WJEC – AS Unit 2 Applied Mathematics A

Sample Assessment Materials

PAPER WITH TEACHER'S SOLUTIONS

Total marks available: 120

Reminder: *Sufficient working must be shown to demonstrate the **mathematical** method employed.*

SECTION A – Statistics

1. The events A, B are such that $P(A) = 0.2$, $P(B) = 0.3$. Determine the value of $P(A \cup B)$ when

   (a) A, B are mutually exclusive, [2]

   (b) A, B are independent, [3]

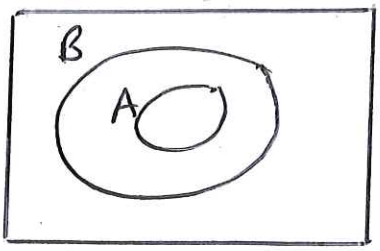
   (c) $A \subset B$. [1]

a) If mutually exclusive $P(A \cup B) = P(A) + P(B)$
$= 0.2 + 0.3 = 0.5$

b) If independent $P(A \cap B) = P(A) \times P(B)$
$= 0.2 \times 0.3$
$= 0.06$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.2 + 0.3 - 0.06 = 0.44$

c) If $A \subset B$ then A is a subset of B.



(you would say "A is contained in B")

$P(A \cup B) = P(B) = 0.3$

2. Dewi, a candidate in an election, believes that 45% of the electorate intend to vote for him. His agent, however, believes that the support for him is less than this. Given that p denotes the proportion of the electorate intending to vote for Dewi,

(a) state hypotheses to be used to resolve this difference of opinion. [1]

They decide to question a random sample of 60 electors. They define the critical region to be $X \leq 20$, where X denotes the number in the sample intending to vote for Dewi.

(b) (i) Determine the significance level of this critical region.

(ii) If in fact p is actually 0.35, calculate the probability of a Type II error.

(iii) Explain in context the meaning of a Type II error.

(iv) Explain briefly why this test is unsatisfactory. How could it be improved while keeping approximately the same significance level? [8]

a) $H_0 : p = 0.45$ $H_1 : p < 0.45$

bi) H_0 is $B(60, 0.45)$

Critical region is defined as $X \leq 20$

Calculator gives $P(X \leq 20) = 0.0446$

$\therefore$ Significance level $= 0.0446$

ii) A type II error is failing to reject a false null hypothesis

$X \sim B(60, 0.35)$ when $X > 21$ ie $1 - P(X \leq 20) = 1 - 0.45156$

$= 0.548$

iii) In this context a type II error is accepting that support for Dewi is 45% when it is actually 35%

iv) The error probability of 0.548 is too high. It could be reduced if a bigger sample was chosen.

3. Cars arrive at random at a toll bridge at a mean rate of 15 per hour.

(a) Explain briefly why the Poisson distribution could be used to model the number of cars arriving in a particular time interval. [1]

(b) Phil stands at the bridge for 20 minutes. Determine the probability that he sees exactly 6 cars arrive. [3]

(c) Using the statistical tables provided, find the time interval (in minutes) for which the probability of more than 10 cars arriving is approximately 0.3. [3]

a) The distribution can be modelled as POISSON, because the arrivals are INDEPENDENT and the MEAN RATE is constant.

b) Mean rate = 15/hour = 5/20 minute $\therefore \lambda = 5$

$X \sim Po(5)$

require $P(X=6)$ use $\frac{e^{-\lambda} \times \lambda^x}{x!}$ (formula book)

$P(X=6) = \frac{e^{-5} \times 5^6}{6!}$

$= 0.1462$

c) Require $P(X > 10) \approx 0.3$

Look in Poisson tables

This means $P(X \leq 10) \approx 0.7$

From tables $P(X \leq 10) = 0.7060$ when $\lambda = 9$

15/hr = 1 every 4 minutes

so if $\lambda = 9$ number of minutes = $9 \times 4 = 36$

Therefore time at bridge = 36 minutes.

4. A researcher wishes to investigate the relationship between the amount of carbohydrate and the number of calories in different fruits. He compiles a list of 90 different fruits, e.g. apricots, kiwi fruits, raspberries.

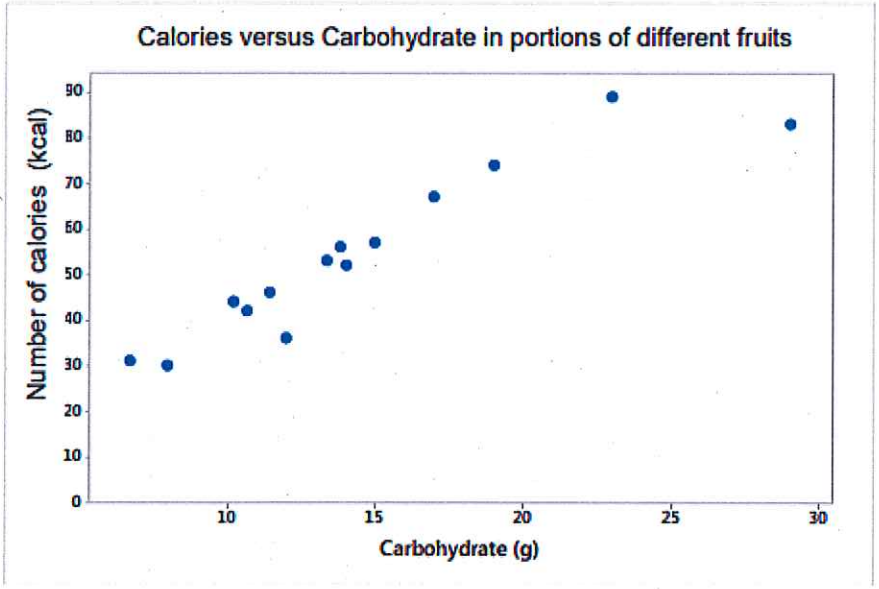
As he does not have enough time to collect data for each of the 90 different fruits, he decides to select a simple random sample of 14 different fruits from the list. For each fruit selected, he then uses a dieting website to find the number of calories (kcal) and the amount of carbohydrate (g) in a typical adult portion (e.g. a whole apple, a bunch of 10 grapes, half a cup of strawberries). He enters these data into a spreadsheet for analysis.

(a) Explain how the random number function on a calculator could be used to select this sample of 14 different fruits. [3]

(b) The scatter graph represents 'Number of calories' against 'Carbohydrate' for the sample of 14 different fruits.

(i) Describe the correlation between 'Number of calories' and 'Carbohydrate'. [1]

(ii) Interpret the correlation between 'Number of calories' and 'Carbohydrate' in this context. [1]



(c) The equation of the regression line for this dataset is:

'Number of calories' = 12.4 + 2.9 × 'Carbohydrate'

(i) Interpret the gradient of the regression line in this context. [1]

(ii) Explain why it is reasonable for the regression line to have a non-zero intercept in this context. [1]

a) Allocate each fruit a numbe 01-90
Generate a random numbe on a calculator using the random numbe functon.
Match this to the numbe allocated to the fruits.
This is the first numbe in your sample.
Do this until you have 14 different fruits (do not allow repeats).

bi) strong positive correlation

ii) The greate the amount of carbohydrate in the fruit Implies a greater numbe of calories.

ci) Every additional gram of carbohydrate corresponds to an increase in calories of 2-9 on areage (MUST SAY ON AVERAGE)

ii) Even if there is no carbohydrate in the fruit, you would expect it to contain some calories from dhe food groups, eg fats.

5. Gareth has a keen interest in pop music. He recently read the following claim in a music magazine.

In the pop industry most songs on the radio are not longer than three minutes.

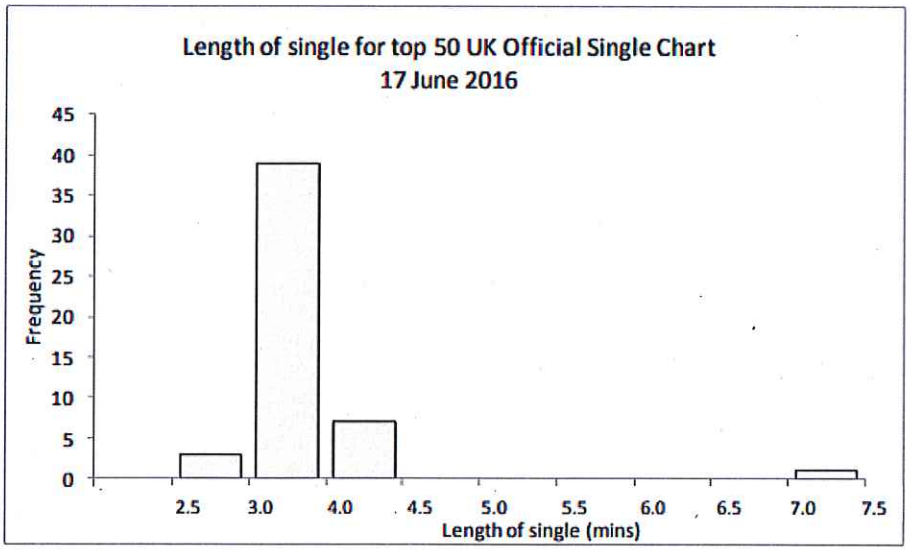
(a) He decided to investigate this claim by recording the lengths of the top 50 singles in the UK Official Singles Chart for the week beginning 17 June 2016. (A 'single' in this context is one digital audio track.)

Comment on the suitability of this sample to investigate the magazine's claim. [1]

(b) Gareth recorded the data in the table below.

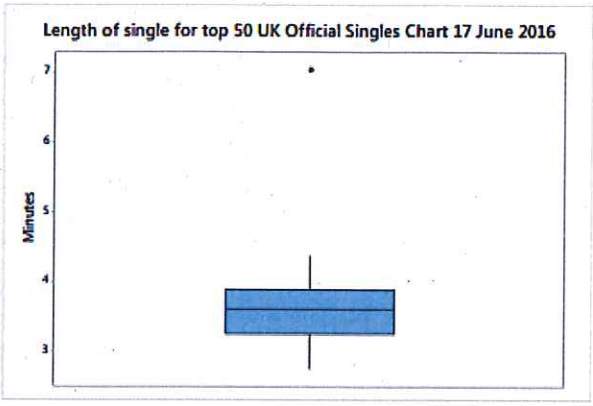
Length of singles for top 50 UK Official Chart singles, 17 June 2016									
2.5–(3.0)	3.0–(3.5)	3.5–(4.0)	4.0–(4.5)	4.5–(5.0)	5.0–(5.5)	5.5–(6.0)	6.0–(6.5)	6.5–(7.0)	7.0–(7.5)
3	17	22	7	0	0	0	0	0	1

He used these data to produce a graph of the distributions of the lengths of singles



State two corrections that Gareth needs to make to the histogram so that it accurately represents the data in the table. [2]

(c) Gareth also produced a box plot of the lengths of singles.



He sees that there is one obvious outlier.

(i) What will happen to the mean if the outlier is removed?

(ii) What will happen to the standard deviation if the outlier is removed? [2]

(d) Gareth decided to remove the outlier. He then produced a table of summary statistics.

(i) Use the appropriate statistics from the table to show, by calculation, that the maximum value for the length of a single is not an outlier.

	Summary statistics Length of single for top 50 UK Official Singles Chart (minutes)							
Length of single	N	Mean	Standard deviation	Minimum	Lower quartile	Median	Upper quartile	Maximum
	49	3.57	0.393	2.77	3.26	3.60	3.89	4.38

(ii) State, with a reason, whether these statistics support the magazine's claim. [4]

(e) Gareth also calculated summary statistics for the lengths of 30 singles selected at random from his personal collection.

	Summary statistics Length of single for Gareth's random sample of 30 singles (minutes)							
Length of single	N	Mean	Standard deviation	Minimum	Lower quartile	Median	Upper quartile	Maximum
	30	3.13	0.364	2.58	2.73	2.92	3.22	3.95

Compare and contrast the distribution of lengths of singles in Gareth's personal collection with the distribution in the top 50 UK Official Singles Chart. [3]

5a) It might not be representative as the UK Singles Chart might include lots of genres of music that are played on the radio, it might not be just 'pop'.

b) No gaps - length of single is a continuous variable
Scale - wrong on the 'length of single axis' at 3.0-4.0.

ci) Mean will reduce

ii) Standard deviation will reduce

di) Outlie if $> Q_3 + 1.5\,IQR$

$Q_3 = 3.89$ $IQR = Q_3 - Q_1 = 3.89 - 3.26 = 0.63$

$Q_3 + 1.5\,IQR = 3.89 + 1.5(0.63) = 4.84$ minutes

∴ Not an outlie because $4.84 > 4.38$

ii) No. If you look at the lower quartile it tells us that 3/4 of the songs are greate than 3.26 minutes. This contradicts his claim that most songs are less than 3 minutes.

Or Median $= 3.6 > 3$ so at least half are $>$ 3 minutes.

e) If you compare the medians, Gareth's singles are on average shorter. The standard deviation for Gareth's singles is smaller. This tells us that the lengths of his singles are less variable. For the chart singles, the mean and median values are close. This tells us that the distribution is quite symmetrical. Whereas with Gareth's, the mean is quite a bit higher than the median. So more than half of his singles are shorter than the mean - positively skewed.

SECTION B – Mechanics

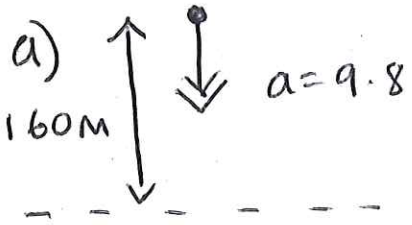
6. A small object, of mass 0.02 kg, is dropped from rest from the top of a building which is160 m high.

(a) Calculate the speed of the object as it hits the ground. [3]

(b) Determine the time taken for the object to reach the ground. [3]

(c) State one assumption you have made in your solution. [1]

a)



$s = 160$
$u = 0$
$v = ?$
$a = 9.8$
$t =$

$v^2 = u^2 + 2as$

$v^2 = 0 + 2(9.8)(160)$

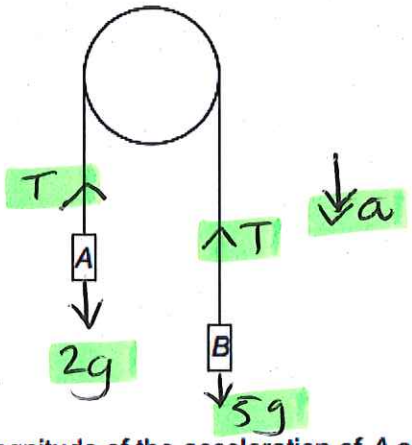
$v = 56 ms^{-1}$

b) $v = u + at$

$56 = 0 + 9.8t$, $t = \frac{40}{7} s$

c)
- object modelled as a particle
- Ignore air resistance
- Ignore all external forces apart from gravity.

7. The diagram below shows two particles *A* and *B*, of mass 2 kg and 5 kg respectively, which are connected by a light inextensible string passing over a fixed smooth pulley. Initially, *B* is held at rest with the string just taut. It is then released.



(a) Calculate the magnitude of the acceleration of *A* and the tension in the string. [6]

(b) What assumption does the word 'light' in the description of the string enable you to make in your solution? [1]

a) $F = ma$

$5g - T = 5a$ +

$T - 2g = 2a$

$3g = 7a$

$a = \frac{3g}{7}$, $a = 4.2 ms^{-1}$

$T = 2a + 2g$

$T = 2(4.2) + 2(9.8)$

$T = 28 N$

b) light → tension constant throughout the string.

8. A particle P, of mass 3 kg, moves along the horizontal x-axis under the action of a resultant force F N. Its velocity v ms^{-1} at time t seconds is given by

$$v = 12t - 3t^2.$$

(a) Given that the particle is at the origin O when $t = 1$, find an expression for the displacement of the particle from O at time t s. [3]

(b) Find an expression for the acceleration of the particle at time t s. [2]

a) $V = 12t^2 - 3t^2$

$x = \int (12t - 3t^2)\, dt$

$x = \frac{12t^2}{2} - \frac{3t^3}{3} + c$

$x = 6t^2 - t^3 + c$

at $t=1$, $x=0$

$\therefore\ 0 = 6 - 1 + c$, $c = -5$

$s = 6t^2 - t^3 - 5$

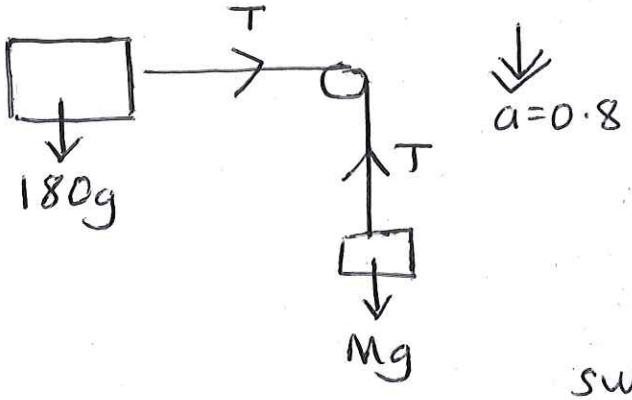
b) $a = \frac{dv}{dt}$

$a = 12 - 6t^2$

9. A truck of mass 180 kg runs on smooth horizontal rails. A light inextensible rope is attached to the front of the truck. The rope runs parallel to the rails until it passes over a light smooth pulley. The rest of the rope hangs down a vertical shaft. When the truck is required to move, a load of M kg is attached to the end of the rope in the shaft and the brakes are then released.

(a) Find the tension in the rope when the truck and the load move with an acceleration of magnitude 0.8 ms^{-2} and calculate the corresponding value of M. [5]

(b) In addition to the assumptions given in the question, write down one further assumption that you have made in your solution to this problem and explain how that assumption affects both of your answers. [3]



$F = ma$

$Mg - T = 0{\cdot}8M$ ①

$T = 180 \times 0{\cdot}8$ ②

$T = 144N$

substitute into ①

$M(9{\cdot}8) - 144 = 0{\cdot}8M$

$9M = 144$

$M = 16$

b) No air resistance

The truck is required to move with an acceleration of $0{\cdot}8 ms^{-2}$. If the air resistance aids motion, we would need a smaller tension. But if it resists motion, T will be greater.

If we conside the M kg load hanging down the vertical shaft, the '$F=ma$' equation will have an extra term opposing motion so we would need to increase M.

10. Two forces **F** and **G** acting on an object are such that

$$\mathbf{F} = \mathbf{i} - 8\mathbf{j},$$
$$\mathbf{G} = 3\mathbf{i} + 11\mathbf{j}.$$

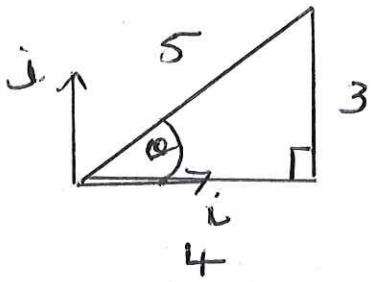
The object has a mass of 3 kg. Calculate the magnitude and direction of the acceleration of the object. [7]

Resultant force $= F + G$

$= (1+3)\underline{i} + (-8+11)\underline{j}$

$= 4\underline{i} + 3\underline{j}$

Magnitude of this force, using pythagoras



$= \sqrt{3^2 + 4^2}$

$= \sqrt{25}$

$= 5\,N$

$F = ma$

$5 = 3(a)$

$a = \frac{5}{3}\,ms^{-2}$

Direction:-

$\tan\theta = \frac{opp}{adj} = \frac{3}{4}$

$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^{\circ}$