WJEC – AS unit 1 Pure Mathematics A

16th May 2018 2 hour 30 minutes

PAPER WITH TEACHER'S SOLUTIONS

Total marks available: 120

Reminder: *Sufficient working must be shown to demonstrate the **mathematical** method employed.*

0 1 Showing all your working, simplify

a) $\dfrac{24\sqrt{a}}{\left(\sqrt{a}+3\right)^2-\left(\sqrt{a}-3\right)^2}$, [3]

b) $\dfrac{3\sqrt{7}+5\sqrt{3}}{\sqrt{7}+\sqrt{3}}$. [4]

a) $\dfrac{24\sqrt{a}}{(a+6\sqrt{a}+9)-(a-6\sqrt{a}+9)} = \dfrac{24\sqrt{a}}{12\sqrt{a}} = 2$

b) $\dfrac{3\sqrt{7}+5\sqrt{3}}{\sqrt{7}+\sqrt{3}} \times \dfrac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}}$

$= \dfrac{21+5\sqrt{21}-3\sqrt{21}-15}{7-\sqrt{21}+\sqrt{21}-3}$

$= \dfrac{6+2\sqrt{21}}{4} = \dfrac{3+\sqrt{21}}{2}$

0 2 The points A and B have coordinates $(-1, 10)$ and $(5, 1)$ respectively. The straight line L has equation $2x - 3y + 6 = 0$.

a) The line L intersects the line AB at the point C. Find the coordinates of C. [5]

b) Determine the ratio in which the line L divides the line AB. [2]

c) The line L crosses the x-axis at the point D. Find the coordinates of D. [1]

d) **i)** Show that L is perpendicular to AB.

ii) Calculate the area of the triangle ACD. [6]

a) Find equation of AB first gradient $= \frac{1-10}{5--1} = \frac{-9}{6} = \frac{-3}{2}$

$(y-y_1) = m(x-x_1)$ $\quad y-1 = -\frac{3}{2}(x-5)$

$2y-2 = -3x+15$, $\quad 2y+3x=17$

equation of L (x2)

equation of AB (x3)

$4x-6y=-12$ +

$9x+6y=51$

$13x=39$

$x=3$

in equation of L, $2(3)-3y+6=0$

$12=3y$, $y=4$

$C=(3,4)$

b) A $(-1,10)$ — across 4, down 6 — C intersection $(3,4)$ — across 2, down 3 — B $(5,1)$

AC : CB

2 : 1

c) The line L crosses the x axis when $y=0$

∴ sub $y=0$ into L $\quad 2x-3(0)+6=0$

$2x=-6$

$x=-3$ $\quad D=(-3,0)$

di) L:- $2x - 3y + 6 = 0$

$3y = 2x + 6$

$y = \frac{2}{3}x + 2$ gradient $= \frac{2}{3}$

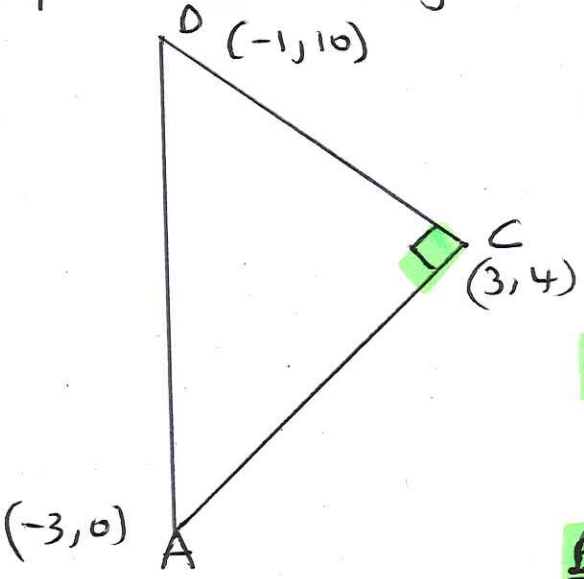
AB:- $2y + 3x = 17$

$2y = -3x + 17$

$y = -\frac{3}{2}x + \frac{17}{2}$ gradient $= -\frac{3}{2}$

product of gradients $= \frac{2}{3} \times \frac{-3}{2} = -1$ ∴ perpendicular

ii) Perpendicular ∴ Right angle at C



Use $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find distance

DC $= \sqrt{(-1-3)^2 + (10-4)^2}$

$= \sqrt{(-4)^2 + 6^2} = \sqrt{52}$

AC $= \sqrt{(-3-3)^2 + (0-4)^2}$

$= \sqrt{(-6)^2 + (-4)^2} = \sqrt{52}$

Area of triangle $= \frac{1}{2}$ base × height

$= \frac{1}{2}\sqrt{52} \times \sqrt{52}$

$= 26$

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0 3 Solve the following equation for values of θ between 0° and 360°.

$$2 - 3\cos^2\theta = 2\sin\theta$$

[6]

Use $\sin^2\theta + \cos^2\theta \equiv 1$

$\cos^2\theta \equiv 1 - \sin^2\theta$

Equation becomes

$2 - 3(1 - \sin^2\theta) = 2\sin\theta$

$2 - 3 + 3\sin^2\theta = 2\sin\theta$

$3\sin^2\theta - 2\sin\theta - 1 = 0$

ac	b
-3	-2

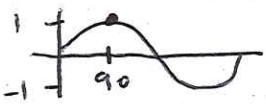
$-3, 1$

$3\sin^2\theta - 3\sin\theta + \sin\theta - 1 = 0$

$3\sin\theta(\sin\theta - 1) + 1(\sin\theta - 1) = 0$

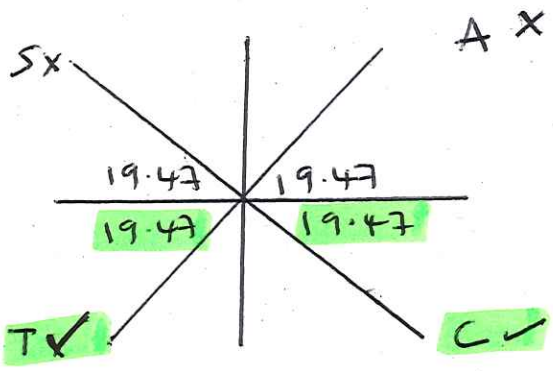
$(\sin\theta - 1)(3\sin\theta + 1) = 0$

$\sin\theta = 1$, $\sin\theta = -\frac{1}{3}$



$\theta = 90°$

$\theta = -19.47°$



$\theta = 90°, 199.47°, 340.53°$

0 4 a) Given that $y = \frac{5}{x} + 6\sqrt[3]{x}$, find $\frac{dy}{dx}$ when $x = 8$. [3]

b) Find $\int \left(5x^{\frac{3}{2}} + 12x^{-5} + 7\right) dx$. [3]

a) Rewrite as $y = 5x^{-1} + 6x^{1/3}$

$\frac{dy}{dx} = -5x^{-2} + \frac{1}{3}(6)x^{-2/3}$

$\frac{dy}{dx} = -\frac{5}{x^2} + 2x^{-2/3}$

Require $\frac{dy}{dx}$ when $x=8$

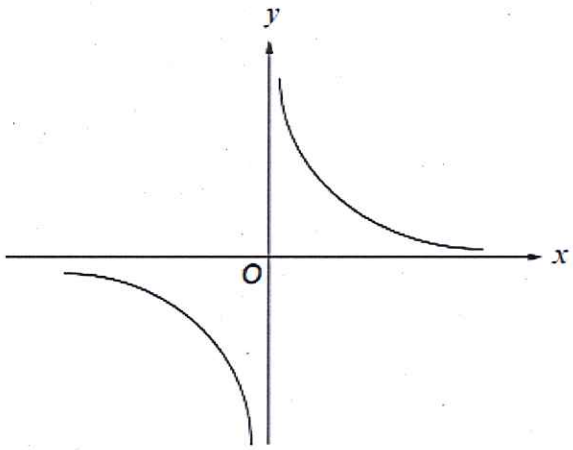
$\frac{dy}{dx} = -\frac{5}{8^2} + 2(8)^{-2/3} = \frac{27}{64}$

b) $\int \left(5x^{3/2} + 12x^{-5} + 7\right) dx$

$= \frac{5x^{5/2}}{5/2} + \frac{12x^{-4}}{-4} + 7x + C$

Must say '+C' or lose 1 mark

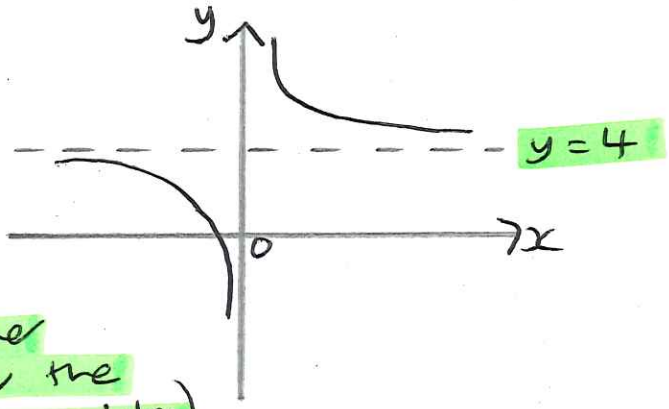
0 5 The diagram below shows a sketch of $y = f(x)$.



a) Sketch the graph of $y = 4 + f(x)$, clearly indicating any asymptotes. [2]

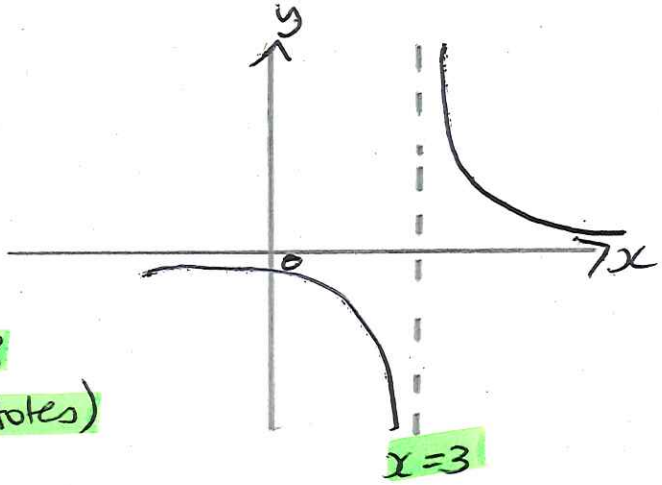
b) Sketch the graph of $y = f(x - 3)$, clearly indicating any asymptotes. [2]

a) $y = 4 + f(x)$
translation, +4
in y direction



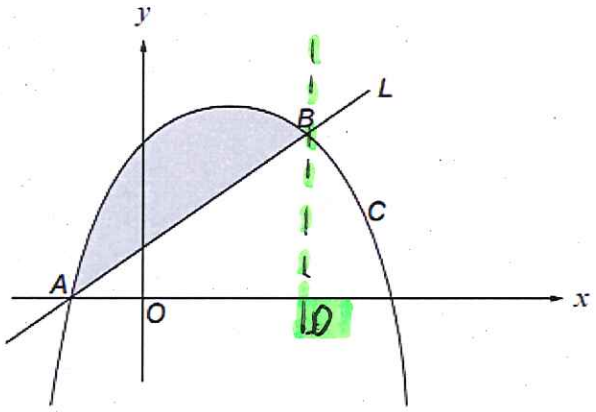
(you must put in $y = 4$ so the examiner knows that you know the location of the asymptotes)

b) $y = f(x - 3)$
translation, +3
in x direction



(again, label $x = 3$ to indicate asymptotes)

0 6



The sketch shows the curve C with equation $y = 14 + 5x - x^2$ and line L with equation $y = x + 2$. The line intersects the curve at the points A and B.

a) Find the coordinates of A and B. **[4]**

b) Calculate the area enclosed by L and C. **[6]**

a) Subs $y = x+2$ into $y = 14+5x-x^2$

$x+2 = 14+5x-x^2$

$x^2-4x-12=0$

$(x-6)(x+2)=0$

when $x=6$, $y=8$

when $x=-2$, $y=0$

$B=(6,8)$

$A=(-2,0)$

b) Required area

$= \int_{-2}^{6} (14+5x-x^2)\,dx$ − area ΔABD

$= \left[14x + \frac{5x^2}{2} - \frac{x^3}{3}\right]_{-2}^{6} - \frac{1}{2}(8\times 8)$

$= \left[\left(14(6) + \frac{5(6)^2}{2} - \frac{6^3}{3}\right) - \left(14(-2) + \frac{5(-2)^2}{2} - \frac{(-2)^3}{3}\right)\right] - 32$

$= \frac{352}{3} - 32 = \frac{256}{3} = 85\frac{1}{3}$

0 7 Prove that

$$\frac{\sin^3\theta + \sin\theta\cos^2\theta}{\cos\theta} \equiv \tan\theta.$$ [3]

$$LHS \equiv \frac{\sin^3\theta + \sin\theta\cos^2\theta}{\cos\theta}$$

$$\equiv \frac{\sin\theta(\sin^2\theta + \cos^2\theta)}{\cos\theta}$$

now, $\sin^2\theta + \cos^2\theta \equiv 1$

$$\therefore \quad LHS \equiv \frac{\sin\theta(1)}{\cos\theta}$$

and, $\frac{\sin\theta}{\cos\theta} \equiv \tan\theta$

$$\therefore \quad LHS \equiv \tan\theta \equiv RHS$$

0 8 Given that $(x - 2)$ and $(x + 2)$ are factors of the polynomial $2x^3 + px^2 + qx - 12$,

a) find the values of p and q, [4]

b) determine the other factor of the polynomial. [1]

a) If $(x-2)$ is a factor $f(2) = 0$

$2(2)^3 + p(2)^2 + q(2) - 12 = 0$

$16 + 4p + 2q - 12 = 0$

$4p + 2q = -4$

$2p + q = -2$ (1)

If $(x+2)$ is a factor $f(-2) = 0$

$2(-2)^3 + p(-2)^2 + q(-2) - 12 = 0$

$-16 + 4p - 2q - 12 = 0$

$4p - 2q = 28$

$2p - q = 14$ (2)

(1) + (2) gives $4p = 12$, $p = 3$

in (1) $6 + q = -2$, $q = -8$

b) $2x^3 + 3x^2 - 8x - 12 \equiv (x+2)(x-2)(ax+b)$

$2x^3 + 3x^2 - 8x - 12 \equiv (x^2 - 4)(ax+b)$

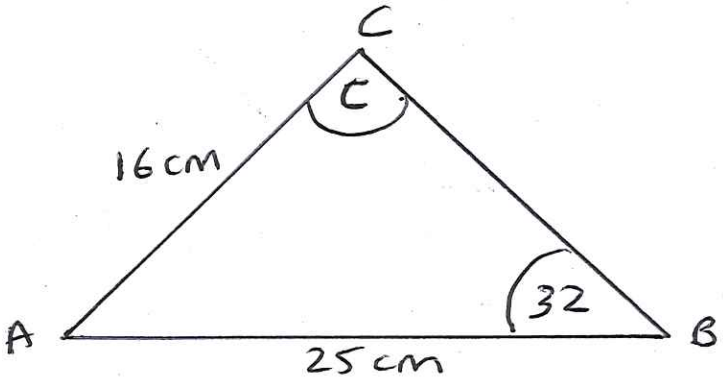
comparing coefficients :- $x^3 \rightarrow 2 = a$

const $\rightarrow -12 = -4b$

$b = 3$

other factor $= (2x+3)$

0 9 The triangle ABC is such that $AC = 16$ cm, $AB = 25$ cm and $A\hat{B}C = 32°$. Find two possible values for the area of the triangle ABC. [5]



Using sine rule $\frac{\sin\hat{C}}{c} = \frac{\sin\hat{B}}{b}$

$\frac{\sin C}{25} = \frac{\sin 32}{16}$

$\sin C = \frac{25 \times \sin 32}{16}$

$\sin C = 0.827\ldots$ $\hat{C} = 55.89°$

Remember, sine is also positive in the second quadrant so there is an alternative $\hat{C} = 180 - 55.89°$

$\hat{C} = 124.11°$

You therefore have two possible angles for A

$\hat{A} = 180° - (32 + 55.89) = 92.11°$

or, $\hat{A} = 180° - (32 + 124.11) = 23.89°$

Area $\Delta = \frac{1}{2} bc \sin\hat{A}$

$= \frac{1}{2} \times 16 \times 25 \times \sin 23.89$ or $\frac{1}{2} \times 16 \times 25 \times \sin 92.11$

$= 81.0\text{ cm}^2$ $= 199.9\text{ cm}^2$

1	0	a)	Use the binomial theorem to expand $(a+\sqrt{b})^4$.	[2]
		b)	Hence, deduce an expression in terms of a and b for $(a+\sqrt{b})^4+(a-\sqrt{b})^4$.	[2]

a) Using Pascal's triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$\therefore (a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$

so, $(a+\sqrt{b})^4 = a^4+4a^3\sqrt{b}+6a^2b+4ab\sqrt{b}+b^2$

b) $(a-\sqrt{b})^4 = a^4-4a^3\sqrt{b}+6a^2b-4ab\sqrt{b}+b^2$

$\therefore (a+\sqrt{b})^4+(a-\sqrt{b})^4 = 2a^4+12a^2b+2b^2$

1	1

a) The vectors $\mathbf{u}$ and $\mathbf{v}$ are defined by $\mathbf{u} = 9\mathbf{i} - 40\mathbf{j}$ and $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$. Determine the range of values for μ such that $\mu|\mathbf{v}| > |\mathbf{u}|$. [3]

b) The point A has position vector $11\mathbf{i} - 4\mathbf{j}$ and the point B has position vector $21\mathbf{i} + \mathbf{j}$. Determine the position vector of the point C, which lies between A and B, such that $AC:CB$ is 2:3. [3]

a) $|\underline{u}|$ = Magnitude of $\underline{u}$

$= \sqrt{9^2 + (-40)^2}$

$= \sqrt{1681} = 41$

$|\underline{v}|$ = Magnitude of $\underline{v}$

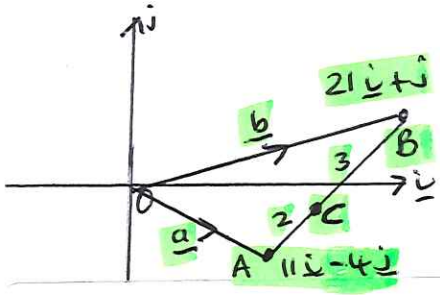
$= \sqrt{3^2 + (-4)^2}$

$= \sqrt{25} = 5$

Require $\mu 5 > 41$

$\mu > \frac{41}{5}$, $\mu > 8.2$

b)



$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \underline{b} - \underline{a}$

$\overrightarrow{OC} = \overrightarrow{OA} + \frac{2}{5}\overrightarrow{AB}$

$\overrightarrow{OC} = \underline{a} + \frac{2}{5}(\underline{b} - \underline{a})$

$\overrightarrow{OC} = \frac{3}{5}\underline{a} + \frac{2}{5}\underline{b}$

but

$\underline{a} = 11\underline{i} - 4\underline{j}$ and $\underline{b} = 21\underline{i} + \underline{j}$

$\therefore \overrightarrow{OC} = \frac{3}{5}(11\underline{i} - 4\underline{j}) + \frac{2}{5}(21\underline{i} + \underline{j})$

$\overrightarrow{OC} = \frac{33}{5}\underline{i} - \frac{12}{5}\underline{j} + \frac{42}{5}\underline{i} + \frac{2}{5}\underline{j}$

$\overrightarrow{OC} = 15\underline{i} - 2\underline{j}$

1	2	Find the values of m for which the equation $4x^2 + 8x - 8 = m(4x - 3)$ has real roots.[5]

$4x^2+8x-8=4mx-3m$

$4x^2+(8-4m)x-8+3m=0$

real roots if $b^2-4ac \geq 0$

$(8-4m)^2-4(4)(-8+3m) \geq 0$

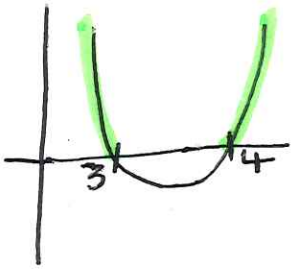
$64-64m+16m^2+128-48m \geq 0$

$16m^2-112m+192 \geq 0 \quad (\div 16)$

$m^2-7m+12 \geq 0$

$(m-3)(m-4) \geq 0$

Critical values are $m=3$, $m=4$



$m \leq 3$ or $m \geq 4$

1 3 A curve C has equation $y = x^3 - 3x^2$.

a) Find the stationary points of C and determine their nature. [7]

b) Draw a sketch of C, clearly indicating the stationary points and the points where the curve crosses the coordinate axes. [3]

c) **Without performing the integration**, state whether $\int_0^3 (x^3 - 3x^2)dx$ is positive or negative, giving a reason for your answer. [1]

a) $\frac{dy}{dx} = 3x^2 - 6x$ stationary values occur when $\frac{dy}{dx} = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0 \rightarrow y = 0$

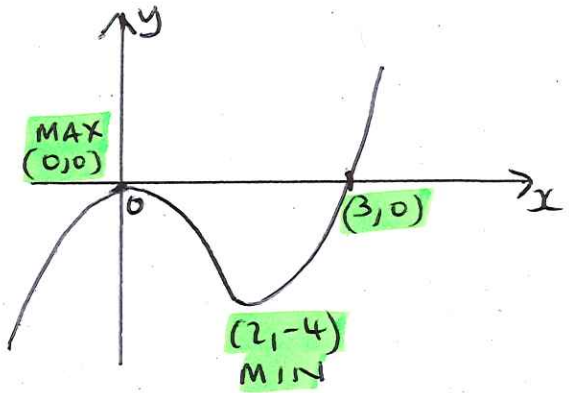
$x = 2 \rightarrow y = -4$

Stationary values at $(0,0)$ & $(2,-4)$

$\frac{d^2y}{dx^2} = 6x - 6$

at $x = 0$ $\frac{d^2y}{dx^2} < 0$ MAXIMUM $(0,0)$

at $x = 2$ $\frac{d^2y}{dx^2} > 0$ MINIMUM $(2,-4)$



cuts x axis at $y = 0$

$0 = x^3 - 3x^2$

$0 = x^2(x-3)$

at $x = 0, x = 3$

c) The integral will be negative because $y \leq 0$ in that relevant interval

or integration gives area and it is a negative area because it is under the x axis.

1 4 In each of the two statements below, c and d are real numbers. One of the statements is true, while the other is false.

A : $(2c - d)^2 = 4c^2 - d^2$, for all values of c and d.

B : $8c^3 - d^3 = (2c - d)(4c^2 + 2cd + d^2)$, for all values of c and d.

a) Identify the statement which is false. Show, by counter example, that this statement is in fact false. [2]

b) Identify the statement which is true. Give a proof to show that this statement is in fact true. [2]

a) Statement A is false

Let $c=2$ and $d=1$

LHS $= (2(2)-1)^2 = 3^2 = 9$

RHS $= 4(2)^2 - (1)^2 = 16 - 1 = 15$

$\therefore$ LHS $\neq$ RHS

b) Statement B is true

RHS $\equiv (2c-d)(4c^2+2cd+d^2)$

$\equiv 8c^3 + 4c^2d + 2cd^2 - 4c^2d - 2cd^2 - d^3$

$\equiv 8c^3 - d^3$

$\equiv$ LHS

1 6 The curve C has equation $y = 7 + 13x - 2x^2$. The point P lies on C and is such that the tangent to C at P has equation $y = x + c$, where c is a constant. Find the coordinates of P and the value of c. [5]

$y = 7 + 13x - 2x^2$

$\frac{dy}{dx} = 13 - 4x$

at the point P, tangent has gradient 1

$\therefore \frac{dy}{dx} = 1$ $\quad 13 - 4x = 1$

$12 = 4x$

$x = 3$

$y = 7 + 13(3) - 2(3)^2$, $y = 28$ $\quad P = (3, 28)$

Equation of tangent $\rightarrow$ $y = x + c$

but at P, $y = 28$, $x = 3$ $\quad \therefore 28 = 3 + c$

$c = 25$

17 a) Solve $2\log_{10}x = 1 + \log_{10}5 - \log_{10}2$. [4]

b) Solve $3 = 2e^{0.5x}$. [2]

c) Express $4^x - 10 \times 2^x$ in terms of y, where $y = 2^x$.

Hence solve the equation $4^x - 10 \times 2^x = -16$. [5]

a) $2\log_{10}x - \log_{10}5 + \log_{10}2 = 1$

$\log_{10}x^2 + \log_{10}2 - \log_{10}5 = 1$ (power law)

$\log_{10}\left(\frac{x^2 \times 2}{5}\right) = 1$ (addition law & subtraction law)

$\log_{10}\left(\frac{2x^2}{5}\right) = \log_{10}10$

$\therefore \frac{2x^2}{5} = 10$, $2x^2 = 50$, $x^2 = 25$

$x = 5$

b) $\frac{3}{2} = e^{0.5x}$

$1.5 = e^{0.5x}$ rewrite as a logarithm

$\ln_e 1.5 = 0.5x$

$\frac{\ln e^{1.5}}{0.5} = x$, $x = 0.811$ (3dp)

c) $4^x - 10 \times 2^x$

$y = 2^x$

$\therefore 4^x - 10 \times 2^x = 2^{2x} - 10(2^x)$

$= y^2 - 10y$

$y^2 - 10y = -16$

$y^2 - 10y + 16 = 0$

$(y-8)(y-2) = 0$ $y = 2$ or 8, but $y = 2^x$

$2^x = 2$ $x = 1$ or $2^x = 8 = 2^3$ $x = 3$

1 8 The coordinates of three points A, B, C are (4, 6), (–3, 5) and (5, –1) respectively.

a) Show that $B\hat{A}C$ is a right angle. [3]

b) A circle passes through all three points A, B, C. Determine the equation of the circle. [5]

gradient of AB $= \frac{6-5}{4--3} = \frac{1}{7}$

gradient of AC $= \frac{6--1}{4-5} = \frac{7}{-1} = -7$

Product of gradients $= \frac{1}{7} \times (-7) = -1$ ∴ perpendicular

Hence, $B\hat{A}C$ is a right angle

b) $\hat{A}$ is a right angle

So $\hat{A}$ must be the angle in a semi circle

So BC must be a diameter

∴ Centre of circle = Midpoint of BC

B (–3, 5) — Centre (1, 2) — C (5, –1)

length of radius

$\sqrt{4^2+3^2} = 5$

Equation of circle $(x-1)^2 + (y-2)^2 = 5^2$

1 5 The value of a car, £V, may be modelled as a continuous variable. At time t years, the value of the car is given by $V = Ae^{kt}$, where A and k are constants. When the car is new, it is worth £30000. When the car is two years old, it is worth £20000. Determine the value of the car when it is six years old, giving your answer correct to the nearest £100. [6]

$V = Ae^{kt}$

at $t=0$, $V=30,000$

$\therefore 30,000 = Ae^{k(0)}$

$A = 30,000$

at $t=2$, $V=20,000$

$\therefore 20,000 = 30,000e^{2k}$

$\frac{2}{3} = e^{2k}$

Rewriting as a logarithm

$\ln_e\left(\frac{2}{3}\right) = 2k$

$k = -0.20273\ldots$

at $t=6$

$V = 30,000e^{6\times(-0.20273\ldots)}$

$V = £8900$ (must give answer to nearest £100)