PARAMETRIC & IMPLICIT DIFFERENTIATION

A2 Unit 3: Pure Mathematics B

WJEC past paper questions: 2010 – 2017

Total marks available 109 (approximately 2 hours 10 minutes)

1. *(a)* Given that

$$y^4 + 4x^2y = 3x^3 - 5x,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y. [4]

(b) Given that $x = 4t + \cos 2t$, $y = \sin 3t$, show that $\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ when $t = \frac{\pi}{12}$. [5]

(Summer 10)

2. *(a)* Given that

$$x^4 + 3x^2y - 2y^2 = 15,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y. [4]

(b) Given that $x = \ln t$, $y = t^3 - 7t$,

find an expression for $\frac{dy}{dx}$ in terms of t, [4]

(January 11)

3. *(a)* Given that

$$2x^3 + x^2 \cos y + y^4 + 2x - 25 = 0,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y. [4]

(b) Given that

$$x = t^3, \quad y = 2t^2 + 5t^4,$$

(i) find and simplify an expression for $\frac{dy}{dx}$ in terms of t,

(ii) show that there is no real value of t for which $\frac{dy}{dx} = 5$. [7]

(Summer 11)

4. Given that $x^2y^2 + x^4 + 6 = 2y^3 + 2x$, find the value of $\frac{dy}{dx}$ at the point (2, 3). [4]

(January 12)

5. *(a)* The curve C is defined by

$$x^3 - 4x^2y = 2y^3 - 3x - 2.$$

Find the value of $\frac{dy}{dx}$ at the point (3, 1). [4]

(b) Given that

$$x = \sin at,\ y = \cos at,$$

where a is a constant, find and simplify

an expression for $\frac{dy}{dx}$ in terms of a and t, [4]

(Summer 12)

6. *(a)* Given that

$$x^3 + 5x^4y - 2y^3 + 7 = 0,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y. [4]

(b) Given that $x = t^3 - 5$, $y = t^4 + 7t^5$,

find an expression for $\frac{dy}{dx}$ in terms of t, [4]

(January 13)

7. Given that, for $t > 0$,

$$x = \ln t, \; y = 5t^4,$$

find and simplify an expression for $\frac{dy}{dx}$ in terms of t, [4]

(Summer 13)

8. The curve C is defined by

$$x^3y^2 = 128.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of x and y. [3]

The point P lies on C and has coordinates (a, b).

(b) Given that the value of $\frac{dy}{dx}$ at the point P is 3,

(i) show that $b = -2a$,

(ii) find the value of a and the value of b. [4]

(Summer 13)

9. The variables x and y are defined parametrically in terms of the variable t. It is known that $x = 2t^3$ and that $\frac{dy}{dx} = 2t + 4t^3$.

(a) Find an expression for $\frac{dx}{dt}$ in terms of t. [1]

(b) Given that $y = 10$ when $t = 1$, find an expression for y in terms of t. [5]

(January 14)

10. The curve C is defined by

$$x^3 - 2x^2y + 3y^2 = 3.$$

Find the value of $\frac{dy}{dx}$ at the point $(-2, -1)$. [4]

(January 14)

11. The curve C is defined by

$$y^4 - 2x^2 + 8xy^2 + 9 = 0.$$

(a) Show that $\frac{dy}{dx} = \frac{x - 2y^2}{y^3 + 4xy}$. [4]

(b) Show that there is no point on C at which $\frac{dy}{dx} = 0$. [4]

(Summer 14)

12. Given that $x = 2e^t - 5$, $y = 8e^{-t} + 3e^t - 4$, find the value of t when $\frac{dy}{dx} = -1$.

Give your answer correct to three decimal places. [7]

(Summer 14)

13. The curve C_1 is defined by

$$x^3 + 2x\cos y + y^2 = 1 + \frac{\pi}{4}^2.$$

Find the value of $\frac{dy}{dx}$ at the point $(1, \frac{\pi}{2})$. [4]

(Summer 15)

14. Given that $x = \tan^{-1} t$, $y = \ln t$, where $t > 0$,

find an expression for $\frac{dy}{dx}$ in terms of t, [4]

(Summer 15)

15. The curve C is defined by

$$x^2 + 3xy + 2y^3 - 2x = 21.$$

The point P has coordinates $(-5, 2)$ and lies on C.

Find the value of $\frac{dy}{dx}$ at P. [4]

(Summer 16)

16. A function is defined parametrically by

$$x = 4\sin 3t, \; y = 2\cos 3t.$$

Find and simplify an expression for $\frac{dy}{dx}$ in terms of t. [4]

(Summer 16)

17. *(a)* Given that

$$x^4 - 3x^2y + 2y^3 - 4x = 7,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y. [4]

(b) Given that $x = 7t + 2t^2$, $y = \frac{4+3t}{7+4t}$,

show that $\frac{dy}{dx} = \frac{k}{(7+4t)^n}$,

where the values of the constants k and n are to be found, [5]

(Summer 17)