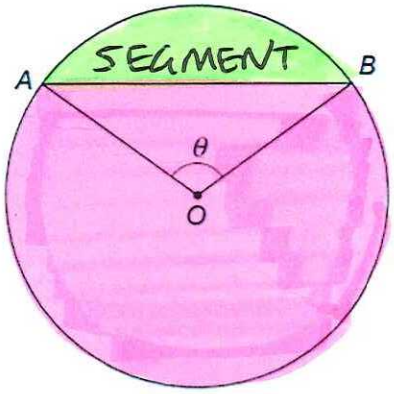
$D(f) = [1, \infty)$, $R(f) = [0, 1)$

Domain of $f^{-1}(x)$ is range of $f(x)$ $D(f^{-1}) = [0, 1)$



b) $ff(x)$ cannot be formed because the range of $f(x)$ is not in the domain of $f(x)$.

1 2 A chord AB subtends an angle θ radians at the centre of a circle. The chord divides the circle into two segments whose areas are in the ratio 1 : 2.



a) Show that $\sin\theta = \theta - \frac{2\pi}{3}$. [4]

b) **i)** Show that θ lies between 2·6 and 2·7.

ii) Starting with $\theta_0 = 2{\cdot}6$, use the Newton-Raphson Method to find the value of θ correct to three decimal places. [6]

a) Area sector AOB $= \frac{1}{2}r^2\theta$

Area triangle AOB $= \frac{1}{2}r^2\sin\theta$

Area shaded segment $= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

Area pink segment $= \pi r^2 -$ area green segment

$= \pi r^2 - \left(\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta\right)$

given, area pink segment $= 2 \times$ area green segment

$$\pi r^2 - \frac{1}{2}r^2\theta + \frac{1}{2}r^2\sin^2\theta = 2\left(\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta\right)$$

$$\pi r^2 = \frac{3}{2}r^2\theta - \frac{3}{2}r^2\sin\theta \quad (\div r^2)$$

$$\pi = \frac{3}{2}\theta - \frac{3}{2}\sin\theta$$

$$\frac{3}{2}\sin\theta = \frac{3}{2}\theta - \pi \quad \left(\times \frac{2}{3}\right)$$

$\sin\theta = \theta - \frac{2\pi}{3}$ as required

bi) Let $f(\theta) = \theta - \sin\theta - \frac{2\pi}{3}$ (remember calculator in radian mode)

$f(2.6) = -0.00989647$ < 0

$f(2.7) = 0.17822501 74$ > 0

change of sign, therefore $2.6 < \theta < 2.7$

bii) $f(\theta) = \theta - \sin\theta - \frac{2\pi}{3}$

$f'(\theta) = 1 - \cos\theta$ and $\theta_{n+1} = \theta_n - \frac{f(\theta_n)}{f'(\theta_n)}$ (Newton Raphson iteration given in formula booklet)

$$\theta_{n+1} = \theta_n - \frac{\theta_n - \sin\theta_n - 2\pi/3}{1 - \cos\theta_n}$$

$\theta_0 = 2.6$

$\theta_1 = 2.6 - \frac{2.6 - \sin 2.6 - 2\pi/3}{1 - \cos 2.6} = 2.60532 96$

$\theta_2 = 2.605325675$

$\theta = 2.605$ (3dp)

1 3 Wildflowers grow on the grass verge by the side of a motorway. The area populated by wildflowers at time t years is $A\,m^2$. The rate of increase of A is directly proportional to A.

a) Write down a differential equation that is satisfied by A. [1]

b) At time $t = 0$, the area populated by wildflowers is $0{\cdot}2\,m^2$. One year later, the area has increased to $1{\cdot}48\,m^2$. Find an expression for A in terms of t in the form pq^t, where p and q are rational numbers to be determined. [7]

a) "The rate of increase of A is directly proportional to A"

$$\frac{dA}{dt} = kA$$

b) $\frac{1}{A}\frac{dA}{dt} = k$, now integrate both sides with respect to t.

$$\int \frac{1}{A}\frac{dA}{dt}\,dt = \int k\,dt$$

$$\int \frac{1}{A}\,dA = \int k\,dt$$

$$\ln A = kt + C$$

given, at $t=0$, $A=0{\cdot}2$

$\ln 0{\cdot}2 = 0 + C$, $C = \ln 0{\cdot}2$

$\ln A = kt + \ln 0{\cdot}2$

$\ln A - \ln 0{\cdot}2 = kt$ (subtraction law)

$$\ln \frac{A}{0{\cdot}2} = kt$$

given, at $t=1$, $A=1{\cdot}48$

$$\ln \frac{1{\cdot}48}{0{\cdot}2} = k$$

$\ln 7{\cdot}4 = k$ (rewrite as a power)

$e^k = 7{\cdot}4$ $K = 2{\cdot}00148$

* rewrite $\ln \frac{A}{0{\cdot}2} = kt$ as a power

$e^{kt} = \frac{A}{0{\cdot}2}$

$0{\cdot}2e^{kt} = A$, $A = 0{\cdot}2(e^k)^t$ and $e^k = 7{\cdot}4$

$A = 0{\cdot}2(7{\cdot}4)^t$

1	4

a) Find $\int (e^{2x} + 6\sin 3x)\,dx$. [2]

b) Find $\int 7(x^2 + \sin x)^6(2x + \cos x)\,dx$. [1]

c) Find $\int \frac{1}{x^2}\ln x\,dx$. [4]

d) Use the substitution $u = 2\cos x + 1$ to evaluate

$$\int_0^{\frac{\pi}{3}} \frac{\sin x}{(2\cos x+1)^2}\,dx.$$ [4]

a) $= \int e^{2x}dx + \int 6\sin 3x\,dx$

$= \frac{e^{2x}}{2} + \frac{6(-\cos 3x)}{3} + C$

$= \frac{1}{2}e^{2x} - 2\cos 3x + C$

b) Note $\int (x^2+\sin x)^6(2x+\cos x)\,dx$ is of the format

$\int [f(x)]^n f'(x)\,dx = \frac{1}{n+1}[f(x)]^{n+1} + C \quad n \neq -1$

$\therefore 7\int (x^2+\sin x)^6(2x+\cos x)\,dx = 7 \cdot \frac{1}{7}(x^2+\sin x)^7 + C$

$= (x^2+\sin x)^7 + C$

c) $\int \frac{1}{x^2}\ln x\,dx$ ($\frac{1}{x^2}$: dv/dx, $\ln x$: u), use integration by parts

$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$ formula booklet

$u = \ln x \quad \frac{du}{dx} = \frac{1}{x} \quad \frac{dv}{dx} = x^{-2} \quad v = \frac{x^{-1}}{-1} = -\frac{1}{x}$

$$\int \frac{1}{x^2}\ln x\,dx = \ln x\left(\frac{-1}{x}\right) - \int\left(\frac{-1}{x}\right)\frac{1}{x}\,dx$$

$$= -\frac{1}{x}\ln x + \int x^{-2}\,dx$$

$$= -\frac{1}{x}\ln x + \frac{x^{-1}}{-1} + C$$

$$= \underline{\underline{-\frac{1}{x}\ln x - \frac{1}{x} + C}}$$

d) $\int_0^{\pi/3} \frac{\sin x}{(2\cos x+1)^2}\,dx$

$u = 2\cos x + 1$

$\frac{du}{dx} = -2\sin x$

$-\frac{1}{2}\,du = \sin x\,dx$

when $x=0$, $u = 2\cos 0 + 1$, $u=3$

when $x = \frac{\pi}{3}$ $u = 2\cos\frac{\pi}{3} + 1$, $u=2$

(careful, calculator in radian mode)

$$= \int_3^2 -\frac{1}{2}\left(\frac{1}{u^2}\right)du$$

$$= -\frac{1}{2}\int_3^2 u^{-2}\,du$$

$$= -\frac{1}{2}\left[\frac{u^{-1}}{-1}\right]_3^2$$

$$= \frac{1}{2}\left[\frac{1}{u}\right]_3^2$$

$$= \frac{1}{2}\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$= \underline{\underline{\frac{1}{12}}}$$

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1	5	Use proof by contradiction to show that $\sqrt{6}$ is irrational.	[5]

Assume that $\sqrt{6}$ is rational

This mean that $\sqrt{6}$ can be written as $\frac{a}{b}$

where a and b are integers with no common factors (except 1).

$\sqrt{6} = \frac{a}{b}$

squaring both sides gives

$6 = \frac{a^2}{b^2}$

$6b^2 = a^2$

$\therefore a^2$ is an even number

$\therefore a$ is an even number

so, $a = 2k$

so, $6b^2 = (2k)^2$

$6b^2 = 4k^2$

$3b^2 = 2k^2$

$\therefore b^2$ is an even number

$\therefore b$ is an even number

so, $b = 2h$

This means a and b have a common factor of 2.

Contradiction. Hence, $\sqrt{6}$ irrational