GCSE MARKING SCHEME

SUMMER 2019

ADDITIONAL MATHEMATICS
9550/01

INTRODUCTION

This marking scheme was used by WJEC for the 2019 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCSE ADDITIONAL MATHEMATICS

SUMMER 2019 MARK SCHEME

		Mark	Comment
1	(a)(i) $(9x+5)(3x-1)$	B2	Mark final answer. *Ignore inclusion of '=0"* B1 $(9x \pm 5)(3x \pm 1)$ or $9x(3x-1)+5(3x-1)$ or or $(x - 1/3)(27x+15)$ or $\frac{(27x-9)(27x+15)}{27}$ or sight of $(9x+5)$ AND $(3x-1)$
	(ii) -5/9 with 1/3 or equivalent or -0.55(5...) or -0.556 with 0.33(3...)	B2	If a restart in (ii) to factorise, do not alter marking in (i), unless the candidate is clearly replacing their answer (i) Ignore sight of "=0" Must be from factorising. STRICT FT for their factors. B1 for each answer Do not accept from the use of the quadratic formula
	(b)(i) $(x+5)^2 \pm \ldots$ …….. $+10$	B1 B1	Sight of $(x+5)^2$. Ignore sight of '=0' Do not accept '= -10' or '=10' *$(x+5)^2+10$, B1, B1 ISW* Allow +35 -25 for 10 provided 10 seen in later working
	(ii) Least value (+)10	B1	Must follow completing the square FT their value but not 35 or - 10
	(iii) (x =) -5	B1	FT from 'their $(x+5)^2$' Do not accept (-5, 10)
	(c) $(x - 22/2)^2 - 121 = -5$ or $(x - 22/2)^2 - 121 + 5 = 0$ $(x-11)^2 = 121 - 5$ $x = 11 \pm 2\sqrt{29}$	M1 M1 A2 12	Allow for sight of $(x - 22/2)^2 - 121 + 5$ or $(x - 22/2)^2 - 116$ FT from 1 slip, e.g. for sight of $(x-11)^2 = 126$ or $(x-11)^2 - 126 = 0$ Allow A2 for $x = 11 \pm\sqrt{116}$ ISW A1 for $11 + \sqrt{116}$ or $11 + 2\sqrt{29}$ or $11 - \sqrt{116}$ or $11 - 2\sqrt{29}$ or FT $11 \pm \sqrt{126}$ or $11 \pm 3\sqrt{14}$ *No working in (c), no marks*
2	(a) $40x^3 + 6x$ (+0)	B3	*Penalise '+c' shown -1 only throughout* B1 for $40x^3$ (not $10\times4x^3$), B1 for $+6x$ (not $3\times2x$), and B1 for +0 (or blank) provided at least 1 other mark awarded. Mark final answer
	(b) $-22x^{-12}$ or $\frac{-22}{x^{12}}$	B1	Mark final answer
	(c) $\frac{7}{8}x^{-1/8}$ or $\frac{7}{8x^{1/8}}$	B1 5	Index needs to be simplified. Mark final answer

3	{ 55(x) -22(x+ 3) + 10(x+5)} (/110) {55x-22x-66+10x+50} (/110) (43x - 16)/110 or showing LHS ≡ RHS	M1 B1 B1 A1 4	Attempt to use common denominator, may be implied by sight of 55(x) -22(x+ 3) + 10(x+5) without sight of /110 May be seen in stages Or equivalent. May be seen in stages, as intention of method B1 for 1 slip (e.g. +66). Must be as a sum of 5 terms. Convincing **must** follow from fully correct working at each stage Allow following sight of 3 separate correct fractions with denominator 110 seen *If no denominator then possible M1 (see note above), B1 B1 A0, however if denominator replaced later all marks are allowable*
4	(a) ($y+\delta y =$) $(x+\delta x)^2 + 7(x+\delta x) + 2$ Intention to subtract ($y=$) x^2+7x+2 to find δy ($\delta y =$) $2x\delta x + (\delta x)^2 + 7\delta x$ Dividing by δx **and** (lim) $\delta x \to 0$ $dy/dx = \lim_{\delta x \to 0} \delta y/\delta x = 2x + 7$	B1 M1 A1 M1 A1 5	Or alternative notation. Allow if final bracket omitted Accept δx^2 as meaning $(\delta x)^2$ FT equivalent level of difficulty CAO. Must follow from correct working and notation All notation throughout the working must be correct in order to award the final A1 Do not accept $dy/dx = \lim_{x \to 0} 2x + 7$ as a final answer *Use of dy/dx throughout max 4 marks only, final A0*

5	Overall strategy that could lead to finding $E\hat{C}B$, e.g. length of 3 sides and then cosine rule $EC^2 = 6.2^2 + 3.7^2$ $BC^2 = 2.5^2 + (8.4 – 6.2)^2$ $EB^2 = 8.4^2 + 3.7^2 + 2.5^2$ With substituted values: $\cos E\hat{C}B = \frac{EC^2 + BC^2 – EB^2}{2 \times EC \times BC}$ i.e. $\cos E\hat{C}B = \frac{52.13 + 11.09 – 90.5}{2 \times 7.22... \times 3.33...}$ (= -0.567...) $E\hat{C}B$ = 124.56(....°) or 124.6° or 125(°) QWC2: Candidates will be expected to • present work clearly, with words explaining process or steps AND • make few if any mistakes in mathematical form, spelling, punctuation and grammar in their answer QWC1: Candidates will be expected to • present work clearly, with words explaining process or steps OR • make few if any mistakes in mathematical form, spelling, punctuation and grammar in their final answer	S1 M1 M1 M1 M2 A1 QWC 2 9	Or full alternative strategy (EC^2 = 52.13, EC =7.22....cm) (BC^2 = 11.09, BC = 3.33....cm) May be shown in stages (e.g. $BF^2 = 8.4^2 + 3.7^2$ then $EB^2 = 2.5^2 + BF^2$) (EB^2 = 90.5, EB = 9.513...cm) OR alternative full method, e.g. finding angles BEC or EBC using cosine rule followed by use of sine rule with $\sin E\hat{C}B$ isolated FT 'their derived lengths' provided at least 2 M marks previously awarded M1 for substituted values: $EB^2 = EC^2 + BC^2 – 2 \times EC \times BC \times \cos E\hat{C}B$ OR for alternative full method without $\sin E\hat{C}B$ isolated CAO, must be from correct working Allow 124.4(... °), 124.48(°), 124.5(°) or 125.39(... °) or 125.4(°) from premature approximation QWC2 Presents relevant material in a coherent and logical manner, using acceptable mathematical form, and with few if any errors in spelling, punctuation and grammar. QWC1 Presents relevant material in a coherent and logical manner but with some errors in use of mathematical form, spelling, punctuation or grammar OR evident weaknesses in organisation of material but using acceptable mathematical form, with few if any errors in spelling, punctuation and grammar. QWC0 Evident weaknesses in organisation of material, and errors in use of mathematical form, spelling, punctuation or grammar.
6	(a) Multiplier $(6-\sqrt{3}) / (6-\sqrt{3})$ Denominator $36 + 6\sqrt{3} - 6\sqrt{3} - 3$ OR $36 - 3$ OR 33 $\frac{12 - 2\sqrt{3}}{33}$ (b)(i) $y^{1/5}/y^{3/4}$ or alternative correct 1st step $y^{-11/20}$ or $1/y^{11/20}$ (ii) Correctly extracting $x^{2/7}$ as a factor, or $\frac{x^{2/7}}{2x^{2/7}} + \frac{6x^{3/7}}{2x^{2/7}}$ $½ + 3x^{1/7}$ or $\frac{1 + 6x^{1/7}}{2}$	M1 A1 A1 B1 B1 M1 A1 7	Allow if the multiplier is stated as $(6-\sqrt{3})$ provided it is used as $(6-\sqrt{3})/(6-\sqrt{3})$ CAO. Mark final answer *Unsupported answer is awarded no marks.* Or equivalent first stage of working with indices CAO. Mark final answer CAO. Mark final answer

7	(a) $(3)^3 + 8(3)^2 - 2(3) + 6$ (= 27 + 72 - 6 + 6) = 99 (b)(i) Substitute x = -3 Showing f(-3) = 0 (ii) $(x + 3)(x^2 + bx + c)$ or intention to divide by (x+ 3) with x^2 shown ((x + 3)) $(x^2 - 2x - 35)$ ((x + 3))(x + 5)(x - 7)	M1 A1 M1 A1 M1 A2 A1 8	Or division method giving $x^2 + 11x$ … Or division method giving $x^2 - 2x$ … Convincing from working shown (not if incorrect working seen), allow $(-3)^3 + (-3)^2 - 41(-3) - 105 = 0$, also allow for sight of $-3^3 + -3^2 - 41\times-3 - 105 = 0$ provided no incorrect calculation is given such as -3^2 as -9 A1 for -2x or -35. Or use of factor theorem A1 (x+5), A1 (x- 7) CAO. Mark final answer, *but ignore attempts to 'solve'*
8	(dy/dx=) $12x^2 - 6x$ dy/dx = 0 or $12x^2 - 6x = 0$ or $12x^2 = 6x$ x = 0 and y = 20 x = ½ and y = 19¾ $d^2y/dx^2 = 24x - 6$ (0, (20)): $d^2y/dx^2 < 0$, point is a maximum (½, (19¾)): $d^2y/dx^2 > 0$, point is a minimum	B1 M1 A1 A1 M1 A1 A1 7	FT their dy/dx form $ax^2 \pm bx$ If A0, A0 here, award A1 for x = 0 with x = ½ *Answer only, no working shown M0 A0 A0* Or first derivative test, interpretation of first derivative test. Or alternative (e.g. full graphical method with explanation) FT for their x value FT for their other x value provided this does not have the same interpretation as the first x value *Answer only, no working shown M0 A0 A0* *If $d^2y/dx^2 = cx + d$ where $c \neq 0$ and test applied correctly then SC2 instead of final A1, A1 (as M1 has not been awarded))provided one minimum and one maximum*
9	$\frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4}$	B1 1	**<u>Working must be shown</u>**
10	(a) $FG^2 = (-4 - 8)^2 + (10 - 28)^2$ $(= 12^2 + 18^2 = 468)$ $FG = 6\sqrt{13}$ (b) Gradient FG (28-10)/(8 - -4) = 18/12 (= 9/6 = 3/2) (c) (-4 + 8)/2 or (10 + 28)/2 Mid point (2, 19) Perpendicular gradient -2/3 (or -6/9 or -12/18) $\frac{y - 19}{x - 2} = \frac{-2}{3}$ or $19 = -2/3 \times 2 + c$ y – 19 = -2/3(x - 2) or 3(y – 19) = -2(x – 2) or 3y = -2x + 61 or c = 20⅓ or c = 61/3 2x + 3y – 61 = 0 or -2x – 3y + 61 = 0	M1 A1 M1 A1 M1 A1 B1 M1 m1 A1 10	Or equivalent. Allow 1 slip or error M1, A0 for answers √468 or 21.6(3...) CAO Do not ignore incorrect cancelling, mark final answer Sight of (2, …) or (… , 19) implies M1 provided no incorrect working is seen FT -1/ 'their answer in (b)' OR for an alternative correct method of finding the equation of a straight line, for the idea of how an equation of a straight line can be found. FT 'their perpendicular gradient' or 'their answer in (b)' AND 'their mid point' or for 'points F or G' used **Do not allow** use gradient from their answer in (b), and/or points F or G as the mid-point of FG. Only FT for 'their perpendicular gradient' (not 'their answer' from (b)) AND 'their mid point' CAO. Must be in this form with '=0' with terms in any order

11	(a) Correct shaped graph with (0°,) 180° & 360° labelled on the x-axis AND 2, 7 & 12 labelled on the y-axis (b) Maximum value 12 AND Minimum value 2	B3 B1 4	Ignore outside the required range *Intention for approximately (0°, 7), (90°, 2), (180°, 7), (270°, 12) and (360°, 7)* B2 awarded a for correct shape graph with conditions: • sin$x$ reflected • with one complete period, labelled 0° to 360° • with difference in y values between maximum and minimum of 10, for their labels OR B1 for a correct shape graph with any 2 of the 3 bullet points above met, OR B1 for a graph with all 3 bullet points above met but joined by straight lines (even if turning points curved), OR B1 for a curved graph through intended points: (0°, 7), (90°, 2), (180°, 7), (270°, 12) and (360°, 7) Accept Maximum (270(°), 12) and Minimum (90(°), 2) Allow unsupported correct responses FT provided at least B2 previously awarded in (a)
12	(a) (dy/dx=) $16x^7 + 8x$ (d^2y/dx^2=) $112x^6 + 8$ (b) $(5/5)\ x^5 + (3/-1)\ x^{-1} + (-2/-2)x^{-2}$ $(= x^5 - 3x^{-1} + x^{-2})$ + c (constant) (c) $6x^2/2 + 10x$ $[\ 6x^2/2 + 10x\ ]^3_2$ and with intention to substitute and subtract $=(6\times3^2/2+ 10\times3) - (6\times2^2/2+ 10\times2)$ $(= 57 - 32)$ $= 25$	B1 B1 B3 B1 B2 M1 A1 A1 11	 FT to 2nd B1 from dy/dx = kx^n (+ …) B1 for each term. Accept unsimplified. ISW Award if at least B1 given for integration B1 for $6x^2/2$ or $10x$ Intention to use 3, 2 (in either order) **and** subtract FT their integration, not the same terms as given or differentiated, this includes if there is only1 term seen. FT for correct use of limits provided working with 2 terms from 'their integration' CAO, not FT. *Answer only, no working shown, M0 A0 A0*
13	(When x = 2) y = 27 (Gradient when x = 2, dy/dx =) 5×2x 20 Equation $\frac{y - 27}{x - 2} = 20$ or $27 = 20\times2 + c$ $y - 27 = 20(x - 2)$ or $c = -13$ $y = 20x - 13$	B1 M1 A1 M1 m1 A1 6	 For differentiation, before substitution of x = 2 FT values for 'their 27' and 'their 20' provided at least one of these is correct. Implies previous M1 CAO. Mark final answer
14	Method to solve simultaneously, e.g. use of y = 2x + 1 or x = (y – 1)/2 into the first equation $x^2 - 7x + 12 = 0$ or $y^2 - 16y + 63 = 0$ $(x - 3)(x - 4)$ (=0) or $(y - 9)(y - 7)$ (=0) (3, 7) and (4, 9)	M1 A1 m1 A1 4	$2x + 1 = x^2 - 5x + 13$ or $y = \frac{(y - 1)^2}{2^2} - 5\frac{(y - 1)}{2} + 13$ Or equivalent but must '=0' or implied in further working OR $x = (7 \pm \sqrt{1})/2$ or $y = (16 \pm \sqrt{4})/2$ FT from their quadratic CAO Need not be in this form, accept x=3, y=7 with x=4, y=9 x & y values must be given Do not accept unsupported responses Do not accept trial & improvement

15	Working to support that (7, 10) and (2, -5) both lie on the curve	B2 2	Working, e.g. • substituting the x-values and correctly finding y-values • substituting coordinates for the points and showing "=55" (Allow sight of -5^2 in working provided -25 is not seen) B1 for either correct working for either point
16	Intention to integrate $-x^3/3 + 8x^2/2 – 12x$ Use of correct limits 6 & 2 in the correct order **and** intention to subtract 32/3 or 10.66(6…) or 10.7	M1 A2 m1 A1 5	Intention to integrate (not using given or differentiated) A1 one term correct. The limits must be used in the correct order CAO. Only allow 10.6 from correct working seen *Answer only gets no marks* *No marks for use of the trapezium rule*

Differentiating from first principles. Marking guide.

Q4.

4	(a) $(y+\delta y =)\quad (x+\delta x)^2 + 7(x+\delta x) + 2$ Intention to subtract $(y=)\ x^2+7x+2$ to find δy $(\delta y =)\quad 2x\delta x + (\delta x)^2 + 7\delta x$ Dividing by δx **and** (lim) $\delta x \to 0$ $dy/dx = \lim_{\delta x \to 0} \delta y/\delta x = 2x + 7$	B1 M1 A1 M1 A1 5	Or alternative notation. Allow if final bracket omitted Accept δx^2 as meaning $(\delta x)^2$ FT equivalent level of difficulty CAO. Must follow from correct working and notation All notation throughout the working must be correct in order to award the final A1 Do not accept $dy/dx = \lim_{x \to 0} 2x + 7$ as a final answer *Use of dy/dx throughout max 4 marks only, final A0*

B1 For sight of $(x+\delta x)^2 + 7(x+\delta x) + 2$ or $(x+h)^2 + 7(x+h) + 2$ or using alternative notation. This mark is given whether $(x+\delta x)^2 + 7(x+\delta x) + 2$ stands alone or is embedded in an expression or a formula.

M1 For the <u>intent</u> to subtract $x^2 + 7x + 2$ from the above.
So $(x+\delta x)^2 + 7(x+\delta x) + 2 - x^2 + 7x + 2$ will gain the M1 even though there are missing brackets.
It can also be awarded to those who have expanded $(x+\delta x)^2 + 7(x+\delta x) + 2$ and then crossed out the x^2 term and the $+7x$ term and 2.
Those who reverse the subtraction will gain M0 <u>unless</u> there is evidence later on of dividing by $-\delta x$.

A1 For sight of $2x\delta x + (\delta x)^2 + 7\delta x$ (Accept δx^2 as meaning $(\delta x)^2$) with no other terms. Treat as a CAO.
$2x + \delta x + 7$ will imply the above if division by δx has already been done.

M1 A FT, if of equivalent difficulty, is possible for this M1 (but not the subsequent A1).
A <u>correct</u> division by δx has to be done
(so if a FT it has to be correct for their $2x\delta x + (\delta x)^2 + 7\delta x$)
<u>**AND**</u> we must see 'lim $\delta x \to 0$' OR '$\delta x \to 0$' OR 'δx tends to 0'.
It is M0 for '$\delta x = 0$' OR '$\delta x \approx 0$' OR ' δx is so small we can forget about it'.

All of the above marks can be gained even if there is no l.h.s. shown.

Final A1. Must be for a 'text book' quality presentation. E.g.
Has to be a correct l.h.s. for each line, 'δy' or '$\delta y/\delta x$'
AND at some point '$dy/dx = \lim_{\delta x \to 0} \delta y/\delta x$' or '$dy/dx = \lim_{\delta x \to 0} 2x + \delta x + 7$'