MOCK EXAM MARKSCHEME

AS Unit 1: Pure Mathematics A

This exam is based on 2018 Legacy WJEC C1 and C2 papers

Time allowed 2 ½ hours **Total Marks Available 121**

1. The points A, B, C and D have coordinates $(-2, 7)$, $(2, -1)$, $(5, 3)$ and $(3, 7)$ respectively.

   (a) (i) Show that AB and DC are parallel.

   (ii) Find the equation of AB. [5]

   (b) The line L has equation $x - 2y + 11 = 0$ and intersects AB at the point E.

   (i) Giving a reason, determine whether or not L is perpendicular to AB.

   (ii) Show that E has coordinates $(-1, 5)$.

   (iii) Calculate the length of EF, where F denotes the midpoint of AB. [8]

1a) Gradient AB $= \frac{\text{change in } y}{\text{change in } x} = \frac{7--1}{-2-2} = \frac{8}{-4} = -2$

Gradient DC $= \frac{\text{change in } y}{\text{change in } x} = \frac{7-3}{3-5} = \frac{4}{-2} = -2$

Gradients are the same ∴ AB & DC are parallel

ii) $(y - y_1) = m(x - x_1)$ where $(x_1, y_1) = (-2, 7)$ & $m = -2$

$y - 7 = -2(x - -2)$ ← examiner accepts unsimplified version

bi) rearrange equation of L → $y = \frac{1}{2}x + \frac{11}{2}$ ∴ gradient $= \frac{1}{2}$

L is perpendicular to AB because $\frac{1}{2} \times -2 = -1$

ii) AB and L intersect at the point E

∴ need to solve simultaneously to find E

AB → $y - 7 = -2x - 4$, $y = -2x + 3$ (1)

L → $y = \frac{1}{2}x + \frac{11}{2}$ (2) equating (1) & (2) gives

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$-2x+3 = \frac{1}{2}x + \frac{11}{2}$

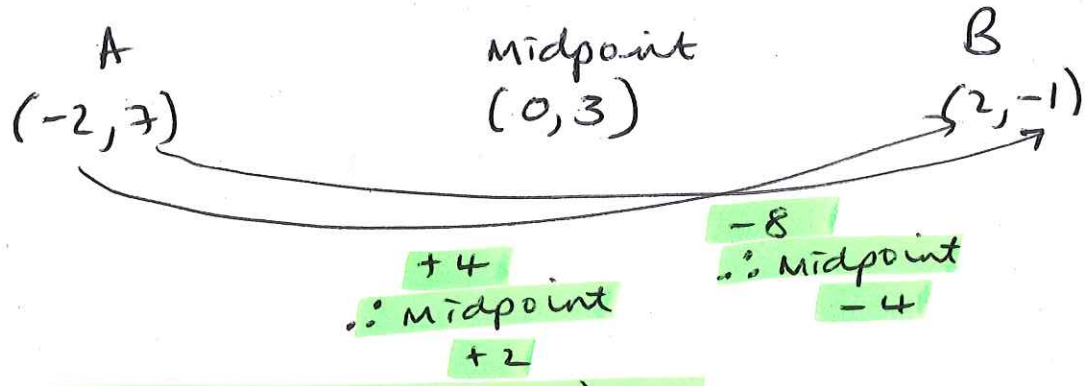
$-2-5 = 2-5x$, $\underline{x=-1}$

in (1) $y=-2x+3$

$y=-2(-1)+3$

$\underline{y=5}$ $\therefore$ $\underline{E=(-1,5)}$

iii)



$\underline{\text{Midpoint of } AB = (0,3) = F}$

$E=(-1,5)$ $F=(0,3)$

$\text{length} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$EF = \sqrt{(0--1)^2+(3-5)^2}$

$EF = \sqrt{1+4}$ $\underline{EF=\sqrt{5}}$

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2. Simplify $\sqrt{500}+\left(\sqrt{12}\times\sqrt{15}\right)-\frac{7\sqrt{60}}{\sqrt{3}}$. [4]

2) $\sqrt{500} = \sqrt{5}\sqrt{100} = 10\sqrt{5}$

$\sqrt{12}\times\sqrt{15} = \sqrt{4}\sqrt{3}\sqrt{5}\sqrt{3} = 2\times3\sqrt{5} = 6\sqrt{5}$

$\frac{7\sqrt{60}}{\sqrt{3}} = \frac{7\sqrt{20}\sqrt{3}}{\sqrt{3}} = 7\sqrt{4}\sqrt{5} = 7\times2\sqrt{5}$

$= 14\sqrt{5}$

∴ Expression becomes

$= 10\sqrt{5} + 6\sqrt{5} - 14\sqrt{5}$

$= 2\sqrt{5}$ (must be fully simplified)

3. The curve C has equation $y = x^2 - 6x + 7$. The point P, whose x-coordinate is 2, lies on C.

(a) Show that the equation of the **normal** to C at P is $y = \frac{1}{2}x - 2$. [6]

(b) The normal to C at P intersects C again at the point Q. Find the coordinates of Q. [4]

3a) find y coordinate when $x = 2$

$y = 2^2 - 6(2) + 7$, $y = -1$ $\quad P = (2, -1)$

Differentiate to find gradient of tangent at P

$\frac{dy}{dx} = 2x - 6$

at $(2, -1)$ $\frac{dy}{dx} = 2(2) - 6 = -2$

∴ gradient of tangent at $(2, -1)$ is -2

∴ gradient of normal at $(2, -1)$ is $\frac{1}{2}$

(because $2 \times -\frac{1}{2} = -1$)

$(y - y_1) = m(x - x_1)$ where $m = \frac{1}{2}$ and $(x_1, y_1) = (2, -1)$

$y - (-1) = \frac{1}{2}(x - 2)$

$y = \frac{1}{2}x - 2$ is the equation of the normal

b) Solve simultaneously $\quad y = \frac{1}{2}x - 2$ and $y = x^2 - 6x + 7$

$x^2 - 6x + 7 = \frac{1}{2}x - 2 \quad (\times 2)$

$2x^2 - 12x + 14 = x - 4$

$2x^2 - 13x + 18 = 0$

$2x^2 - 4x - 9x + 18 = 0$

xac	+b
36	−13

$-9, -4$

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$$2x(x-2) - 9(x-2) = 0$$
$$(2x-9)(x-2) = 0$$

either $x = \frac{9}{2}$ or $x = 2$

$x = \frac{9}{2}$ → x coordinate of point Q

$x = 2$ → x coordinate of point P

$$y = \frac{1}{2}x - 2$$
$$y = \frac{1}{2}\left(\frac{9}{2}\right) - 2$$
$$y = \frac{1}{4}$$

$$Q = \left(\frac{9}{2}, \frac{1}{4}\right)$$

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4. *(a)* Express $4x^2 + 40x - 69$ in the form $a(x + b)^2 + c$, where the values of the constants a, b and c are to be found. [3]

(b) **Using your answer to part *(a)*,** solve the equation

$$4x^2 + 40x - 69 = 0.$$ [3]

4a) $4x^2 + 40x - 69$

$= 4(x^2 + 10x) - 69$

$= 4[(x+5)^2 - 25] - 69$

$= 4(x+5)^2 - 100 - 69$

$= 4(x+5)^2 - 169$

$a = 4$

$b = 5$

$c = -169$

b) Equation becomes

$4(x+5)^2 - 169 = 0$

$4(x+5)^2 = 169$

$(x+5)^2 = \frac{169}{4}$, square root both sides

$x + 5 = \pm\sqrt{\frac{169}{4}}$

$x + 5 = \pm\frac{13}{2}$

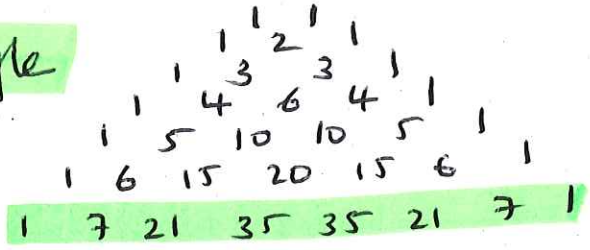
$x = -5 + \frac{13}{2}$, $x = -5 - \frac{13}{2}$

$x = \frac{3}{2}$ $x = -\frac{23}{2}$

5. *(a)* Using the binomial theorem, write down and simplify the first four terms in the expansion of $\left(1-\frac{x}{2}\right)^7$ in ascending powers of x. [4]

(b) The coefficient of x^2 in the expansion of $(1 + 4x)^n$ is 3360. Given that n is a positive integer, find the value of n. [3]

5a) Using Pascal's Triangle



$(1+x)^7 = 1 + 7x + 21x^2 + 35x^3 + \cdots$

$\left(1-\frac{x}{2}\right)^7 = 1 - \frac{7x}{2} + \frac{21x^2}{4} - \frac{35x^3}{8}$

b) Because the powe is 'n' we cannot use pascal's triangle. Use formula booklet, page 2.

$(1+x)^n$ has x^2 term $\frac{n(n-1)x^2}{2}$

$\therefore$ $(1+4x)^n$ has x^2 term $\frac{n(n-1)(4x)^2}{2} = 8n(n-1)x$

$\therefore$ $8n(n-1) = 3360 \quad (\div 8)$

$n(n-1) = 420$

$n^2 - n - 420 = 0$

$(n-21)(n+20) = 0$

$n = 21$ or $n = -20$ (given n positive)

$\therefore$ $n = 21$

6. Find the range of values of x satisfying the inequality

$$9x^2 + 16x - 4 > 0.$$ [3]

6)

$9x^2 + 16x - 4 > 0$

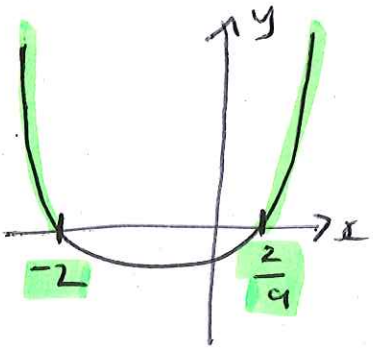
$9x^2 + 18x - 2x - 4 > 0$

$9x(x+2) - 2(x+2) > 0$

$(x+2)(9x-2) > 0$

ax^2	b
-36	16

$18, -2$



Critical values are $x = -2$, $x = \frac{2}{9}$

We require y to be positive

$x < -2$ or $x > \frac{2}{9}$

careful
Must be two seperate, strict inequalities, if you say the word 'and' you lose 1 mark. It must be or.

7. (a) Given that $y = 9x^2 - 7x - 8$, find $\frac{dy}{dx}$ from first principles. [5]

(b) Given that $y = \frac{k}{x} + 14\sqrt{x}$ and that $\frac{dy}{dx} = 2$ when $x = 9$, find the value of the constant k. [4]

7a) $y + \delta y = 9(x+\delta x)^2 - 7(x+\delta x) - 8$

$y + \delta y = 9x^2 + 18x\delta x + 9(\delta x)^2 - 7x - 7\delta x - 8$

subtract y from both sides

$\delta y = 18x\delta x + 9(\delta x)^2 - 7\delta x$

divide through by δx

$\frac{\delta y}{\delta x} = 18x + 9\delta x - 7$

let $\delta x \to 0$

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 18x - 7$

b) $y = kx^{-1} + 14x^{1/2}$

$\frac{dy}{dx} = -kx^{-2} + 7x^{-1/2}$ $\left(= -\frac{k}{x^2} + \frac{7}{\sqrt{x}}\right)$

$2 = -\frac{k}{9^2} + \frac{7}{3}$

$-\frac{1}{3} = \frac{-k}{81}$, $k = 27$

8. *(a)* (i) Find one real root of the equation $8x^3 + 7x^2 - 13x + 10 = 0$.

(ii) Show that the root you have found is the only real root of the equation

$$8x^3 + 7x^2 - 13x + 10 = 0.$$ [7]

8a)i) $f(1) = 8(1)^3 + 7(1)^2 - 13(1) + 10 \neq 0$

$f(-1) = 8(-1)^3 + 7(-1)^2 - 13(-1) + 10 \neq 0$

$f(2) = 8(+2)^3 + 7(+2)^2 - 13(+2) + 10 \neq 0$

$f(-2) = 8(-2)^3 + 7(-2)^2 - 13(-2) + 10 = 0$ $\therefore (x+2)$ is a factor and $x = -2$ is a real root

ii) Comparing coefficients

$$8x^3 + 7x^2 - 13x + 10 = (x+2)(ax^2 + bx + c)$$

x^3 → $8 = a$

constant → $10 = 2c$, $c = 5$

x^2 → $7 = 2a + b$

$7 = 16 + b$, $b = -9$

$\therefore (x+2)(8x^2 - 9x + 5) = 0$

consider the discriminant of the quadratic factor to see if it has any real roots

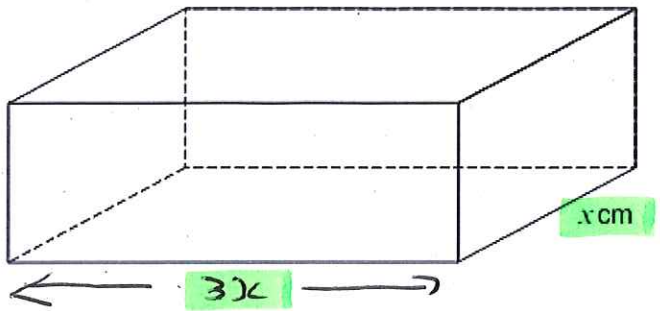
$b^2 - 4ac = (-9)^2 - 4(8)(5) = -79$

$b^2 - 4ac < 0$

$\Rightarrow$ There are no real roots to the quadratic factor $\therefore$ $x = -2$ is the only real root of the original cubic.

9

A closed box, in the form of a cuboid, is such that the length of its base is three times the width of its base. The volume of the box is 6000 cm^3. The total length of the twelve edges of the box is denoted by L cm.



(a) Show that $L = 16x + \frac{8000}{x^2}$, where x cm denotes the width of the base. [3]

(b) Find the minimum value of L, showing that the value you have found is a minimum value. [5]

9a) $V = L \times W \times H$

$6000 = 3x \times x \times H$, $H = \frac{6000}{3x^2}$, $H = \frac{2000}{x^2}$

Total length of edges = L

$L = 4 \times (L + W + H)$

$L = 4 \times \left(x + 3x + \frac{2000}{x^2}\right)$

$L = 16x + \frac{8000}{x^2}$

b) $L = 16x + 8000x^{-2}$

$\frac{dL}{dx} = 16 - 16000x^{-3}$

Stationary values occur when $\frac{dL}{dx} = 0$

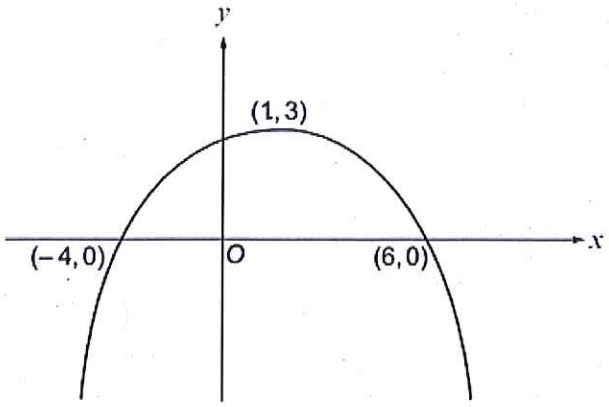
ie $16 = \frac{16000}{x^3}$, $x^3 = 1000$, $x = 10$

$\frac{d^2L}{dx^2} = 48000x^{-4}$

at $x = 10$, $\frac{d^2L}{dx^2} > 0$, $\therefore$ MINIMUM at $x = 10$

at $x = 10$, $L = 16(10) + \frac{8000}{10^2} = 240$

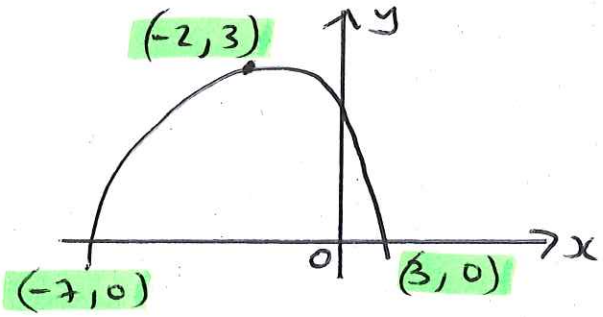
10. The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the points $(-4, 0)$ and $(6, 0)$ and has a maximum point at $(1, 3)$.



(a) Sketch the graph of $y = f(x + 3)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. [3]

(b) Gwen is asked by her teacher to draw the graph of $y = f(ax)$ for various values of the constant a. Two of Gwen's graphs pass through the point $(2, 0)$.
Find the value of a corresponding to each of these two graphs. [2]

10a) Translation 3 to the left along x axis



b) $y = f(3x)$ is one because this is a $\times \frac{1}{3}$ stretch along x axis and

$(6,0) \rightarrow (2,0)$

$y = f(-2x)$ is the other, because this is a reflection across the y axis (x values 'flip over') and a $\times \frac{1}{2}$ stretch

$(-4,0) \rightarrow (4,0) \rightarrow (2,0)$

$a = -2, \ a = 3$

11.

(a) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$10\sin^2\theta + 3\sin\theta = 4\cos^2\theta - 2.$$ [6]

(b) Find all values of ϕ in the range $0° \leqslant \phi \leqslant 360°$ satisfying

$$\frac{3}{\cos\phi} - \frac{5}{\sin\phi} = 0.$$ [3]

11a) use $\cos^2\theta \equiv 1-\sin^2\theta$, equation becomes

$10\sin^2\theta + 3\sin\theta = 4(1-\sin^2\theta) - 2$

$14\sin^2\theta + 3\sin\theta - 2 = 0$

xac	+b
-28	3

$-4, 7$

$14\sin^2\theta + 7\sin\theta - 4\sin\theta - 2 = 0$

$7\sin\theta(2\sin\theta + 1) - 2(2\sin\theta + 1) = 0$

$(7\sin\theta - 2)(2\sin\theta + 1) = 0$

$\sin\theta = \frac{2}{7}$, $\sin\theta = -\frac{1}{2}$

$\theta = 16.6°$ $\theta = -300$

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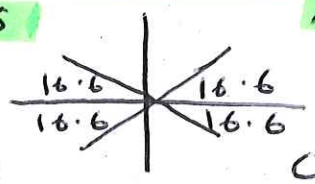

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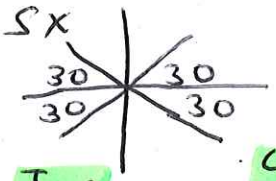


S X T✓ C ✓

$\theta = 16.6°, 163.4°, 210°, 330°$

b) $\frac{3}{\cos\theta} - \frac{5}{\sin\theta} = 0$

$\frac{3}{\cos\theta} = \frac{5}{\sin\theta}$

$3\sin\theta = 5\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{5}{3}$

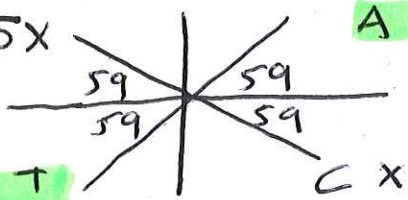
$\tan\theta = \frac{5}{3}$

$\left(\tan\theta \equiv \frac{\sin\theta}{\cos\theta}\right)$

$\theta = \text{shift tan}\left(\frac{5}{3}\right)$

$\theta = 59.0°$ A ✓

5X



✓T C X

$\theta = 59.0°, 239.0°$

12. Find all values of x satisfying the equation

$$\log_a(11x^2 + 16x + 5) - \log_a(4x^2 + 1) = 3\log_a 2. \quad [5]$$

12) Using subtraction law & power law

$$\log_a\left(\frac{11x^2+16x+5}{4x^2+1}\right) = \log_a 2^3$$

$$\therefore \quad \frac{11x^2+16x+5}{4x^2+1} = 8$$

$$11x^2+16x+5 = 8(4x^2+1)$$

$$11x^2+16x+5 = 32x^2+8$$

$$21x^2-16x+3 = 0$$

x ac	$+b$	
63	-16	$-7, -9$

$$21x^2-7x-9x+3=0$$

$$7x(3x-1)-3(3x-1)=0$$

$$(3x-1)(7x-3)=0$$

$$x=\frac{1}{3}, \quad x=\frac{3}{7}$$

13. Given that n is an even number, prove that $9n^2 + 6n$ has a factor of 12. [3]

13) If n is an even number, n can be written as $2m$

$n = 2m$

$$\therefore \quad 9n^2 + 6n = 9(2m)^2 + 6(2m)$$
$$= 36m^2 + 12m$$
$$= 12(3m^2 + m)$$

Hence, 12 is a factor of $9n^2 + 6n$ if n is even

14 The circle C_1 has centre $A(2, -1)$ and passes through the point $P(6, 1)$.

(a) (i) Show that the equation of C_1 is given by

$$x^2 + y^2 - 4x + 2y - 15 = 0.$$

(ii) Given that the point Q is such that PQ is a diameter of C_1, find the coordinates of Q.

(iii) Find the equation of the tangent to C_1 at P. [9]

(b) The circle C_2 has centre $B(-4, 7)$ and radius $\sqrt{8}$. Find the shortest distance between C_1 and C_2. Give your answer correct to one decimal place. [3]

14ai) Equation of circle given by $(x-a)^2+(y-b)^2=r^2$

where (a,b) = centre & r = radius

$\therefore$ $(x-2)^2+(y--1)^2=r^2$

Find r using distance $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

radius $=\sqrt{(6-2)^2+(1--1)^2}=\sqrt{20}$

$(x-2)^2+(y+1)^2=20$

$x^2-4x+4+y^2+2y+1=20$

$x^2+y^2-4x+2y-15=0$

(ii) Q $(-2,-3)$ A $(2,-1)$ P $(6,1)$ by inspection

iii) Find gradient of radius AP first

$=\frac{\text{change in } y}{\text{change in } x}=\frac{1--1}{6-2}=\frac{2}{4}=\frac{1}{2}$

$\therefore$ gradient of tangent $=-2$ (perpendicular)

Now, use $(y-y_1)=m(x-x_1)$

where $m=-2$ and $P=(x_1,y_1)=(6,1)$

$y-1=-2(x-6)$ or, $y=-2x+13$

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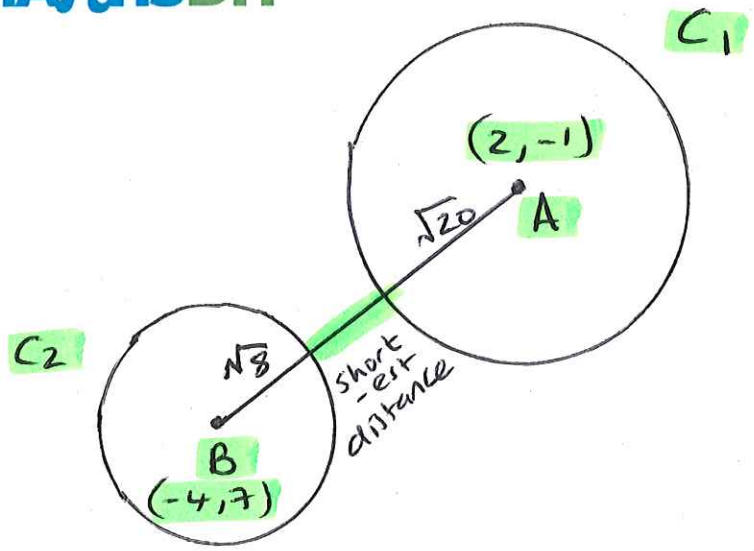
b)



First, work out distance AB

$$AB = \sqrt{(-4-2)^2 + (7--1)^2}$$

$$AB = \sqrt{36+64}, \quad \underline{AB = 10}$$

Now, shortest distance between the circles

= distance between the centres − sum of radii

$$= 10 - (\sqrt{8} + \sqrt{20}) = \underline{2.7}$$

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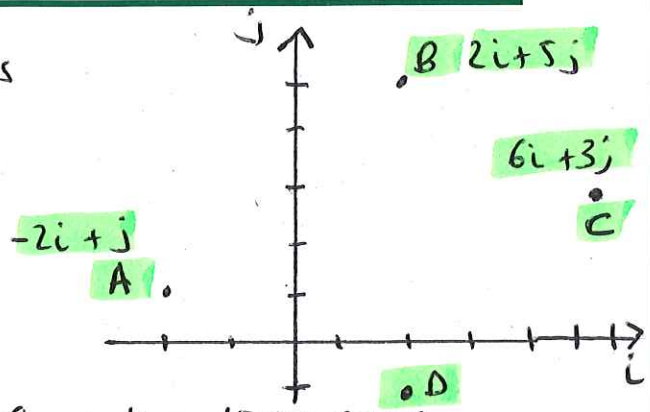
15. The points *A*, *B* and *C* have position vectors $-2\mathbf{i} + \mathbf{j}$, $2\mathbf{i} + 5\mathbf{j}$ and $6\mathbf{i} + 3\mathbf{j}$ respectively.

M is the midpoint of *BC*.

(a) Find the position vector of the point D such that $\overrightarrow{BC} = \overrightarrow{AD}$. [3]

(b) Find the magnitude of $\overrightarrow{AM}$. [3]

15) Sometimes a diagram really helps



midpoint of $\overrightarrow{BC}$ can be found from the diagram to be $4\underline{i} + 4\underline{j}$

a) D can be found informally from the diagram to be $2\underline{i} - \underline{j}$

or $\overrightarrow{BC} = 4\underline{i} - 2\underline{j} = \overrightarrow{AD}$

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

$4\underline{i} - 2\underline{j} = \overrightarrow{OD} - (-2\underline{i} + \underline{j})$, $\overrightarrow{OD} = 2\underline{i} - \underline{j}$

b) $\overrightarrow{OM} = 4\underline{i} + 4\underline{j}$ (Midpoint of $\overrightarrow{BC}$ above)

$\overrightarrow{AM} = \overrightarrow{OM} - \overrightarrow{OA} = 4\underline{i} + 4\underline{j} - (-2\underline{i} + \underline{j})$

$= 6\underline{i} + 3\underline{j}$

Magnitude of $\overrightarrow{AM} = |\overrightarrow{AM}| = \sqrt{6^2 + 3^2}$

$= \sqrt{45}$

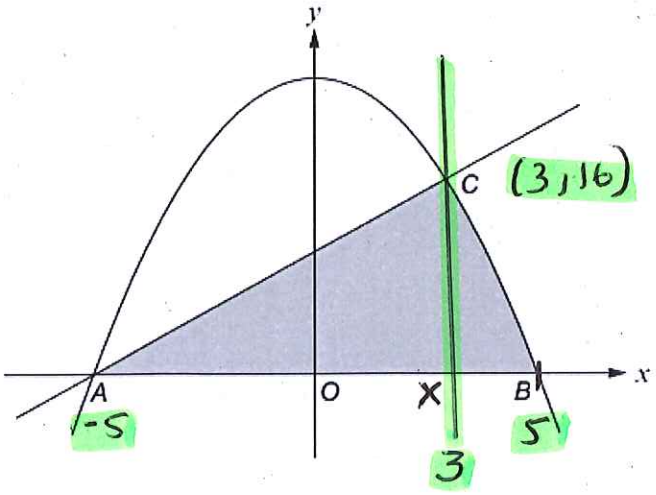
$= 3\sqrt{5}$

16.

(a) Find $\int\left(\sqrt[3]{x}-\frac{4}{x^{\frac{7}{2}}}\right)dx$. [2]

$= \int\left(x^{1/3} - 4x^{-7/2}\right)dx$

(b)



The diagram shows a sketch of the curve $y = 25 - x^2$ and the line $y = 2x + 10$. The curve and the line intersect at the points A and C. The curve intersects the x-axis at the points A and B. The coordinates of A, B and C are (– 5, 0), (5, 0) and (3, 16) respectively. Find the area of the shaded region. [6]

16 a) $\frac{x^{4/3}}{4/3} - \frac{4x^{-5/2}}{-5/2} + C$ No need to simplify

b) See diagram above.

Required Area

$=$ area of $\Delta AXC + \int_3^5 (25 - x^2)\,dx$

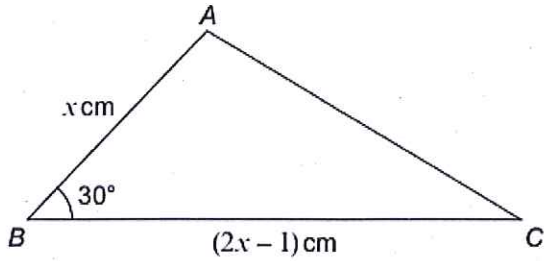
$= \frac{8 \times 16}{2} + \left[25x - \frac{x^3}{3}\right]_3^5$

$= 64 + \left[\left(25(5) - \frac{5^3}{3}\right) - \left(25(3) - \frac{3^3}{3}\right)\right]$

$= 64 + \left[\frac{250}{3} - 66\right] = \frac{244}{3}$

17.

(a) The diagram below shows a sketch of the triangle ABC with $AB = x$ cm, $BC = (2x - 1)$ cm and $A\hat{B}C = 30°$. The area of triangle ABC is 11·25 cm^2.



(i) Write down and simplify a quadratic equation satisfied by x.
Hence show that $x = 5$.

(ii) Find the length of AC. Give your answer correct to one decimal place. [6]

17a) Area $\triangle ABC = \frac{1}{2}ac\sin\hat{B}$

$11.25 = \frac{1}{2}(2x-1)(x)\sin 30$ (×2)

$22.5 = (2x^2 - x)(\frac{1}{2})$ (×2)

$45 = 2x^2 - x$

$2x^2 - x - 45 = 0$

× ac	+ b
−90	−1

$-10, 9$

$2x^2 - 10x + 9x - 45 = 0$

$2x(x-5) + 9(x-5) = 0$

$(2x+9)(x-5) = 0$

$x = -\frac{9}{2}$ (discard) negative length

$x = 5$ as required

b) cosine rule

$b^2 = a^2 + c^2 - 2ac\cos\hat{B}$

$AC^2 = 5^2 + 9^2 - 2(5)(9)\cos 30$

$AC^2 = 28.057\ldots$

$AC = 5.3$ cm (1dp)