

THE NORMAL DISTRIBUTION

A2 Unit 4: Applied Mathematics B

Section A: Statistics

1. **WJEC past paper questions: 2010 – 2017**

Total marks available 93 (approximately 1 hour 50 minutes)

Arwyn collects data about household expenditure on food. He records the weekly expenditure on food for 80 randomly selected households from across Wales.

Cost, x (£)	$x < 40$	$40 \leq x < 50$	$50 \leq x < 60$	$60 \leq x < 70$	$70 \leq x < 80$	$80 \leq x < 90$	$x \geq 90$
Number of households	5	11	16	18	15	9	6

- a) Explain why a normal distribution may be an appropriate model for the weekly expenditure on food for this sample. [1]

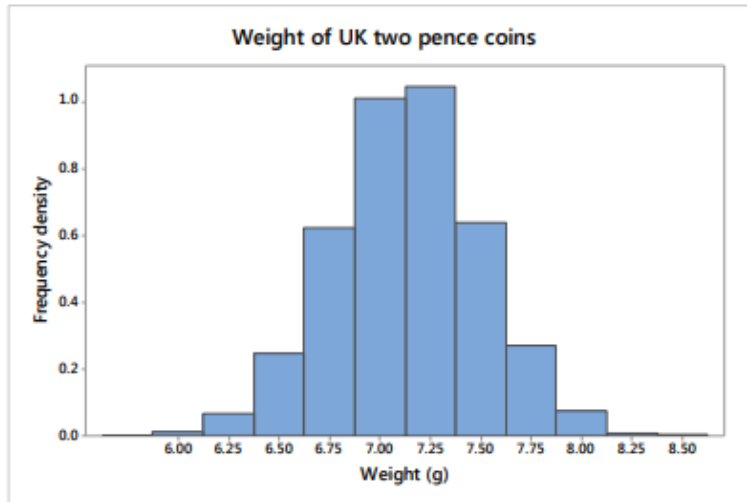
Arwyn uses the distribution $N(64, 15^2)$ to model expenditure on food.

- b) Find the number of households in the sample that this model would predict to have weekly food expenditure in the range
- i) $60 \leq x < 70$,
 - ii) $x \geq 90$. [4]
- c) Use your answers to part (b)
- i) to comment on the suitability of this model,
 - ii) to explain how Arwyn could improve the model by changing one of its parameters. [2]
- d) Arwyn's friend Colleen wishes to use the improved model to predict household expenditure on food in Northern Ireland. Comment on this plan. [1]

(WJEC 2018)

2. Automatic coin counting machines sort, count and batch coins. A particular brand of these machines rejects 2p coins that are less than 6.12 grams or greater than 8.12 grams.

- (a) The histogram represents the distribution of the weight of UK 2p coins supplied by the Royal Mint. This distribution has mean 7.12 grams and standard deviation 0.357 grams.



Explain why the weight of 2p coins can be modelled using a normal distribution. [1]

- (b) Assume the distribution of the weight of 2p coins is normally distributed. Calculate the proportion of 2p coins that are rejected by this brand of coin counting machine. [2]

- (c) A manager suspects that a large batch of 2p coins is counterfeit. A random sample of 30 of the suspect coins is selected. Each of the coins in the sample is weighed. The results are shown in the summary statistics table.

Summary statistics						
Weights (in grams) for a random sample of 30 UK 2p coins						
Mean	Standard deviation	Minimum	Lower quartile	Median	Upper quartile	Maximum
6.89	0.296	6.45	6.63	6.88	7.08	7.48

- i) What assumption must be made about the weights of coins in this batch in order to conduct a test of significance on the sample mean? State, with a reason, whether you think this assumption is reasonable. [2]

(WJEC SAMPLE)

- 3.
- (i) The variable X has the distribution $N(20, 9)$.
 - (a) Find $P(X > 25)$. [1]
 - (b) Given that $P(X > a) = 0.2$, find a . [1]
 - (c) Find b such that $P(20 - b < X < 20 + b) = 0.5$. [3]
 - (ii) The variable Y has the distribution $N(\mu, \frac{\mu^2}{9})$. Find $P(Y > 1.5\mu)$. [3]

(OCR 2018)

4. A market gardener records the masses of a random sample of 100 of this year's crop of plums. The table shows his results.

Mass, m grams	$m < 25$	$25 \leq m < 35$	$35 \leq m < 45$	$45 \leq m < 55$	$55 \leq m < 65$	$65 \leq m < 75$	$m \geq 75$
Number of plums	0	3	29	36	30	2	0

- (a) Explain why the normal distribution might be a reasonable model for this distribution. [1]

The market gardener models the distribution of masses by $N(47.5, 10^2)$.

- (b) Find the number of plums in the sample that this model would predict to have masses in the range:
 - (i) $35 \leq m < 45$ [2]
 - (ii) $m < 25$. [2]
- (c) Use your answers to parts (b)(i) and (b)(ii) to comment on the suitability of this model. [1]

The market gardener plans to use this model to predict the distribution of the masses of next year's crop of plums.

- (d) Comment on this plan. [1]

(OCR SPECIMEN)

5. (a) The length, in cm, of a copper pipe has a normal distribution with mean μ and variance σ^2

It is found that 12% of the pipes exceed 17.5 cm and that 25% of the pipes are less than 16.8 cm.

- (i) Find the values of μ and σ^2 [8]

The copper pipe required for a certain bathroom unit must be less than 17.2 cm.

- (ii) Calculate the percentage of copper pipes that will be too long for the bathroom unit. [5]

- (b) Give a reason why the normal distribution is important in statistics. [1]

(CCEA 2019)

6.

A machine produces metal disks.

The diameters of the disks are normally distributed with mean 25 cm and standard deviation 0.4 cm.

A disk is rejected if its diameter is more than 25.5 cm or less than 24.7 cm.

- (i) Find the percentage of the disks that are accepted. [10]

The machine setting is altered so that larger disks can be made.

The standard deviation does not change but the mean diameter does.

- (ii) Find the new mean diameter if 4% of the disks produced are larger than 30 cm in diameter. [5]

(CCEA SPECIMEN)