CONDITIONAL PROBABILITY

A2 Unit 4: Applied Mathematics B

Section A: Statistics

WJEC Past paper questions: 2010 - 2018

Total marks available 180 (approximately 3 hours 40 minutes)

1. Events A and B are such that

$$P(A) = 0{\cdot}2, \quad P(B) = 0{\cdot}4, \quad P(A \cup B) = 0{\cdot}52.$$

(a) Show that A and B are independent. [5]

(b) Calculate the probability of exactly one of the two events occurring. [2]

(c) Given that exactly one of the two events occurs, calculate the probability that A occurs. [3]

(Jan 10)

2. A bag contains three balls numbered 1, 2 and 3 respectively. Jim selects one of these balls at random and he notes the number on the selected ball. He then tosses that number of fair coins.

(a) Calculate the probability that no head is obtained. [5]

(b) Given that no head was obtained, find the probability that he tossed 2 coins. [3]

(Jan 10)

3. Jack is taking part in a quiz programme. For each question in the quiz, four alternative answers are given, only one of which is correct. Jack has probability 0·6 of knowing the correct answer to a question, and when he does not know the correct answer he chooses one of the four answers at random.

(a) Calculate the probability that Jack gives the correct answer to a question. [3]

(b) Given that Jack gave the correct answer to a question, find the probability that he knew the correct answer. [3]

(Summer 10)

4. In a certain country, 80% of the defendants being tried in the Law Courts actually committed the crime. For those who committed the crime, the probability of being found guilty is 0·9. For those who did not commit the crime, the probability of being found guilty is 0·05.

(a) Find the probability that a randomly chosen defendant is found guilty. [3]

(b) Given that a randomly chosen defendant is found guilty, find the probability that this defendant committed the crime. [3]

(Jan 11)

5. A box contains three coins. Two of these coins are fair and the third coin is double-headed so that when tossed a head is always obtained. One of these coins is selected at random and tossed three times.

(a) Find the probability that three heads are obtained. [4]

(b) Given that three heads are obtained, find the probability that the double-headed coin was selected. [3]

(c) The selected coin is tossed a fourth time. Find the probability that a head is obtained. [2]

(Summer 11)

6. The events A and B are such that

$$P(A) = 0{\cdot}4,\ P(B) = 0{\cdot}2 \text{ and } P(A|B) = 0{\cdot}3.$$

Calculate

(a) $P(A \cap B)$, [2]

(b) $P(A \cup B)$, [2]

(c) $P(B|A)$. [2]

(Jan 12)

7. Each of three boxes contains 3 cards. Box A contains 1 red card, Box B contains 2 red cards and Box C contains 3 red cards. One of the boxes is selected at random and a card is chosen at random from that box.

(a) Find the probability that a red card is chosen. [3]

(b) Given that a red card is chosen, find the probability that Box A was selected. [3]

(Jan 12)

8. The events A and B are such that

$$P(A) = 0{\cdot}5,\ P(B) = 0{\cdot}3.$$

(a) Evaluate $P(A \cup B)$ when

(i) A, B are mutually exclusive,
(ii) A, B are independent. [5]

(b) Given that $P(A \cup B) = 0{\cdot}7$, find the value of $P(B|A)$. [3]

(Summer 12)

9. In a certain population, 60% are male and 40% are female. It is known that 8% of males are colour-blind and 3% of females are colour-blind. A member of the population is selected at random.

(a) Find the probability that this person is colour-blind. [3]

(b) Given that this person is colour-blind, find the probability that the person is female. [3]

(Summer 12)

10. In a mass screening programme, a new diagnostic test is being used to detect the presence or otherwise of a certain disease. When the person being tested has the disease, the test gives a positive result with probability 0·96. When the person being tested does not have the disease, the test gives a positive result with probability 0·01. It is known that 2% of the population have this disease. The test is given to a randomly chosen member of the population.

(a) Find the probability that a positive result is obtained. [3]

(b) Given that a positive result is obtained, find the probability that

(i) this person has the disease,
(ii) a positive result will be obtained if a second test is given to this person. [6]

(Jan 13)

11. Box A contains four balls numbered 1, 2, 3, 4 respectively, Box B contains three balls numbered 1, 2, 3 respectively and Box C contains two balls numbered 1, 2 respectively. Gwen selects one of these boxes at random and then selects a ball at random from that box.

(a) Determine the probability that a ball numbered 1 is selected. [3]

(b) Given that a ball numbered 1 is selected, determine the probability that Box A was selected. [3]

(Summer 13)

12. The events A and B are such that

$$P(A) = 0{\cdot}5,\ P(B) = 0{\cdot}2,\ P(A|B) = 0{\cdot}4.$$

(a) Calculate

(i) $P(A \cap B)$,

(ii) $P(B|A)$. [4]

(b) Giving a reason, state whether or not A and B are mutually exclusive. [1]

(Jan 14)

13. Three drawers each contain 4 coins. Drawer A contains 4 gold coins. Drawer B contains 3 gold coins and 1 silver coin. Drawer C contains 2 gold coins and 2 silver coins. David selects one of these drawers at random and then selects 2 coins at random from that drawer without replacement.

(a) Determine the probability that he selects 2 gold coins. [5]

(b) Given that he selects 2 gold coins, determine the probability that Drawer A was selected. [3]

(Jan 14)

14. The events A and B are such that

$$P(A) = 0{\cdot}3,\ P(B) = 0{\cdot}4,\ P(A \cup B) = 0{\cdot}5.$$

(a) Determine whether or not A and B are independent. [3]

(b) Evaluate $P(A|B')$. [3]

(Summer 14)

15. A purse contains three fair coins and one double-headed coin. A coin is selected at random from the purse and tossed.

(a) Find the probability that a head is obtained. [3]

(b) Given that a head is obtained,

(i) determine the probability that the double-headed coin was selected,

(ii) find the probability that a head will be obtained if the selected coin is tossed a second time. [6]

(Summer 14)

16. Ann and Brenda each have a calculator which can generate a single digit random number from the set {1, 2, 3, 4, 5, 6, 7, 8}. They each generate a random number on their calculator.

(a) Find the probability that the two numbers are equal. [2]

(b) Find the probability that the sum of the two numbers is 12. [3]

(c) Given that the sum of the two numbers is 12, find the probability that the two numbers are equal. [2]

(Summer 14)

17. The events A and B are such that

$$P(A) = 0{\cdot}4,\ P(B) = 0{\cdot}5 \text{ and } P(A \cup B) = 2 \times P(A \cap B).$$

(a) Show that $P(A \cap B) = 0{\cdot}3$. [2]

(b) Evaluate $P(A|B)$. [2]

(c) Evaluate $P(B|A')$. [3]

(Summer 15)

18. At a certain university, 60% of the students are male and 40% are female. It is known that 75% of the male students own a bicycle and 30% of the female students own a bicycle. One of the students is selected at random.

(a) Calculate the probability that the selected student

(i) is a male student who owns a bicycle,

(ii) owns a bicycle. [5]

(b) Given that the selected student owns a bicycle, calculate the probability that this student is female. [3]

(Summer 15)

19. The events A and B are such that

$$P(A) = 0{\cdot}3,\ P(B) = 0{\cdot}4.$$

Evaluate $P(A \cup B)$ in each of the following cases.

(a) A and B are mutually exclusive. [2]

(b) A and B are independent. [3]

(c) $P(A|B) = 0{\cdot}25$. [4]

(Summer 16)

20. In a certain population, 45% are male and 55% are female. It is known that 3% of the males have red hair while 5% of the females have red hair. One of the members of the population is selected at random.

(a) Calculate the probability that the selected person has red hair. [3]

(b) Given that the selected person has red hair, calculate the probability that this person is female. [3]

(Summer 16)

21. The events A and B are such that

$$P(A) = 0{\cdot}2,\ P(B) = 0{\cdot}3,\ P(A \cup B) = 0{\cdot}4.$$

(a) Show that A and B are not independent. [3]

(b) Determine the value of

(i) $P(A'|B)$,

(ii) $P(A \cup B')$. [6]

(Summer 17)

22. It is known that 5% of animals of a certain species have a particular disease. A diagnostic test can be applied to animals of this species to indicate whether or not they have this disease. When applied to an animal which has this disease, the test gives a positive response with probability 0·96. When applied to an animal which does not have this disease, the test gives a positive response with probability 0·02.

(a) The test is given to a randomly chosen animal.

(i) Calculate the probability that a positive response is obtained. [3]

(ii) Given that a positive response is obtained, find the probability that this animal has the disease. [3]

(b) A randomly chosen animal gave a positive response when tested. It is tested again.

(i) Find the probability that it gives a second positive response.

(ii) Given that this second response is positive, calculate the probability that this animal has the disease. [4]

(Summer 17)

23. The independent events A and B are such that

$$P(A \cup B) = 0{\cdot}9, \quad P(A \cap B) = 0{\cdot}4, \quad P(A) > P(B).$$

(a) Determine the values of $P(A)$ and $P(B)$. [7]

(b) Determine the value of $P(A|A \cup B)$. [3]

(Summer 18)

24. Jim uses the following method to decide how to travel to work. He throws a fair six-sided dice and if he obtains a 6, he runs to work; otherwise he cycles to work. If he runs to work, the probability that his journey takes longer than 20 minutes is 0·6. If he cycles to work, the probability that his journey takes longer than 20 minutes is 0·24. On a randomly chosen day,

(a) calculate the probability that his journey takes longer than 20 minutes, [3]

(b) given that his journey took longer than 20 minutes, calculate the probability that he cycled to work. [3]

(Summer 18)