MOCK EXAM WITH TEACHER'S SOLUTIONS

AS Unit 2: Applied Mathematics A

Time allowed 1 hour 45 minutes **Total Marks Available 78**

Section A: Statistics 44 marks **Section B: Mechanics 34 marks**

Reminder: *Sufficient working must be shown to demonstrate the* ***mathematical*** *method employed*

Section A: Statistics

1. A researcher records the blood glucose levels of a group of females.
The results, in mmol/litre are represented in the box plot in **Fig. 2** below.

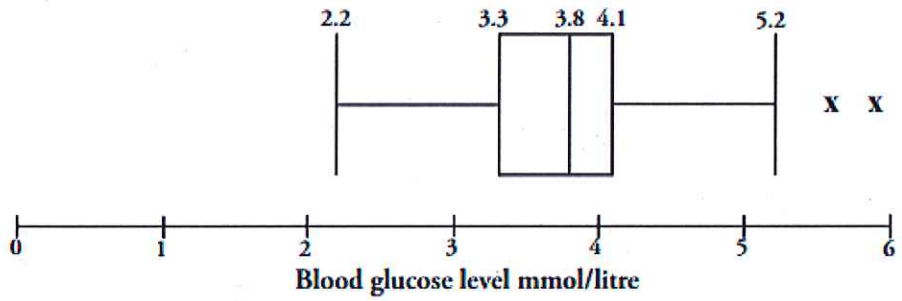


Fig.2

The researcher decides it is appropriate to clean the data by removing any outliers, which are indicated by **x** on the box plot.

If the outliers are removed from the original set of results, describe the effect this will have on

(i) the median, [1]

(ii) the standard deviation. [1]

a) The median could remain the same or be reduced.
The standard deviation will be reduced.

2. **(i)** State **two** of the conditions necessary for a random variable to be modelled by a binomial distribution. [2]

Customers at a petrol station can pay by cash or by credit card.
The probability that a customer pays by cash is 0.3

(ii) 10 customers are selected at random.

Calculate the probability that exactly 2 customers pay by cash. [4]

i) • independent trials
• fixed number of trials
• probability of success the same for each trial
(any two)

ii) $X \sim B(10, 0.3)$

Formula booklet page 9.

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$P(X = 2) = {}^{10}C_2 \, 0.3^2 \, 0.7^8$$

$$= 0.233 \quad (3dp)$$

3. The independent events A and B are such that

$$P(A \cup B) = 0{\cdot}9, \quad P(A \cap B) = 0{\cdot}4, \quad P(A) > P(B).$$

(a) Determine the values of $P(A)$ and $P(B)$. [7]

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0{\cdot}9 = P(A) + P(B) - 0{\cdot}4$

$P(A) + P(B) = 1{\cdot}3$ (1)

Independent events $\therefore$ $P(A) \times P(B) = P(A \cap B)$

$P(A) \times P(B) = 0{\cdot}4$ (2)

From (1) $P(A) = 1{\cdot}3 - P(B)$

substitute this into (2)

$(1{\cdot}3 - P(B))(P(B)) = 0{\cdot}4$

let $P(B) = x$ (it will be 'nicer' to solve)

$(1{\cdot}3 - x)(x) = 0{\cdot}4$

$1{\cdot}3x - x^2 = 0{\cdot}4$

$0 = x^2 - 1{\cdot}3x + 0{\cdot}4 \quad (\times 10)$

$0 = 10x^2 - 13x + 4$

x	$+$
ac	b
$+40$	-13

$-8, -5$

$0 = 10x^2 - 8x - 5x + 4$

$0 = 2x(5x - 4) - 1(5x - 4)$

$0 = (5x - 4)(2x - 1)$ either $x = 0{\cdot}8$ or $x = 0{\cdot}5$

given $P(A) > P(B)$ $\therefore$ $P(B) = 0{\cdot}5$

$\therefore$ $P(A) = 0{\cdot}8$

4. Cars arrive at a car wash in such a way that the number arriving during an interval of length t minutes has a Poisson distribution with mean $0{\cdot}25t$.

(a) Find the probability that

(i) exactly 4 cars arrive between 9:00 a.m. and 9:15 a.m.,

(ii) more than 6 cars arrive between 10:00 a.m. and 10:20 a.m. [6]

(b) The probability that less than 10 cars arrive during an interval of length t minutes is equal to 0·7166. Find the value of t. [3]

a) $X \sim Po(0.25t)$

i) 15 minutes, $0.25 \times 15 = 3.75$

$X \sim Po(3.75)$

page 9 formula book

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$P(X=4) = \frac{e^{-3.75} \times 3.75^4}{4!} = 0.194$$

ii) 20 minutes, $0.25 \times 20 = 5$

$X \sim Po(5)$, $P(X>6) = 1 - P(X \leq 6)$

$= 1 - 0.7622$

$= 0.2378$ (from tables)

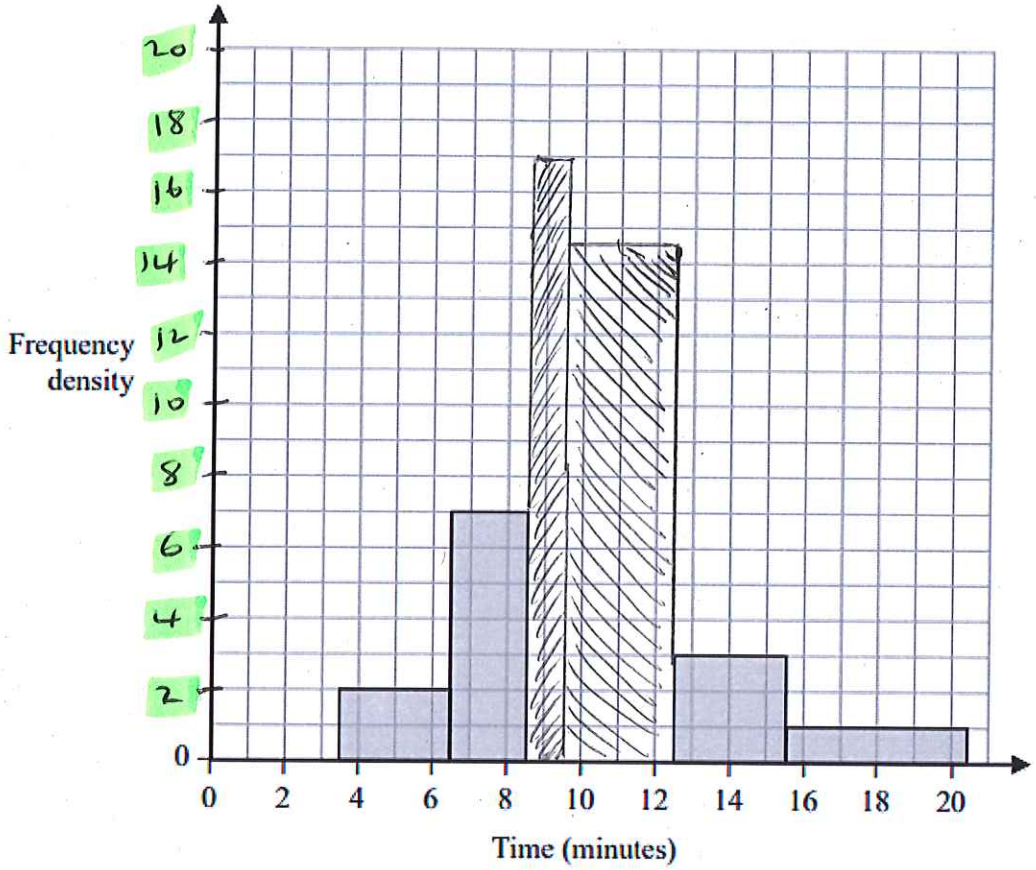
b) Require

$P(X<10) = 0.7166$

ie $P(X \leq 9) = 0.7166$, but don't know t by inspection, when $\lambda = 8$ we get required value.

$\therefore 8 = 0.25t$, $t = 32$ minutes

5. The partially completed histogram and the partially completed table show the time, to the nearest minute, that a random sample of motorists was delayed by roadworks on a stretch of motorway.

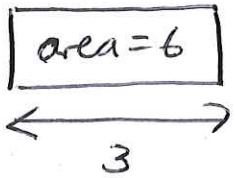


Delay (minutes)	Number of motorists
4 – 6	6
7 – 8	14
9	17
10 – 12	45
13 – 15	9
16 – 20	5

Estimate the percentage of these motorists who were delayed by the roadworks for between 8.5 and 13.5 minutes.

(5)

4-6 Minutes is actually
3.5 - 6.5 Minutes and the frequency = 6

∴ area = 6 

height must be 2

We can now label the frequency density column.

7-8 Minutes is actually
6.5 - 8.5 Minutes, width = 2, area = 7 x 2 = 14
frequency = 14

16-20 minutes is actually
15.5 - 20.5, width = 5, area = 5 x 1 = 5
frequency = 5

∴ Total Number of Motorists
= 6 + 14 + 17 + 45 + 9 + 5
= 96

Number delayed 8.5 - 13.5 Minutes = 17 + 45 + 3*
= 65

(* Careful, between 12.5 - 15.5 there are 9 vehicles so this means we estimate between 12.5 and 13.5 as 3 vehicles)

% delayed between 8.5 - 13.5 Minutes = $\frac{65}{96} \times 100$

= 67.7%

6. Sara was studying the relationship between rainfall, r mm, and humidity, h%, in the UK.
She takes a random sample of 11 days from May 1987 for Leuchars from the large data set.

She obtained the following results.

h	93	86	95	97	86	94	97	97	87	97	86
r	1.1	0.3	3.7	20.6	0	0	2.4	1.1	0.1	0.9	0.1

Sara examined the rainfall figures and found

$$Q_1 = 0.1 \qquad Q_2 = 0.9 \qquad Q_3 = 2.4$$

A value that is more than 1.5 times the interquartile range (IQR) above Q_3 is called an outlier.

(a) Show that $r = 20.6$ is an outlier.

(1)

(b) Give a reason why Sara might:

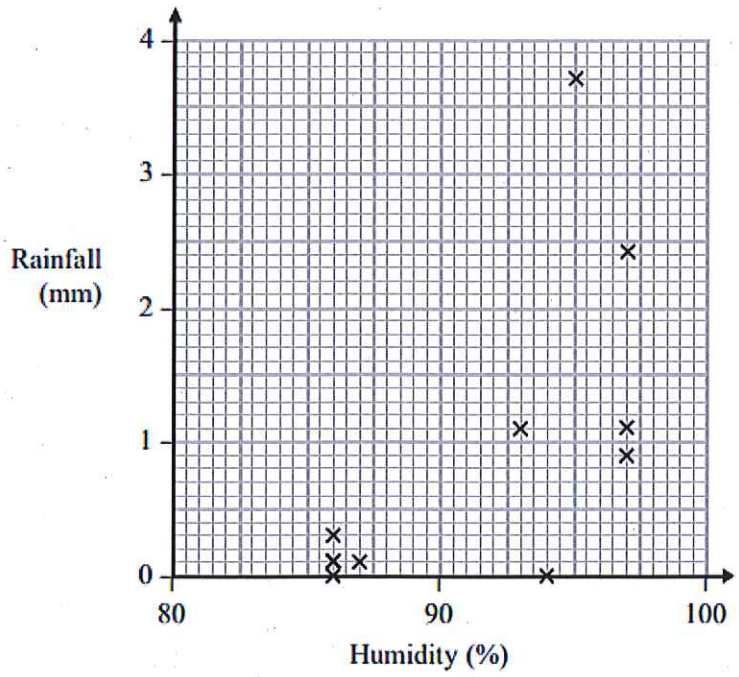
(i) include

(ii) exclude

this day's reading.

(2)

Sara decided to exclude this day's reading and drew the following scatter diagram for the remaining 10 days' values of r and h.



(c) Give an interpretation of the correlation between rainfall and humidity.

(1)

The equation of the regression line of r on h for these 10 days is $r = -12.8 + 0.15h$

(d) Give an interpretation of the gradient of this regression line.

(e) (i) Comment on the suitability of Sara's sampling method for this study. (1)

(ii) Suggest an improvement that Sara could make to her sampling method. (1)

a) $r = 20.6$ is an outlier if it is greater than $Q_3 + 1.5\,IQR$

Arrange in order

0, 0, 0.1, 0.1, 0.3, 0.9, 1.1, 1.1, 2.4, 3.7, 20.6

0.1 ↓ Lower quartile Q_1 | 0.9 ↓ Median Q_2 | 2.4 ↓ Upper quartile Q_3

$Q_3 + 1.5\,IQR = 2.4 + 1.5(2.4 - 0.1) = 5.85$

$20.6 > 5.85 \therefore$ 20.6 is an outlier.

bi) It could be an accurate piece of data and we want true and accurate findings so it should be included

ii) It could be an error in collecting data and it would really skew the results.

c) as humidity increases, rainfall increases.

d) A 10% increase in humidity gives a 1.5mm increase in rainfall on average. (or, each percentage point increase in humidity results in an extra 0.15mm of rainfall on average)

ei) It is not a good method because it only uses 11 days in 1 month in 1 place. It might not be representative (eg seasonal changes, changes in location in UK).

ii) Include more months, more days, more UK locations.

7. The discrete random variable $X \sim B(40, 0.27)$

Find $P(X \geqslant 16)$

(2)

Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager suspects that there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(b) Write down the hypotheses that should be used to test the manager's suspicion.

(1)

(c) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's suspicion. You should state the probability of rejection in each tail, which should be less than 0.05

(3)

(d) Find the actual significance level of a test based on your critical region from part (c).

(1)

One afternoon the manager observes that 12 of the 20 customers who bought baked beans, bought their beans in single tins.

(e) Comment on the manager's suspicion in the light of this observation.

(1)

Later it was discovered that the local scout group visited the supermarket that afternoon to buy food for their camping trip.

(f) Comment on the validity of the model used to obtain the answer to part (e), giving a reason for your answer.

(1)

a) $P(X \geq 16) = 1 - P(X \leq 15) = 1 - 0.94907 = 0.0509$

b) $H_0: p = 0.3$, $H_1: p \neq 0.3$

c) $H_0 \sim B(20, 0.3)$, $P(Y \leq 2) = 0.0355$

$P(Y \leq 9) = 0.9520$

In lower tail, critical value = 2, critical region $Y \leq 2$

In upper tail, critical value = 10, critical region $Y \geq 10$

∴ critical region is $Y \leq 2$ OR $Y \geq 10$

d) actual significance level $= 0.0355 + 0.0480$

$= 0.0835$

(because $P(Y \geq 10) = 1 - 0.9520 = 0.0480$)

e) Assuming that the 20 customers were a random sample, 12 is in the critical region. This is therefore enough evidence to reject H_0. There is evidence to assume the manage is correct. His suspicion is supported that there is a change in the proportion of customers buying single tins.

f) The 20 customers should be a random sample so the validity is questionable because a scout group is not representative. It would therefore not be a valid rejection of H_0.

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Section B: Mechanics

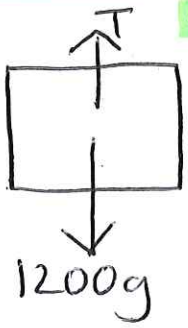
8. *(a)* A lift, of mass 1200 kg, is moving upwards. Find the tension in the lift cable when the lift is moving with

(i) an acceleration of $2\,ms^{-2}$,

(ii) constant speed. [4]

(b) A person of mass M kg stands in a lift which is moving downwards with an acceleration of $3\,ms^{-2}$. The reaction of the floor of the lift on the person is 442 N. Determine the value of M. [3]

a) i)



$a = 2ms^{-2}$ ↑

$F = ma$ $\quad a = 2ms^{-2}$

$T - 1200g = 1200(2)$

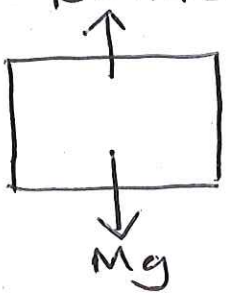
$T = 14160\,N$

ii) Constant speed = zero acceleration

$T = 1200g$

$T = 11760\,N$

b)



$a = 3ms^{-2}$ ↓

$F = ma$

$Mg - 442 = 3M$

$(9.8 - 3)M = 442$

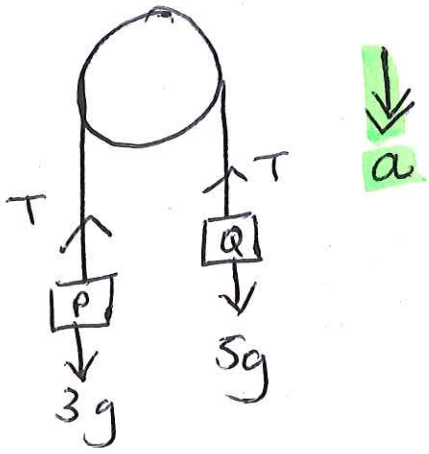
$M = 442 / 6.8$

$M = 65\,N$

9. Two particles P and Q, of masses 3kg and 5kg respectively, are attached one to each end of a light inextensible string which passes over a smooth peg. Initially, the particles are held at rest with the string just taut and with both hanging parts of the string vertical. The particles are then released from rest.

(a) Find the magnitude of the acceleration of P and the tension in the string. [7]

(b) What assumption does the word 'light', in the description of the string, enable you to make in your solution? [1]



$F = ma$

$T - 3g = 3a$

$5g - T = 5a$ +

$2g = 8a$

$a = \frac{2 \times 9.8}{8}$

$a = 2.45 ms^{-2}$

$T - 3(9.8) = 3(2.45)$

$T = 36.75 N$

b) Light string means the tension is the same throughout the string.

10. A raindrop *A* falls freely from rest from the top of a cliff. After it has fallen a distance 0·1 m, a second raindrop *B* begins to fall from rest from the top of the same cliff. The height of the cliff is 40 m.

(a) Find the velocity of *A* at the instant *B* begins to fall. [3]

(b) Find the velocity of *A* at the instant it reaches the ground. [2]

(c) Calculate the distance between the raindrops when the first raindrop *A* hits the ground. [7]

a) $s = 0.1$, $u = 0$, $v = ?$, $a = 9.8$, $t =$

$v^2 = u^2 + 2as$

$v^2 = 0 + 2(9.8)(0.1)$

$v^2 = 1.96$, $v = 1.4 ms^{-1}$

b) $s = 40$, $u = 0$, $v = ?$, $a = 9.8$, $t =$

$v^2 = u^2 + 2as$

$v^2 = 0 + 2(9.8)(40)$

$v^2 = 784$, $v = 28 ms^{-1}$

c) strategy ① Work out journey time of A from 0·1m → ground
② This is the total journey time for B
③ Use this journey time to see how far B travels.
④ This is subtracted from 40m to get the distance between the raindrops.

① Journey time for A from 0·1m → ground

$s = 39.9$, $u = 1.4$, $v = 28$, $a = 9.8$, $T = ?$

$v = u + at$

$28 = 1.4 + 9.8t$

$\frac{28 - 1.4}{9.8} = t$ $t = \frac{19}{7}$ seconds

② ∴ Total journey time for raindrop $B = \frac{19}{7}$ seconds

$s = ?$, $u = 0$, $v =$, $a = 9.8$, $T = \frac{19}{7}$

$s = ut + \frac{1}{2}at^2$

$s = 0 + \frac{1}{2}(9.8)\left(\frac{19}{7}\right)^2$

$s = 36.1m$

③ B travels 36·1m

④ Distance between A and B = 40m − 36·1m = 3·9m

12. A particle P moves in a straight line, starting from a point O. The velocity of P, measured in m s^{-1}, at time t s after leaving O is given by

$$v = 0.6t - 0.03t^2.$$

(i) Verify that, when $t = 5$, the particle is 6.25 m from O. Find the acceleration of the particle at this time. **[4]**

$v = 0.6t - 0.03t^2$

$x = \int (0.6t - 0.03t^2)\,dt$

$x = \frac{0.6t^2}{2} - \frac{0.03t^3}{3} + c$

$x = 0.3t^2 - 0.01t^3 + c$

'starts from O' $\therefore$ at $t=0$, $x=0$ $\therefore$ $c=0$

$x = 0.3t^2 - 0.01t^3$

at $t=5$ $x = 0.3(5)^2 - 0.01(5)^3 = 6.25\text{m}$

acceleration $= \frac{dv}{dt} = 0.6 - 0.03(2)t$

$= 0.6 - 0.06t$

at $t=5$, acceleration $= 0.6 - 0.06(5)$

$= 0.3\,\text{ms}^{-2}$

11. Two forces $(2\mathbf{i} - 11\mathbf{j})$ N and $(6\mathbf{i} + 7\mathbf{j})$ N act on a body of mass 4 kg.

(i) Find the acceleration of the body. [3]

Resultant force $= 2\underline{i} - 11\underline{j} + 6\underline{i} + 7\underline{j}$

$= 8\underline{i} - 4\underline{j}$

$F = ma$

$8\underline{i} - 4\underline{j} = 4a$

$\frac{8\underline{i} - 4\underline{j}}{4} = a$, $a = 2\underline{i} - \underline{j}$ ms^{-2}