WJEC: A2 Unit 3 Pure Mathematics B

6th June 2018 2 hour 30 minutes

PAPER WITH TEACHER'S SOLUTIONS

Total marks available: 120

Reminder: *Sufficient working must be shown to demonstrate the **mathematical** method employed.*

0 1 Solve the equation

$$|2x + 1| = 3|x - 2|.$$ [4]

Either $2x+1 = 3(x-2)$

$2x+1 = 3x-6$

$1+6 = x$

$x = 7$

or $2x+1 = -3(x-2)$

$2x+1 = -3x+6$

$5x = 6-1$

$5x = 5$

$x = 1$

OR Squaring gives

$(2x+1)^2 = 9(x-2)^2$

$4x^2+2x+2x+1 = 9x^2-18x-18x+36$

$0 = 5x^2-40x+35$

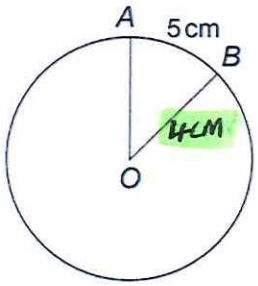
$5x^2-40x+35 = 0 \quad (\div 5)$

$x^2-8x+7 = 0$

$(x-7)(x-1) = 0$

$x = 7$, $x = 1$

0 2 The diagram below shows a circle centre O, radius 4 cm. Points A and B lie on the circumference such that arc AB is 5 cm.



a) Calculate the angle subtended at O by the arc AB. [2]

b) Determine the area of the sector OAB. [2]

a) Arc length $s = r\theta$

$5 = 4\theta$

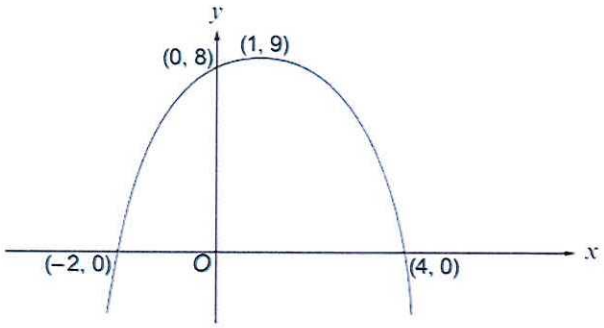
$\frac{5}{4} = \theta$, $\theta = 1.25^c$

b) Area sector $A = \frac{1}{2} r^2 \theta$

$A = \frac{1}{2} \times 4^2 \times 1.25$

$A = 10 cm^2$

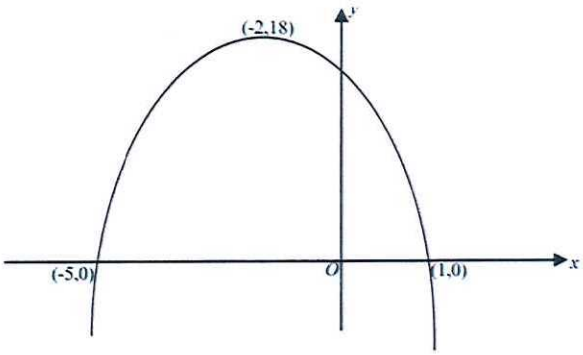
0 3 The diagram below shows a sketch of the graph of $y = f(x)$. The graph passes through the points $(-2, 0)$, $(0, 8)$, $(4, 0)$ and has a maximum point at $(1, 9)$.



a) Sketch the graph of $y = 2f(x + 3)$. Indicate the coordinates of the stationary point and the points where the graph crosses the x-axis. [3]

b) Sketch the graph of $y = 5 - f(x)$. Indicate the coordinates of the stationary point and the point where the graph crosses the y-axis. [3]

a)



$y = 2f(x+3)$

translation, 3 to the left along x axis

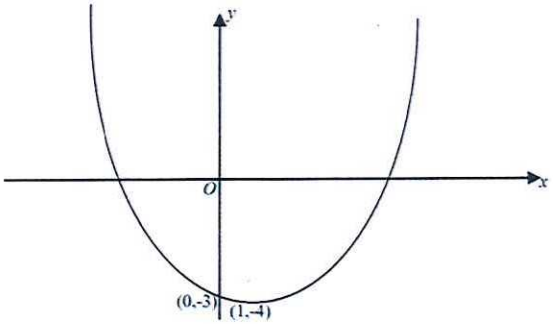
stretch x2 y direction

B1 correct shape (hill) axes required

B1 (-2, 18) as max

B1 (-5, 0), (1, 0)

b)



$y = 5 - f(x)$

$y = -f(x) + 5$

reflection across x axis

y values change

translation up 5 y axis

B1 correct shape(cup) axes required

B1 (1, -4) as min

B1 (0, -3)

0 4 Solve the equation

$$2\tan^2\theta + 2\tan\theta - \sec^2\theta = 2,$$

for values of θ between 0° and 360°. [5]

Use $\sec^2\theta \equiv 1+\tan^2\theta$ to eliminate $\sec^2\theta$

$2\tan^2\theta + 2\tan\theta - \sec^2\theta = 2$

$2\tan^2\theta + 2\tan\theta - (1+\tan^2\theta) = 2$

$\tan^2\theta + \tan\theta - 3 = 0$

$(\tan\theta - 1)(\tan\theta + 3) = 0$

$\tan\theta = 1$, $\tan\theta = -3$

$\theta = 45°, 225°$ $\theta = -71.57°, 108.43°, 288.43$

$0 \leq \theta \leq 360°$

$\theta = 45°, 108.43°, 225°, 288.43°$

(Note, no need to draw a CAST diagram, as tan repeats every 180°, so just ±180 to the initial solution given on the calculator, to get all the solutions in the required range.)

0 5 a) Show that

$$\frac{3x}{(x-1)(x-4)^2} \equiv \frac{A}{(x-1)} + \frac{B}{(x-4)} + \frac{C}{(x-4)^2}, \quad \times (x-1)(x-4)^2$$

where A, B and C are constants to be found. [3]

b) Evaluate $\int_5^7 \frac{3x}{(x-1)(x-4)^2}\,dx$, giving your answer correct to 3 decimal places. [5]

a) $3x = A(x-4)^2 + B(x-1)(x-4) + C(x-1)$

Choose values of x to make terms on the right zero.

$x=4$ $\quad 12 = 3C$, $C=4$

$x=1$ $\quad 3 = 9A$, $A=\frac{1}{3}$

substitute in $x=0$ to find B

$0 = 16A + 4B - C$

$0 = \frac{16}{3} + 4B - 4$

$-\frac{4}{3} = 4B$, $B=-\frac{1}{3}$

$$= \frac{1}{3(x-1)} - \frac{1}{3(x-4)} + \frac{4}{(x-4)^2}$$

$4(x-4)^{-2}$

b) Required integral $I = \int_5^7 \frac{1}{3(x-1)}\,dx - \int_5^7 \frac{1}{3(x-4)}\,dx + \int_5^7 \frac{4}{(x-4)^2}\,dx$

$$= \frac{1}{3}\Big[\ln|x-1|\Big]_5^7 - \frac{1}{3}\Big[\ln|x-4|\Big]_5^7 + 4\left[\frac{(x-4)^{-1}}{-1}\right]_5^7$$

$$= \frac{1}{3}\left[\ln 6 - \ln 4\right] - \frac{1}{3}\left[\ln 3 - \ln 1\right] - 4\left[\frac{1}{3} - 1\right]$$

$\ln 1 = 0$

$$= \frac{1}{3}\left[\ln 6 - \ln 4 - \ln 3\right] + \frac{8}{3}$$

$$= \frac{1}{3}\left[\ln\left(\frac{6}{4}\right) - \ln 3\right] + \frac{8}{3} = \frac{1}{3}\left(\ln\left(\frac{6/4}{3}\right)\right) + \frac{8}{3}$$

$$= \frac{1}{3}\left(\ln\left(\frac{1}{2}\right) + 8\right) = 2.436$$

0 6 Write down the first three terms in the binomial expansion of $(1-4x)^{\frac{1}{2}}$ in ascending powers of x. State the range of values of x for which the expansion is valid. By writing $x = \frac{1}{13}$ in your expansion, find an approximate value for $\sqrt{13}$ in the form $\frac{a}{b}$, where a, b are integers. [5]

Consider binomial expansion of $(1+x)^n$, page 2 formula booklet :-

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\cdot 2}x^2 + \cdots + \frac{n(n-1)\cdots(n-r+1)}{1\cdot 2\cdots r}x^r + \cdots$$

valid, $|x|<1$, $n \in \mathbb{R}$

substitute into this, $n = -\frac{1}{2}$ and $x = (-4x)$

$$(1-4x)^{-1/2} = 1 - \frac{1}{2}(-4x) + \frac{(-\frac{1}{2})(-\frac{3}{2})(-4x)^2}{2} + \cdots$$

{ only need first 3 terms }

$$= 1 + 2x + 6x^2 + \cdots$$

valid, $|4x|<1$

$|x| < \frac{1}{4}$

when $x = \frac{1}{13}$, expansion becomes

$$\left(1 - \frac{4}{13}\right)^{-1/2} = 1 + 2\left(\frac{1}{13}\right) + 6\left(\frac{1}{13}\right)^2 + \cdots$$

$$\left(\frac{9}{13}\right)^{-1/2} = 1 + \frac{2}{13} + \frac{6}{169} + \cdots$$

$$\sqrt{\frac{13}{9}} \approx \frac{15}{13} + \frac{6}{169}$$

$$\frac{\sqrt{13}}{\sqrt{9}} \approx \frac{195}{169} + \frac{6}{169} = \frac{201}{169}$$

$$\frac{\sqrt{13}}{3} \approx \frac{201}{169}$$

$$\sqrt{13} \approx 3 \times \frac{201}{169}$$

$$\sqrt{13} \approx \frac{603}{169}$$

0 7 Use small angle approximations to find the small negative root of the equation

$$\sin x + \cos x = 0{\cdot}5.$$ [3]

Remember your small angle approximations:-

- $\sin\theta \approx \theta$
- $\cos\theta \approx 1 - \frac{\theta^2}{2}$
- $\tan\theta \approx \theta$

$\therefore$ $\sin x + \cos x = 0.5$

becomes

$$x + 1 - \frac{x^2}{2} = 0.5$$

$$0 = \frac{x^2}{2} - x - 0.5 \qquad (\times 2)$$

$$0 = x^2 - 2x - 1$$

Won't factorise, use quadratic formula

$$x = \frac{2 \pm \sqrt{2^2 - 4(-1)}}{2}$$

$$x = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = \frac{2(1 \pm \sqrt{2})}{2}$$

$$x = 1 \pm \sqrt{2}$$

Require small negative root :- $x = 1 - \sqrt{2}$

$$x = -0.4142$$

0	8	Find seven numbers which are in arithmetic progression such that the last term is 71 and the sum of all of the numbers is 329. [5]

Seventh term = 71

$a+6d=71$ ①

$S_7 = 329$

use, $S_n = \frac{n}{2}[2a+(n-1)d]$

$329 = \frac{7}{2}[2a+6d]$

$94 = 2a+6d$

$47 = a+3d$ ②

① - ② gives $24 = 3d$

$d=8$

in ①, $a+6(8)=71$

$a = 71-48$

$a=23$

23, 31, 39, 47, 55, 63, 71

0 9

a) Explain why the sum to infinity of a geometric series with common ratio r only converges when $|r|<1$. [1]

b) A geometric progression V has first term 2 and common ratio r. Another progression W is formed by squaring each term in V. Show that W is also a geometric progression. Given that the sum to infinity of W is 3 times the sum to infinity of V, find the value of r. [6]

c) At the beginning of each year, a man invests £5000 in a savings account earning compound interest at the rate of 3% per annum. The interest is added at the end of each year. Find the total amount of his savings at the end of the 20th year correct to the nearest pound. [3]

a) The sum of a geometric series is given by $S_n = \frac{a(1-r^n)}{1-r}$

As $n \to \infty$, whether the series converges or not, depends upon r^n. If $-1<r<1$, then $r^n \to 0$ as $n \to \infty$.

$S_\infty = \frac{a(1-0)}{1-r} = \frac{a}{1-r}$, and the series converges.

b) V:- $2, 2r, 2r^2, \ldots 2r^k, 2r^{k+1}, \ldots$

W:- $(2^2), (2r)^2, (2r^2)^2, \ldots (2r^k)^2, (2r^{k+1})^2, \ldots$

$= 4, 4r^2, 4r^4, \ldots 4r^{2k}, 4r^{2k+2}, \ldots$

Common ratio? $\frac{4r^2}{4} = r^2$, $\frac{4r^4}{4r^2} = r^2$, $\frac{4r^{2k+2}}{4r^{2k}} = r^2$

Therefore, W is a Geometric Progression with common ratio r^2.

Consider V, $S_\infty = \frac{a}{1-r} = \frac{2}{1-r}$

Consider W, $S_\infty = \frac{a}{1-r}$ where $r = r^2$, $= \frac{4}{1-r^2}$

Given sum to infinity of W = 3 × sum to infinity of V

$\therefore \frac{4}{(1-r^2)} = \frac{2}{1-r} \times 3$ ($r \neq 1$ given sum to infinity exists)

$\frac{4}{(1-r)(1+r)} = \frac{6}{(1-r)}$

$4 = 6+6r$

$-2 = 6r$, $r = -\frac{1}{3}$

c) Consider how much money he would have at the end of the first three years

End of 1st year :- 5000×1.03

End of 2nd year :- $5000 \times 1.03 + 5000 \times 1.03^2$

End of 3rd year :- $5000 \times 1.03 + 5000 \times 1.03^2 + 5000 \times 1.03^3$

$a = 5000 \times 1.03$, $r = 1.03$, $n = 20$

$S_{20} = \frac{a(1-r^{20})}{1-r} = \frac{5000 \times 1.03 (1-1.03^{20})}{(1-1.03)}$

$= £138,382$ (nearest pound)

10 The equation of a curve C is given by the parametric equations

$$x = \cos 2\theta, \quad y = \cos\theta.$$

a) Find the Cartesian equation of C. [2]

b) Show that the line $x - y + 1 = 0$ meets C at the point P, where $\theta = \frac{\pi}{3}$, and at the point Q, where $\theta = \frac{\pi}{2}$. Write down the coordinates of P and Q. [5]

c) Determine the equations of the tangents to C at P and Q. Write down the coordinates of the point of intersection of the two tangents. [7]

a) $x = \cos 2\theta$, and $\cos 2\theta \equiv \cos^2\theta - \sin^2\theta$ (formula booklet)

$= \cos^2\theta - (1 - \cos^2\theta)$

$= 2\cos^2\theta - 1$

$\therefore x = 2\cos^2\theta - 1$

and $y = \cos\theta$

$\therefore x = 2y^2 - 1$, $2y^2 = x + 1$

b) $x - y + 1 = 0$, and $x = \cos 2\theta$ and $y = \cos\theta$ when the curve and line meet.

$\therefore$ substitute $x = \cos 2\theta$ & $y = \cos\theta$ into the equation of the line:-

$\cos 2\theta - \cos\theta + 1 = 0$

substitute in $\cos 2\theta \equiv 2\cos^2\theta - 1$ to get a quadratic

$2\cos^2\theta - 1 - \cos\theta + 1 = 0$

$2\cos^2\theta - \cos\theta = 0$

$\cos\theta(2\cos\theta - 1) = 0$

$\cos\theta = 0$ $\quad$ $\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{2}$ $\quad$ $\theta = \frac{\pi}{3}$ as required.

at P, $\theta = \pi/3$

$x = \cos(2\pi/3)$

$x = -1/2$

$y = \cos(\pi/3)$

$y = \frac{1}{2}$

at Q, $\theta = \pi/2$

$x = \cos(2\pi/2)$

$x = -1$

$y = \cos(\pi/2)$

$y = 0$

$P = (-\frac{1}{2}, \frac{1}{2})$

$Q = (-1, 0)$

c) $\frac{dy}{d\theta} = -\sin\theta$ $\frac{dx}{d\theta} = -2\sin 2\theta$ $\frac{dy}{dx} = \frac{dy}{d\theta} / \frac{dx}{d\theta}$

$\frac{dy}{dx} = \frac{-\sin\theta}{-2\sin 2\theta}$ and $\sin 2\theta \equiv 2\sin\theta\cos\theta$

$\frac{dy}{dx} = \frac{-\sin\theta}{-4\sin\theta\cos\theta}$ $\frac{dy}{dx} = \frac{1}{4\cos\theta}$ ← gives gradient of tangent

at P $\theta = \frac{\pi}{3}$, gradient of tangent $= \frac{1}{4\cos\pi/3} = \frac{1}{2}$

using $y - y_1 = m(x - x_1)$

$y - \frac{1}{2} = \frac{1}{2}(x - -\frac{1}{2})$

$y - \frac{1}{2} = \frac{1}{2}x + \frac{1}{4}$

$4y - 2 = 2x + 1$

$4y = 2x + 3$ is the equation of the tangent at P.

at Q $\theta = \pi/2$, gradient of tangent $= \frac{1}{4\cos\pi/2} = ??$

undefined

This must mean that the tangent at Q is a vertical line because the denominator is zero; because vertical lines have no change in x. We know the x coordinate at Q is −1.

∴ $x = -1$ is the equation of the tangent at Q.

Point of intersection must be when $x = -1$

$4y = 2(-1) + 3$

$4y = 1$

$y = 1/4$

$(-1, \frac{1}{4})$

1 1 Prove by contradiction that, for every real number x such that $0 \leqslant x \leqslant \frac{\pi}{2}$,

$$\sin x + \cos x \geqslant 1.$$ [4]

Suppose that $\sin x + \cos x \geq 1$ is not true.

Then, there exists an x in the given domain for which

$$\sin x + \cos x < 1$$

$$(\sin x + \cos x)^2 < 1^2$$

expanding gives

$$\sin^2 x + 2\sin x \cos x + \cos^2 x < 1^2$$

but $\sin^2 x + \cos^2 x = 1$

$$\therefore 1 + 2\sin x \cos x < 1$$

$$\text{so, } \sin x \cos x < 0$$

As $\sin x \geq 0$ and $\cos x \geq 0$, $0 \leq x \leq \pi/2$

this is impossible.

Hence, $\sin x + \cos x < 1$ cannot be true.

Hence, $\sin x + \cos x \geq 1$ for $0 \leq x \leq \pi/2$

1 2 a) Given that f is a function,

i) state the condition for f^{-1} to exist,

ii) find $ff^{-1}(x)$. [2]

b) The functions g and h, are given by

$$g(x) = x^2 - 1,$$

$$h(x) = e^x + 1.$$

i) Suggest a domain for g such that g^{-1} exists.

ii) Given the domain of h is $(-\infty, \infty)$, find an expression for $h^{-1}(x)$ and sketch, using the same axes, the graphs of $h(x)$ and $h^{-1}(x)$. Indicate clearly the asymptotes and the points where the graphs cross the coordinate axes.

iii) Determine an expression for $gh(x)$ in its simplest form. [8]

ai) f^{-1} is the inverse function, this exists if and only if f is both one to one and onto.

ii) $ff^{-1}(x) = x$

(eg consider $f(x) = 3x$, this would make $f^{-1}(x) = \frac{x}{3}$, using a flow diagram approach

$f(x)$ $x \rightarrow [\times 3] \rightarrow 3x$

then, $f^{-1}(3x)$ $3x \rightarrow [\div 3] \rightarrow x$)

b) i) We need to make sure the function is one to one.

Domain of g could be $[0, \infty)$ (or, $(-\infty, 0]$ or a subset of one of these)

ii) $h(x) = e^x + 1$, domain of h $(-\infty, \infty)$

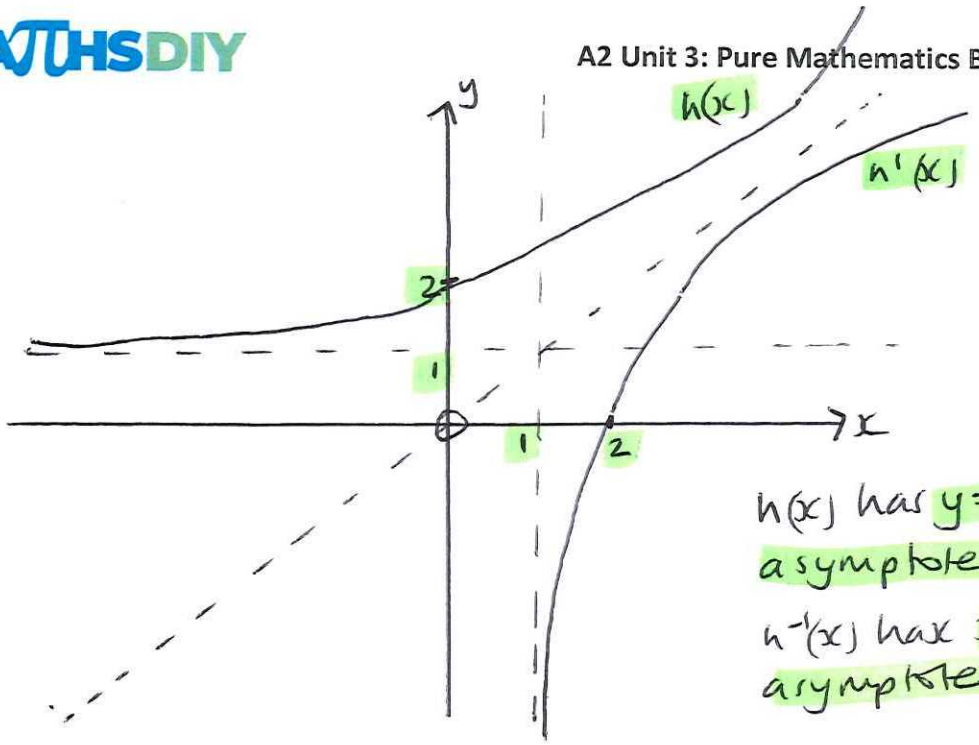
Let, $y = e^x + 1$

$y - 1 = e^x$

rewrite as a logarithm, $\ln(y-1) = x$

now, interchange x and y :- $y = \ln(x-1)$

$h^{-1}(x) = \ln(x-1)$



h(x) has y=1 as an asymptote

h⁻¹(x) has x=1 as an asymptote

iii) $g(x) = x^2 - 1$

$h(x) = e^x + 1$

$$gh(x) = (e^x+1)^2 - 1$$
$$= (e^x+1)(e^x+1) - 1$$
$$= e^{2x} + e^x + e^x + 1 - 1$$
$$= e^{2x} + 2e^x$$

1	3

a) Express $8\sin\theta - 15\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$. [3]

b) Find all values of θ in the range $0° < \theta < 360°$ satisfying

$$8\sin\theta - 15\cos\theta - 7 = 0.$$ [3]

c) Determine the greatest value and the least value of the expression

$$\frac{1}{8\sin\theta - 15\cos\theta + 23}.$$ [2]

a)

Using $\sin(\theta - \alpha) = \sin\theta\cos\alpha - \cos\theta\sin\alpha$

$R\sin(\theta - \alpha) = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$

$8\sin\theta - 15\cos\theta = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$

Equating $\sin\theta$ terms:- $8 = R\cos\alpha$ (1)

Equating $\cos\theta$ terms:- $15 = R\sin\alpha$ (2)

(1)^2 + (2)^2 gives $8^2 + 15^2 = R^2\cos^2\alpha + R^2\sin^2\alpha$

$289 = R^2$ (because $\sin^2\alpha + \cos^2\alpha \equiv 1$)

$R = 17$

(2)/(1) $\frac{R\sin\alpha}{R\cos\alpha} = \frac{15}{8}$

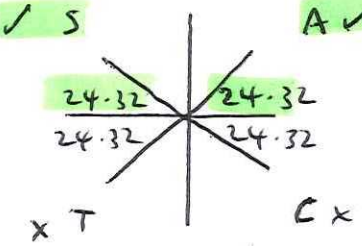
$\tan\alpha = 15/8$

$\alpha = 61.93°$

$= 17\sin(\theta - 61.93°)$

b) Equation becomes $17\sin(\theta - 61.93°) - 7 = 0$

$\sin(\theta - 61.93°) = 7/17$



$\theta - 61.93° = 24.32°, 155.68°$

$\theta = 24.32 + 61.93,\ 155.68 + 61.93$

$\theta = 86.2°,\ 217.6°$

c) $\frac{1}{17\sin(\theta - 61.93°) + 23}$

least value occurs when denominator as big as possible $\frac{1}{17 + 23} = \frac{1}{40}$

greatest value occurs when denominator as small as possible $\frac{1}{-17 + 23} = \frac{1}{6}$

1	4	Evaluate

a) $\int_1^2 x^3 \ln x \, dx$. [6]

b) $\int_0^1 \frac{2+x}{\sqrt{4-x^2}} dx$. [6]

a) use integration by parts

$$\int_1^2 x^3 \ln x \, dx$$

$\frac{dv}{dx}$ u

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx \quad \text{(formula booklet)}$$

$u = \ln x$, $\frac{du}{dx} = \frac{1}{x}$, $\frac{dv}{dx} = x^3$, $v = \frac{1}{4}x^4$

$$\int_1^2 x^3 \ln x \, dx = \left[\ln x \left(\tfrac{1}{4}x^4\right)\right]_1^2 - \int_1^2 \tfrac{1}{4}x^4 \cdot \tfrac{1}{x} dx$$

$$= \left[\tfrac{1}{4}x^4 \ln x\right]_1^2 - \tfrac{1}{4}\int_1^2 x^3 dx$$

$$= \left[\tfrac{1}{4}x^4 \ln x\right]_1^2 - \tfrac{1}{4}\left[\frac{x^4}{4}\right]_1^2$$

$$= (4\ln 2 - 0) - \left(\tfrac{1}{4}\left[4 - \tfrac{1}{4}\right]\right)$$

$$= 4\ln 2 - \frac{15}{16}$$

$$= 1.835 \quad (3\text{dp})$$

b) Use the substitution $x = 2\sin\theta$

$\frac{dx}{d\theta} = 2\cos\theta$

$dx = 2\cos\theta\, d\theta$

$\sqrt{4-x^2}$

$= \sqrt{4-4\sin^2\theta}$

$= \sqrt{4(1-\sin^2\theta)}$

$= \sqrt{4\cos^2\theta}$

$= 2\cos\theta$

When $x=0$, $0 = 2\sin\theta$, $\sin\theta = 0$, $\theta = 0$

When $x=1$, $1 = 2\sin\theta$, $\sin\theta = \frac{1}{2}$, $\theta = \pi/6$

(careful, calculator in radian mode)

$$\int_0^1 \frac{2+x}{\sqrt{4-x^2}}\,dx = \int_0^{\pi/6} \frac{2+2\sin\theta}{2\cos\theta} \cdot 2\cos\theta\, d\theta$$

$$= \int_0^{\pi/6} (2+2\sin\theta)\, d\theta$$

$$= 2\int_0^{\pi/6} (1+\sin\theta)\, d\theta$$

$$= 2\left[\theta - \cos\theta\right]_0^{\pi/6}$$

$$= 2\left[\left(\frac{\pi}{6} - \cos\frac{\pi}{6}\right) - (0-\cos 0)\right]$$

$$= 2\left[\frac{\pi}{6} - \frac{\sqrt{3}}{2} + 1\right]$$

$$= 1.315 \quad (3dp)$$

1 5 The variable y satisfies the differential equation

$$2\frac{dy}{dx} = 5 - 2y, \qquad \text{where } x \geqslant 0.$$

Given that $y = 1$ when $x = 0$, find an expression for y in terms of x. [5]

Rewrite as $\frac{2}{5-2y} \cdot \frac{dy}{dx} = 1$

Seperating the variables and integrating gives

$$\int \frac{2}{5-2y}\, dy = \int dx$$

$$\frac{2\ln|5-2y|}{-2} = x + c$$

$$-\ln|5-2y| = x + c$$

given, $y = 1$ when $x = 0$

$$-\ln|3| = c$$

giving $-\ln|5-2y| = x - \ln|3|$

$$-\ln|5-2y| + \ln 3 = x$$

$$-\ln\left|\frac{5-2y}{3}\right| = x$$

$$\ln\left|\frac{5-2y}{3}\right| = -x$$

rewrite as a power, $e^{-x} = \frac{5-2y}{3}$

$$3e^{-x} = 5 - 2y$$

$$2y = 5 - 3e^{-x}$$

$$y = \frac{5-3e^{-x}}{2}$$

16 a) Differentiate the following functions with respect to x, simplifying your answer wherever possible.

i) $e^{3\tan x}$,

ii) $\frac{\sin 2x}{x^2}$. [5]

b) A function is defined implicitly by

$$3x^2y + y^2 - 5x = 5.$$

Find the equation of the normal at the point (1, 2). [6]

ai) Let $u = 3\tan x$, $y = e^{3\tan x}$

$y = e^u$

$\frac{dy}{du} = e^u$ $\frac{du}{dx} = 3\sec^2 x$

$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$ (Chain rule)

$\frac{dy}{dx} = e^u \cdot 3\sec^2 x$

$\frac{dy}{dx} = 3\sec^2 x\, e^{3\tan x}$

ii) $y = \frac{\sin 2x}{x^2} = \frac{u}{v}$

$u = \sin 2x$ $\frac{du}{dx} = 2\cos 2x$

$v = x^2$ $\frac{dv}{dx} = 2x$

Quotient Rule

$\frac{dy}{dx} = \frac{v\,du/dx - u\,dv/dx}{v^2}$

$\frac{dy}{dx} = \frac{(x^2)2\cos 2x - \sin 2x(2x)}{(x^2)^2}$

$\frac{dy}{dx} = \frac{2x^2\cos 2x - 2x\sin 2x}{x^4} = \frac{2x\cos 2x - 2\sin 2x}{x^3}$

b) $3x^2y + y^2 - 5x = 5$

differentiating with respect to x gives

$$6xy + 3x^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 5 = 0$$

$$\frac{dy}{dx}(3x^2+2y) = 5 - 6xy$$

$$\frac{dy}{dx} = \frac{5-6xy}{3x^2+2y}$$

at $(1,2)$ $\frac{dy}{dx} = \frac{5-6(1)(2)}{3+2(2)} = -1$

$\therefore$ Gradient of tangent at $(1,2)$ is -1

$\therefore$ Gradient of normal at $(1,2)$ is 1 (product of gradients of tangent & normal is -1)

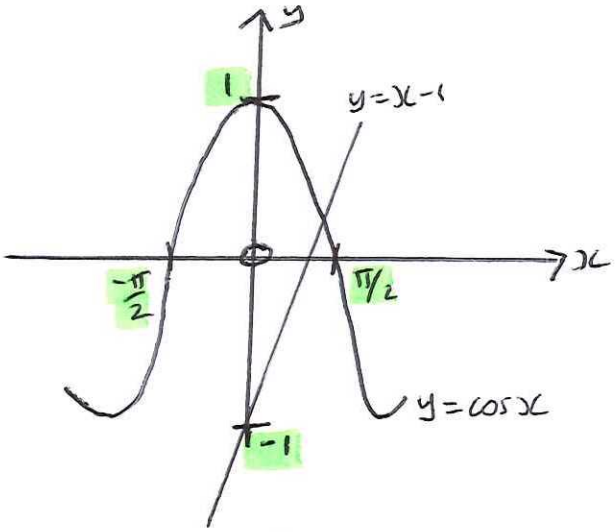
Using $(y-y_1) = m(x-x_1)$

$y - 2 = 1(x-1)$

$y = x+1$ is the equation of the normal.

1	7	By drawing suitable graphs, show that $x - 1 = \cos x$ has only one root. Starting with $x_0 = 1$, use the Newton-Raphson method to find the value of this root correct to two decimal places. [6]

Need to draw/sketch the graphs $y=x-1$, $y=\cos x$. Need radian measure.



$f(x) = x - 1 - \cos x$

$f'(x) = 1 + \sin x$

and $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

(Newton Raphson iteration, given in formula booklet)

$$x_{n+1} = x_n - \frac{x_n - 1 - \cos x_n}{1 + \sin x_n}$$

$x_0 = 1$

$x_1 = 1.293407993$

$x_2 = 1.283435773$

$x_3 = 1.283428742$

Root is 1.28 (correct 2dp)