WJEC – AS unit 2 Applied Mathematics A

23rd May 2018 1 hour 45 minutes

PAPER WITH TEACHER'S SOLUTIONS

Section A Statistics 45 marks

Section B Mechanics 35 marks **Total marks available: 80**

Reminder: *Sufficient working must be shown to demonstrate the **mathematical** method employed.*

Section A: Statistics

0	1	The random variable X has the binomial distribution B(16, 0·3). Showing your calculation, find $P(X = 7)$. [2]

$X \sim B(16, 0.3)$

$P(X=7) = {}^{16}C_7\, 0.3^7 0.7^9$ *

$= 0.10096$

$= 0.1010$ (4dp)

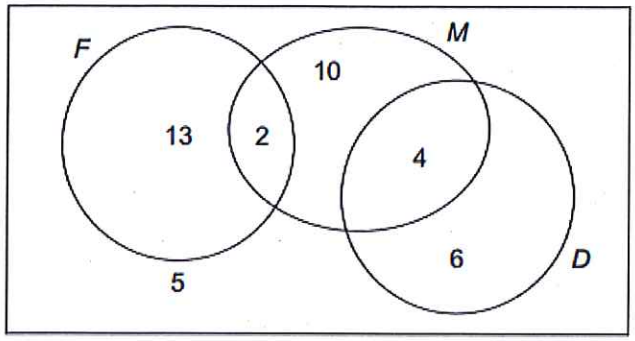
No calculation = no marks

* formula is on page 9 of formula booklet

If Binomial $B(n, p)$ $P(X=x)$

$$\binom{n}{x} p^x (1-p)^{n-x}$$

0 2 The Venn diagram shows the subjects studied by 40 sixth form students. F represents the set of students who study French, M represents the set of students who study Mathematics and D represents the set of students who study Drama.
The diagram shows the number of students in each set.



a) Explain what $M \cap D'$ means in this context. [1]

b) One of these students is chosen at random. Find the probability that this student studies

i) exactly two of these subjects,

ii) Mathematics or French or both. [3]

c) Determine whether studying Mathematics and studying Drama are statistically independent for these students. [3]

a) The set of students who study maths but do not study drama.

b) i) $\frac{2+4}{40} = \frac{6}{40}$

ii) P(maths or French or both) $= \frac{13+2+10+4}{40} = \frac{29}{40}$

c) $P(M \cap D) = \frac{4}{40} = \frac{1}{10}$ → from Venn diagram

If independent $P(M \cap D) = P(M) \times P(D)$

and $P(M) \times P(D) = \frac{16}{40} \times \frac{10}{40} = \frac{160}{1600} = \frac{1}{10}$ which is the value from the Venn diagram. So yes, they are statistically independent.

0 3 Naomi produces oak tabletops, each of area $4{\cdot}8\,m^2$. Defects in the oak tabletops occur randomly at a rate of $0{\cdot}25$ per m^2.

a) Find the probability that a randomly chosen tabletop will contain at most 2 defects. [3]

b) Find the probability that, in a random sample of 7 tabletops, exactly 4 will contain at most 2 defects each. [3]

a) Let the random variable X be the number of defects per tabletop.

$X \sim Po(0.25/m^2)$

area $= 4.8\,m^2$ so $0.25 \times 4.8 = 1.2$

$X \sim Po(1.2)$

$P(X \leq 2) = 0.8795$

b) $X \sim B(7, 0.8795)$

$P(X=4) = {}^{7}C_{4} \times 0.8795^4 \times 0.1205^3$

$= 0.0366$ (4dp)

0 4 Edward can correctly identify 20% of types of wild flower. He studies some books to see if he can improve how often he can correctly identify types of wild flower. He collects a random sample of 10 types of wild flower in order to test whether or not he has improved.

a) **i)** Write suitable hypotheses for this test.

ii) State a suitable test statistic that he could use. [2]

b) Using a 5% level of significance, find the critical region for this test. [3]

c) State the probability of a Type I error for this test and explain what it means in this context. [2]

d) Edward correctly identifies 4 of the 10 types of wild flower he collected. What conclusion should Edward reach? [2]

ai) Let p = probability that Edward correctly identifies types of wild flower.

$H_0 : p = 0.2$

$H_1 : p > 0.2$

ii) The number of times he correctly identifies a type of wild flower from the ten types of wild flower.

b) Under H_0, $X \sim B(10, 0.2)$ require 5% significance

From tables $P(X \leq 4) = 0.9672$

$\therefore$ Critical value is $X = 5$

Critical region is $X \geq 5$

c) $P(X \geq 5) = 1 - 0.9672 = 0.0328$

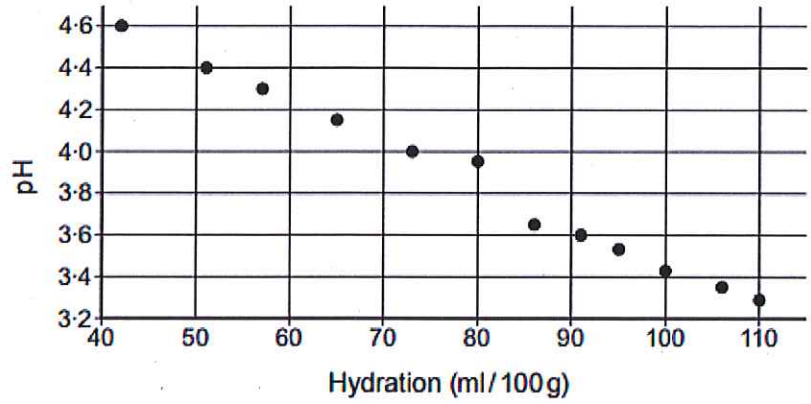
$\therefore$ Probability of a type I error is 0.0328, and this would be the probability that we conclude that Edward has improved his ability to identify wild flowers when he has not.

d) It is not in the critical region therefore do not reject H_0. There is insufficient evidence to conclude that he has improved his ability to correctly identify wild flowers.

0 5 A baker is aware that the pH of his sourdough, y, and the hydration, x, affect the taste and texture of the final product. The hydration is measured in ml of water per 100 g of flour (ml / 100 g). The baker researches how the pH of his sourdough changes as the hydration changes.

The results of his research are shown in the diagram below.

How pH changes with hydration



a) Describe the relationship between pH and hydration. [2]

b) The equation of the regression line for y on x is

$$y = 5{\cdot}4 - 0{\cdot}02x.$$

i) Interpret the gradient and intercept of the regression line in this context.

ii) Estimate the pH of the sourdough when the hydration is 20 ml / 100 g. Comment on the reliability of this estimate. [4]

a) Strong linear relationship or the higher the hydration the lower the pH.

b) i) gradient: For every additional ml of water per 100g of flour the pH decreases by 0.02 on average (must say on average)

intercept: at zero hydration, pH would be 5.4

ii) when $x=20$, $y=5.4-0.02(20)$ $y=5$

All of the data above is for x values between 40–110. $x=20$ is outside of this range. We are therefore using extrapolation which makes it less reliable.

0 6 Basel is a keen learner of languages. He finds a website on which a large number of language tutors offer their services. Basel records the cost, in dollars, of a one hour lesson from a random sample of tutors. He puts the data into a computer program which gives the following summary statistics.

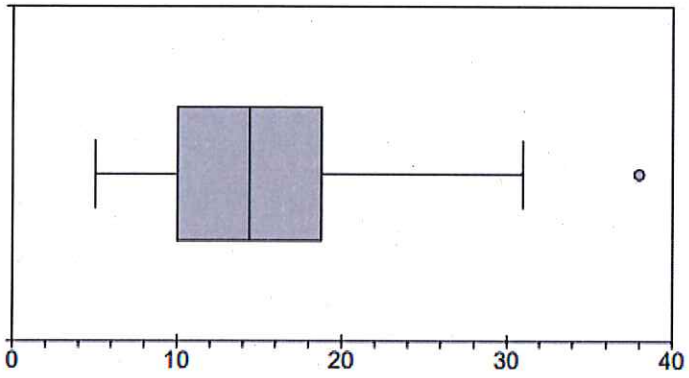
```
Cost per 1 hour lesson
Min.     :10.0
1st Qu.  :16.0
Median   :17.2
Mean     :19.8
3rd Qu.  :21.0
Max.     :40.0
```

a) Showing all calculations, comment on any outliers for the cost of a one hour lesson with a language tutor. [4]

b) Describe the skewness of the data and explain what it means in this context. [2]

Dafydd is also a keen learner of languages. He takes his own random sample of the cost, in dollars, for a one hour lesson. He produces the following box plot.

Cost in dollars for a one hour lesson



c) **i)** What will happen to the mean if the outlier is removed?

ii) What will happen to the median if the outlier is removed? [2]

d) Compare and contrast the distributions of the cost of one hour language lessons for Dafydd's sample and Basel's sample. [2]

a) Inter quartile range $= Q_3 - Q_1$

$= 21 - 16 = 5$

Is 40 an outlier?

$Q_3 + 1.5\,IQR = 21 + 1.5(5) = 28.5$

Therefore, any values above 28.5 are outliers. So, 40 is an outlier, but there

May be more.

Is 10 an outlier?

$Q_1 - 1.5\,IQR = 16 - 1.5(5) = 8.5$

$10 > 8.5$

So 10 isn't an outlier and we can conclude that there are no outlies at the lower end.

b) Positive skew

- A few tutors are very expensive
- The bulk of the tutors are relatively cheap.

ci) Mean will decrease

ii) Median may stay the same or may decrease.

d) • The median for Dafydd ≈ 14, whereas Basel's Median is 17.2. This shows an average that Dafydd's lessons are cheaper.

- The IQR for Dafydd ≈ 9.5 whereas for Basel it is 5. This shows that Dafydd's lessons are more spread out / variable in cost.
- Or, both are positively skewed

Careful in d) just saying "range bigger for Dafydd" or "Basel's median is higher" gets zero marks.

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Section B: Mechanics

0 7 A particle moves along the horizontal x-axis so that its velocity v ms^{-1} at time t seconds is given by

$$v = 6t^2 - 8t - 5.$$

At time $t = 1$, the particle's displacement from the origin is -4 m. Find an expression for the displacement of the particle at time t seconds. [3]

$V = 6t^2 - 8t - 5$ and $x = -4m$ when $t = 1$

$x = \int (6t^2 - 8t - 5)\,dt$

$x = \frac{6t^3}{3} - \frac{8t^2}{2} - 5t + C$ $= 2t^3 - 4t^2 - 5t + C$

at $t = 1$, $x = -4m$,

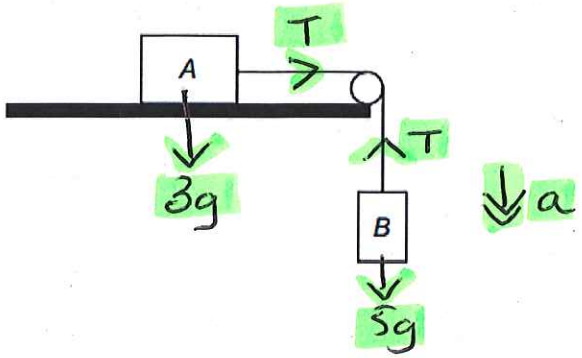
$-4 = 2(1)^3 - 4(1)^2 - 5(1) + C$

$-4 = -7 + C$

$C = 3$

$x = 2t^3 - 4t^2 - 5t + 3$

0 8 The diagram shows two objects *A* and *B*, of mass 3 kg and 5 kg respectively, connected by a light inextensible string passing over a light smooth pulley fixed at the end of a smooth horizontal surface. Object *A* lies on the horizontal surface and object *B* hangs freely below the pulley.



Initially, *B* is supported so that the objects are at rest with the string just taut. Object *B* is then released.

a) Find the magnitude of the acceleration of *A* and the tension in the string. [6]

b) State briefly what effect a rough pulley would have on the tension in the string. [1]

a) $F=ma$

object B → $5g - T = 5a$

object A → $T = 3a$ +

$5g = 8a$

$a = \frac{5g}{8}$

$a = 6.125\ ms^{-2}$

$T = 3a$, $T = 3(6.125)$, $T = 18.375\ N$

b) rough pulley would mean that the tension on either side of the pulley would not be the same.

0 9 Three forces **L**, **M** and **N** are given by

$$\mathbf{L} = 2\mathbf{i} + 5\mathbf{j},$$

$$\mathbf{M} = 3\mathbf{i} - 22\mathbf{j},$$

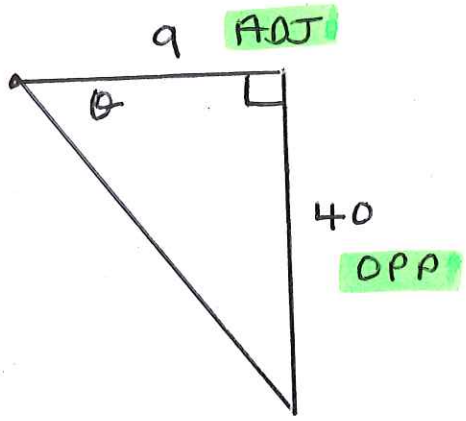
$$\mathbf{N} = 4\mathbf{i} - 23\mathbf{j}.$$

Find the magnitude and direction of the resultant of the three forces. [6]

Resultant force $= (2+3+4)\underline{i} + (5-22-23)\underline{j}$

$= 9\underline{i} - 40\underline{j}$

$|R|$



$|R| = \sqrt{(9^2) + 40^2}$

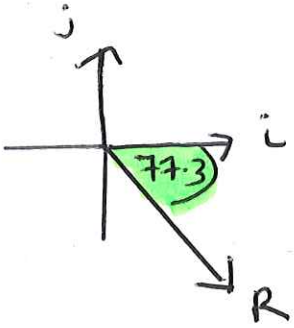
$|R| = \sqrt{1681}$

$|R| = 41N$

$\tan\theta = \frac{OPP}{ADJ}$

$\theta = \tan^{-1}\left(\frac{40}{9}\right)$

$\theta = 77.32°$



Direction $= -77.32°$

or $282.68°$

(Tip :- not 77° 'below the axis' / 'below the horizontal'. Give direction as specified above)

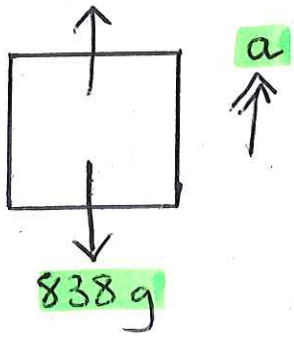
1 0 A person, of mass 68 kg, stands in a lift which is moving upwards with constant acceleration. The lift is of mass 770 kg and the tension in the lift cable is 8000 N.

a) Determine the acceleration of the lift, giving your answer correct to two decimal places. [3]

b) State whether the lift is getting faster, staying at the same speed or slowing down. [1]

c) Calculate the magnitude of the reaction of the floor of the lift on the person. [3]

a)



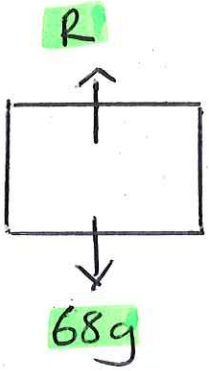
$F = ma$

$8000 - 838g = 838a$

$a = -0.253 \text{ ms}^{-2}$

b) The acceleration is negative which shows the lift is slowing down.

c)



$a = -0.253 \text{ ms}^{-2}$

$F = ma$

$R - 68g = 68(-0.253)$

$R = 649.16 \text{ N}$

1 1 A vehicle moves along a straight horizontal road. Points *A* and *B* lie on the road. As the vehicle passes point *A*, it is moving with constant speed 15 ms^{-1}. It travels with this constant speed for 2 minutes before a constant deceleration is applied for 12 seconds so that it comes to rest at point *B*.

a) Find the distance *AB*. [3]

The vehicle then reverses with a constant acceleration of 2 ms^{-2} for 8 seconds, followed by a constant deceleration of 1·6 ms^{-2}, coming to rest at the point *C*, which is between *A* and *B*.

b) Calculate the time it takes for the vehicle to reverse from *B* to *C*. [4]

c) Sketch a velocity-time graph for the motion of the vehicle. [3]

d) Determine the distance *AC*. [2]

a) During 'constant speed' speed = $\frac{\text{distance}}{\text{time}}$

distance = speed × time

$= 15 \times 120$

$= 1800m$

During 'deceleration'

$s = ?$
$u = 15$
$v = 0$
$a =$
$t = 12$

$s = \frac{1}{2}(u+v)t$

$s = \frac{1}{2}(0+15)12$

$s = 90m$

$AB = 1890m$

b) Constant acceleration first

$s =$
$u = 0$
$v = ?$
$a = 2$
$t = 8$

$v = u + at$

$v = 0 + 2(8)$

$v = 16ms^{-1}$

(The vehicle reverses from rest for 8 seconds and reaches a speed of $16ms^{-1}$ before the deceleration)

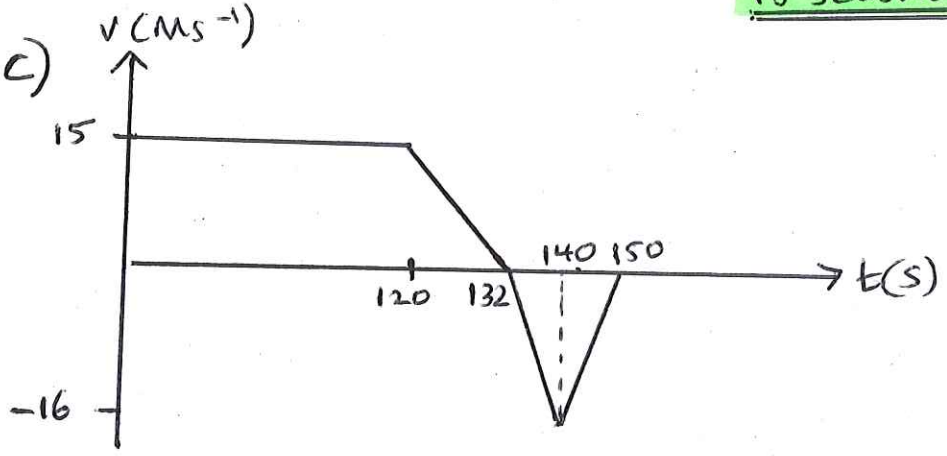
During deceleration

$s =$
$u = 16$
$v = 0$
$a = -1.6$
$t = ?$

$v = u + at$
$0 = 16 - 1.6t$
$t = 10$ seconds

Time to reverse from B to C $= 8 + 10$
$= 18$ seconds

c)



d) Distance AB = 1890m

Distance BC $= \frac{1}{2}$ base x height
$= \frac{1}{2}(18 \times 16) =$ 144m

∴ Distance AC $=$ AB $-$ BC
$= 1890 - 144$
$=$ 1746m

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