GCSE
3310U60-1
MATHEMATICS – NUMERACY
UNIT 2: CALCULATOR-ALLOWED
HIGHER TIER
THURSDAY, 10 MAY 2018 – MORNING
1 hour 45 minutes

ADDITIONAL MATERIALS
A calculator will be required for this paper.
A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES
Use black ink or black ball-point pen. Do not use gel pen or correction fluid.
You may use a pencil for graphs and diagrams only.
Write your name, centre number and candidate number in the spaces at the top of this page.
Answer all the questions in the spaces provided.
If you run out of space, use the continuation page at the back of the booklet. Question numbers must be given for the work written on the continuation page.
Take π as 3.14 or use the π button on your calculator.

INFORMATION FOR CANDIDATES
You should give details of your method of solution when appropriate.
Unless stated, diagrams are not drawn to scale.
Scale drawing solutions will not be acceptable where you are asked to calculate.
The number of marks is given in brackets at the end of each question or part-question.
In question 1(a), the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing.
Formula List - Higher Tier

Area of trapezium = \( \frac{1}{2} (a + b)h \)

Volume of prism = area of cross-section \times length

Volume of sphere = \( \frac{4}{3} \pi r^3 \)
Surface area of sphere = \( 4\pi r^2 \)

Volume of cone = \( \frac{1}{3} \pi r^2 h \)
Curved surface area of cone = \( \pi rl \)

In any triangle \( ABC \)
- Sine rule \( \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \)
- Cosine rule \( a^2 = b^2 + c^2 - 2bc \cos A \)
- Area of triangle = \( \frac{1}{2} ab \sin C \)

The Quadratic Equation
The solutions of \( ax^2 + bx + c = 0 \) where \( a \neq 0 \) are given by
\[ x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a} \]

Annual Equivalent Rate (AER)
AER, as a decimal, is calculated using the formula \( \left(1 + \frac{i}{n}\right)^n - 1 \), where \( i \) is the nominal interest rate per annum as a decimal and \( n \) is the number of compounding periods per annum.
1. In October 2011, a charge of 5p for a carrier bag was introduced in Wales. Money raised from this charge is given to charity.

For the period 1st October 2011 to 31st January 2015, it was estimated that a total of between £16.8 million and £21.9 million was donated to charity. This is as a result of people buying 5p carrier bags.

(a) In this part of the question, you will be assessed on the quality of your organisation, communication and accuracy in writing. **MUST EXPLAIN CAREFULLY**

Calculate an estimate of how much per month was given to charity between 1st October 2011 and 31st January 2015.

You must show all your working. [4 + 2 OCW]

Need to calculate number of months first.

1st Oct 11 = 1st Oct 14 = 31st Jan 15

3 years + 4 months = 40 months

3 \times 12 = 36 months

Estimate of 16.8 million £ to 21.9 million £

Take the Mean \( \frac{16.8 + 21.9}{2} = 19.35 \text{ million} \)

19.35 million = 19,350,000

\[ \text{Amount/month} = \frac{19,350,000}{40} = \£483,750 \]

(examiner accepts £400,000 - £550,000 depending on your workings/methods)
(b) Over time, there has been a reduction in the use of 5p carrier bags. This is because more people are using their own bags.

What impact might this have had on the amount given to charity for the month of September 2014 when compared with September 2012? [1]

The amount given to charity will be **less in 2014 than 2012 as the carrier bag consumption is less.**
2. (a) Megan and Rhodri both set out at the same time from home to go to the swimming pool. Rhodri travels by car. Megan cycles straight through the park.

Diagram not drawn to scale

Rhodri's journey by car is 5.5 miles. His average speed for the journey is 22 mph.

Megan's average speed on her bike is 12 mph. Megan arrives at the swimming pool 5 minutes before Rhodri.

Calculate the distance Megan cycles. Give your answer in miles. You must show all your working. [5]

Rhodri's journey takes:
\[
\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{5.5}{22} = 0.25\text{ hours} = \frac{1}{4}\text{ of }60\text{ minutes} = 15\text{ minutes}
\]

Megan's journey must be 10 minutes = \(\frac{1}{6}\) hour

Megan's distance:
\[
\text{Distance} = \text{Speed} \times \text{Time} = 12 \times \frac{1}{6} = 2\text{ miles}
\]

Distance Megan cycles is \(\frac{2}{\text{miles}}\).
(b) Gary travelled a distance of 231 km in 3 hours and 30 minutes. Calculate Gary's average speed in km/h. Circle your answer.

\[
\text{Speed} = \frac{\text{distance}}{\text{time}}
\]

\[
= \frac{231}{3.5} = 66 \text{ km/h}
\]
3. Yared is going to make a door wedge.

(a) The cross-section of the wedge is shown below. The horizontal length is 12 cm and the vertical height is 3 cm.

(i) Calculate the length $x$.

Give your answer correct to 3 significant figures.

\[ x^2 = 12^2 + 3^2 \]

\[ x^2 = 153 \]

\[ x = \sqrt{153} = 12.369 \text{ cm} \]

Must be 3sf as said so in question

(ii) The wedge must fit under Yared's door. The angle $y$ must be less than 15°.

Show that this wedge will fit under Yared's door. You must show all your working.

Sine, Cosine, Tangent

SOH CAH TOA

We have opp & adj so use TAN

\[ \tan y = \frac{\text{opp}}{\text{adj}} \]

\[ \tan y = \frac{\frac{3}{12}}{12} \]

\[ y = \tan^{-1}\left(\frac{\frac{3}{12}}{12}\right) = 14.0^\circ < 15^\circ \]

So yes the wedge fits under Jared's door.
(b) Yared decides to make a larger wedge that is mathematically similar to the one shown in part (a).
This wedge is to have a vertical height of 4.5 cm.

![Diagram](image)

*Diagram not drawn to scale*

Calculate the horizontal length of this door wedge.

**Comparing widths**

\[
\text{Width } 3\text{ cm } \Rightarrow 4.5\text{ cm}
\]

**Scale factor** = \( \times 1.5 \)

**Horizontal length** = \( 12 \times 1.5 \)

\[= 18\text{ cm} \]

The wedge will be **18 cm** cm long
4. A grass racetrack is shown in the diagram below. This is the region shaded in the diagram. Each end of the grass racetrack is created from semicircles. The inner semicircles have a radius of 15 m. The outer semicircles have a radius of 20 m. Each of the straight sections of the racetrack has a length of 65 metres.

![Diagram](Diagram not drawn to scale)

(a) What is the total area of grass in the two straight sections of the racetrack? You must show all your working.

We need two of these areas

\[ \text{area} = (65 \times 5) \times 2 \]
\[ = 650 \text{ m}^2 \]
(b) Calculate the area of the grass racetrack. You must show all your working.

If you put the two ends together you get

we require this middle which is

area \[ \text{area bigger circle} - \text{area smaller circle} \]

\[ = \pi r^2 - \pi r_1^2 \]

\[ = \pi \times 10^2 - \pi \times 15^2 \]

\[ = \pi \times 100 - \pi \times 225 \]

\[ = 349.7787 \ldots \text{ m}^2 \]

**Total area of the whole racetrack**

\[ = 349.7787 + 650 \]

\[ = 1200 \text{ m}^2 \text{ (nearest m}^2 \text{)} \]

(accepts 1199 -1200 or 175\pi +650)

(c) The grass is to be treated with a fertiliser.
It costs 20p to treat each 3 m\(^2\) of grass.
How much will it cost to treat the grass racetrack?
Give your answer correct to the nearest pound.
You must show all your working.

\[ \frac{1200 \times 0.2}{3} = \ell 80 \]

Cost is £ \[80 \]
5. Hot water is often stored in cylinders. The water in the cylinder is heated for use in the shower.

A plumbing engineer wants to calculate how long a shower can be used continuously before the water runs cold. He uses the following formulae:

\[ C = \frac{H(X - M)}{M - Y} \quad \text{and} \quad T = \frac{C + H}{F} \]

Where:
- \( C \) is the additional volume of water that feeds into the cylinder, in litres.
- \( H \) is the volume of hot water that the cylinder holds, in litres.
- \( M \) is the temperature of the water in the shower, in °C.
- \( X \) is the temperature of the hot water in the cylinder, in °C.
- \( Y \) is the temperature of the cold water that feeds into the cylinder, in °C.
- \( T \) is the time spent using the shower before the water runs cold, in minutes.
- \( F \) is the rate of flow of water in the shower, in litres per minute.

Daisy’s cylinder holds 300 litres of hot water.
The temperature of the hot water in her cylinder is 60°C.
The temperature of the cold water that feeds into Daisy’s cylinder is 8°C.
The water in Daisy’s shower is set at a temperature of 32°C.
Her shower has a rate of flow of 26 litres per minute.
Use the formulae to calculate:

- the additional volume of water that feeds into Daisy's cylinder, in litres,
- the number of minutes Daisy's shower will run continuously before the water runs cold.

\[ C = \frac{H(x-m)}{M-Y} \]

\[ C = \frac{300(60-32)}{32-8} \]

\[ C = 350 \text{ litres} \]

[Now put this into \( T = \frac{C+H}{F} \)]

\[ T = \frac{350 + 300}{26} \]

\[ T = 25 \text{ minutes} \]
6. Dr Khan and her daughter Faryl have different opinions about the mean temperature in their hallway.

Dr Khan and Faryl recorded the temperature in the hallway at 4 p.m. each day during the 30 days of April.

(a) In her note pad, Dr Khan summarised the temperatures in a grouped frequency table.

Unfortunately, Dr Khan has torn the page containing the table from her note pad and has lost some of the original data.

<table>
<thead>
<tr>
<th>Temperature, $t$ (°C)</th>
<th>Number of days</th>
</tr>
</thead>
<tbody>
<tr>
<td>$20 \leq t &lt; 21$</td>
<td>$20.5$</td>
</tr>
<tr>
<td>$21 \leq t &lt; 22$</td>
<td>$21.5$</td>
</tr>
<tr>
<td>$22 \leq t &lt; 23$</td>
<td>$22.5$</td>
</tr>
<tr>
<td>$23 \leq t &lt; 24$</td>
<td>$23.5$</td>
</tr>
</tbody>
</table>

Calculate an estimate of the mean temperature at 4 p.m. for these 30 days in Dr Khan's hallway.  

\[
\text{mean} = \frac{(4 \times 20.5) + (8 \times 21.5) + (8 \times 22.5) + (10 \times 23.5)}{30}
\]

\[
= \frac{82 + 172 + 180 + 235}{30} = \frac{669}{30}
\]

\[
= 22.3
\]

Estimate of the mean temperature at 4 p.m. for April in the hallway is $22.3^\circ C$.  

(b) What assumption have you made in calculating an estimate of the mean temperature at 4 p.m. for April in Dr Khan's hallway?

- Used the midpoints to estimate (assumed) this would provide the best estimate for the mean.

(c) Faryl recorded the same temperatures as her mother at 4 p.m. each day during April. She found that the actual mean temperature in the hallway during April was lower than the correctly calculated estimate of the mean.

Explain how this can be true.

This would happen if a lot of the actual recorded temperatures were less than the midpoint values.

Or

The temperatures tended to be towards the lower ends of the groups.

(Caution: Here you cannot say (midpoints were used) or (Faryl used exact values), you must explain more fully than that.)
7. Gwen fills a 10-litre bucket with water from a tap. She turns the tap until it is fully open. The bucket fills up with water, and when Gwen thinks it is close to being full, she slowly closes the tap. The bucket is full after 20 seconds.

The graph below shows the volume of water in the bucket during the 20 seconds.

Volume of water (litres) vs Time, \( t \) (seconds)

(a) After how many seconds did Gwen start to close the tap?

9.5 seconds  

(accept 9 - 9.5)

(b) Estimate at what rate water is entering the bucket at time \( t = 10 \) seconds. Give your answer in litres per second.

Need to work out gradient by drawing a tangent at \( t = 10 \)

\[ \text{gradient} = \frac{\text{change in } y}{\text{change in } x} = \frac{4.7}{15} = 0.31 \text{litres/second} \]
(c) When the tap is fully open, water flows out at 2 litres per second.

(i) Express 2 litres per second in **gallons per minute**. You must show all your working.

Remember
1 gallon = 8 pints

\[
\frac{2 \text{ litres}}{\text{second}} \quad \Rightarrow \quad \text{need to know this conversion}
\]

\[
1 \text{ litre} = 1.75 \text{ pints}
\]

\[
\therefore \quad \frac{2 \text{ litres}}{\text{second}} \approx 2 \times 1.75 \text{ pints/second}
\]

\[
= 3.5 \text{ pints/second}
\]

\[
= 3.5 \times 60 = 210 \text{ pints/minute}
\]

\[
= 210 \div 8 \text{ gallons/minute}
\]

\[
= 26.25 \text{ gallons/minute}
\]

(ii) Can a fully open tap fill a 90-gallon tank in under 3 1/2 minutes? You must show all your working.

Rate of water flow is 26.25 gallons/minute.
So see how much in 3 1/2 mins

\[
26.25 \times 3.5 = 91.875
\]

So yes, a 90-gallon tank can be filled in under 3 1/2 minutes.
8. The Headteacher of Ysgol Castell Gwyn wants to display pictures, drawn by pupils, along one side of a corridor. The pictures are to be in one row with no gaps between them, as shown in the diagram below.

![Diagram of pictures in a row with dimensions and labels]

The pictures are all square, with sides of length 15 cm, correct to the nearest 0.5 cm. The length of the corridor wall is 2010 cm, correct to the nearest 10 cm.

Calculate the smallest number of pictures and the greatest number of pictures that can be fitted in the row.

**Smallest number of pictures when wall small and pictures big**

\[
\text{Number of pictures} = \frac{2005}{15.25} = 131.4\overline{2}
\]

≈ 131 pictures

**Greatest number of pictures when wall big and pictures small**

\[
\text{Number of pictures} = \frac{2015}{14.75} = 136.8\overline{1}
\]

≈ 136 pictures

<table>
<thead>
<tr>
<th>Smallest number of pictures</th>
<th>Greatest number of pictures</th>
</tr>
</thead>
<tbody>
<tr>
<td>131</td>
<td>136</td>
</tr>
</tbody>
</table>
9. (a) Circle either TRUE or FALSE for each statement given below.

<table>
<thead>
<tr>
<th>STATEMENT</th>
<th>TRUE</th>
<th>FALSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A nominal annual interest rate is not the same as an AER.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A savings account offers a nominal annual interest rate of 2%, with interest paid monthly. After a year, any investment will have increased in value by exactly 2%.</td>
<td>TRUE</td>
<td>FALSE</td>
</tr>
<tr>
<td>A savings account offers an AER of 2.4%, with interest paid monthly. The monthly interest rate the account offers will be exactly 0.2%.</td>
<td>TRUE</td>
<td>FALSE</td>
</tr>
<tr>
<td>£100 is invested in a savings account that pays monthly interest at a rate of 1%. There are no further transactions into or out of the account. The amount in the account after a year will be £112.</td>
<td>TRUE</td>
<td>FALSE</td>
</tr>
</tbody>
</table>

Remember the nominal interest rate doesn't take into account the fact that the amount in the account changes every month as you are accumulating interest. That is why we use AERs to compare. These take into account the interest that is paid throughout the year.

(b) Benjamin invests £1000 into an account that pays interest every 6 months. He does not make any further payments into the account, and does not withdraw any money either. After a year, there is £1036 in the account.

Calculate how much was in the account after 6 months. Give your answer correct to the nearest penny. You must show all your working.

Use formula on page 2 to find i, the nominal interest rate.

\[
\text{interest} = 63.6 \quad \text{so} \quad AER = \frac{36}{100} \times 100 = 3.6\% = 0.036
\]

\[
0.036 = \left(1 + \frac{i}{2}\right)^2 - 1
\]

\[
1.036 = \left(1 + \frac{i}{2}\right)^2
\]

\[
\sqrt{1.036} = 1 + \frac{i}{2}
\]

\[
i = 0.03608 \quad \text{(annual)} \quad \text{or} \quad \frac{1}{2} \Rightarrow 0.0184 \quad \text{(6 monthly)}
\]

\[
\therefore \text{amount after 6 months} = 100 + 18.4 = £118.4
\]
9b) Easier Solution

Think of this as a reverse compound interest question.

Eg, £1000 in bank for 2 years at 5%.

We'd do $1000 \times (1.05)^2 = 1102.50$

In 9b) we've got the final amount and we need to find the multiplier.

Let the multiplier be $x$

\[
1000 \times x = 1036
\]

\[
x = \frac{1036}{1000} = 1.01784...
\]

This means an interest rate of

\[
1.01784... - 1 \times 100 = 0.78408... \%
\]

is paid 6 monthly.

So, Amount after 6 months would have been

\[
1.01784 \times 1000 = £1017.84
\]

must give final answer to nearest penny or you will lose 1 mark.
10. (a) A company makes plastic shelf supports for use in kitchen cupboards. A shelf support is made by attaching a cylinder to a right-angled triangular prism.

![Diagram](https://via.placeholder.com/150)

Diagram not drawn to scale

The cylinder has a diameter of 6 mm and a length of 9 mm. The prism has dimensions $CD = 8$ mm, $DE = 8$ mm, $CE = 11.3$ mm and $BC = 10$ mm.

The company sells the shelf supports in packs of 500. It needs to know the volume of plastic in 500 shelf supports. Calculate the volume of 500 shelf supports.

**Volume of cylinder**

$$V_{cylinder} = \pi r^2 h = \pi \times 3^2 \times 9 = 81\pi \text{ mm}^3$$

**Volume of triangular prism**

$$V_{prism} = \frac{b \times h \times \text{length}}{2} = \frac{8 \times 8 \times 10}{2} = 320 \text{ mm}^3$$

**Total volume of 500 shelf supports**

$$= 500 \times (320 + 81\pi)$$

$$= 287,234 \text{ mm}^3$$

(examiner accepts $287,170 - 287,251$)

Volume of 500 shelf supports = 

Turn over.
The company also makes metal door handles for kitchen cupboards. One of the door handles it makes is shown below. It is formed by joining two cylinders. One of the cylinders has a diameter of 4 cm and a length of 1.2 cm. The other cylinder has a diameter of 1.8 cm and a length of 3 cm.

Diagram not drawn to scale

At present, the company paints all the surfaces of the handle with a protective finish after the two cylinders have been joined together.

The shaded circular face is pressed against a cupboard door when fitted. In future, the company is not going to paint this shaded circular face. This is to reduce costs.

Calculate the percentage reduction in the area that is painted.  

\[
\text{Curved surface area } A = \pi d \times \text{length} = \pi \times 1.8 \times 3 = 5.4 \pi
\]

\[
\text{Curved surface area } B = \pi d \times \text{length} = \pi \times 4 \times 1.2 = 4.8 \pi
\]

\[
\text{Area shaded circular face } = \pi r^2 = \pi \times 0.9^2 = 0.81 \pi
\]
Area circular face on $B = \pi r^2 = \pi \times 2^2 = 4\pi$

"Donut" face on $B$ because 2 cylinders stuck together = $4\pi - 0.81\pi = 3.19\pi$

"Initially painted total of $
\begin{align*}
5 \cdot 4\pi + 4.8\pi + 4\pi + 3.19\pi + 0.81\pi \\
= 18.2\pi
\end{align*}
$
Now we are going to stop painting the $0.81\pi$.

$\therefore$ % reduction = $\frac{\text{change}}{\text{original}} \times 100$

$= \frac{0.81\pi}{18.2\pi} \times 100$

$= 4.45\%$
11. A sensor can detect any movement up to a distance of 6.5 m.

Diagram not drawn to scale

(a) A storeroom is in the shape of a cuboid, as shown below. The sensor is placed at A, so that
- it is aimed directly at B, where BD = 2 m,
- the front of the sensor is 20 cm from A along the line AB.

Diagram not drawn to scale

Will the sensor be able to detect movement at B? You must show all your working.

Calculate BE using Pythagoras first in \( \triangle BOE \)

\[
BE^2 = BD^2 + DE^2
\]

\[
BE^2 = 2^2 + 5.5^2 = 34.25
\]

\[BE = 5.852 \ldots m\]
Now use Pythagoras in \( \triangle BAE \)

\[
AE^2 + BE^2 = AB^2
\]

\[
3.2^2 + (5.852\ldots)^2 = AB^2
\]

\[
AB^2 = 44.49
\]

\[
AB = 6.6700\ldots
\]

\[
AB = 6.67 \text{ m (nearest cm)}
\]

Front of sensor is 20 cm from A along AB

6.67 cm - 20 cm = 6.47 cm = 6.47 m

B is 6.47 m away from front of sensor and sensor can detect 6.5 m away so \text{YES} it will

(b) Show that \( \hat{B}AE = 61.3^\circ \), correct to 1 decimal place.

\[
\tan \hat{A} = \frac{\text{opp}}{\text{adj}}
\]

\[
\tan \hat{A} = \frac{5.852\ldots}{3.2}
\]

\[
\hat{A} = \sinh^{-1}(5.852\ldots) \quad \frac{3.2}{3.2}
\]

\[
\hat{B}AE = 61.3^\circ \quad (1dp)
\]

END OF PAPER