ITERATION & LOCATION OF ROOTS

A2 Unit 3: Pure Mathematics B

WJEC past paper questions: 2010 – 2017

Total marks available 90 (approximately 1 hour 50 minutes)

1. Show that the equation

$$4x^3 - 2x - 5 = 0$$

has a root α between 1 and 2.

The recurrence relation $x_{n+1} = \left(\frac{2x_n + 5}{4}\right)^{\frac{1}{3}}$

with $x_0 = 1.2$, may be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this value is the value of α correct to five decimal places. (7)

(Summer 10)

2. **You may assume** that the equation $6x^4 + 7x - 3 = 0$ has a root α between 0 and 1. The recurrence relation

$$x_{n+1} = \frac{3 - 6x_n^4}{7}$$

with $x_0 = 0.4$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to four decimal places and show this is the value of α correct to four decimal places. (5)

(January 11)

3. (a) Show that $f(x) = 11\tan^{-1}2x - 3x^2$ has a stationary value when x satisfies

$$12x^3 + 3x - 11 = 0.$$ (3)

(b) **You may assume** that the equation $12x^3 + 3x - 11 = 0$ has a root α between 0 and 1. The recurrence relation

$$x_{n+1} = \left(\frac{11 - 3x_n}{12}\right)^{\frac{1}{3}}$$

with $x_0 = 0.9$, may be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and show this is the value of α correct to five decimal places. (5)

(Summer 11)

4. (a) A function is defined parametrically by

$$x = 3t^2, \quad y = t^6 - 4t^3.$$

(i) Find $\frac{dy}{dx}$ in terms of t.

(ii) Given that $\frac{dy}{dx} = \frac{7}{2}$, show that $2t^4 - 4t - 7 = 0$. (5)

(b) Show that the equation

$$2t^4 - 4t - 7 = 0$$

has a root α between 1 and 2.

The recurrence relation

$$t_{n+1} = \left(\frac{4t_n + 7}{2}\right)^{\frac{1}{4}}$$

with $t_0 = 1.6$, can be used to find α. Find and record the values of t_1, t_2, t_3, t_4. Write down the value of t_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. (7)

(January 12)

5. Show that the equation

$$\cos x - 5x + 2 = 0$$

has a root α between 0 and $\frac{\pi}{4}$.

The recurrence relation

$$x_{n+1} = \frac{1}{5}(2 + \cos x_n)$$

with $x_0 = 0.6$, can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. (7)

(Summer 12)

6. (a) On the same diagram, sketch the graphs $y = \ln x$ and $y = 11 - 2x$.
Deduce the number of roots of the equation

$$\ln x + 2x - 11 = 0.$$ (3)

(b) **You may assume** that the equation

$$\ln x + 2x - 11 = 0.$$

has a root α between 4 and 5.

The recurrence relation

$$x_{n+1} = \frac{11 - \ln x_n}{2}$$

with $x_0 = 4.7$, can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. (5)

(January 13)

7. **You may assume** that the equation

$$x^2 + e^x - 3 = 0$$

has a root α between -2 and -1.
The recurrence relation

$$x_{n+1} = -(3 - e^{x_n})^{\frac{1}{2}}$$

with x_0 = -1.5, can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. (5)

(Summer 13)

8. **You may assume** that the equation $x^3 + 7x^2 - 3 = 0$ has a root α between 0 and 1.
The recurrence relation

$$x_{n+1} = \sqrt{\frac{3}{x_n + 7}}$$

with x_0 = 1, can be used to find α. Find and record the values of x_1, x_2, x_3, x_4.
Write down the value of x_4 correct to five decimal places and show this is the value of α correct to five decimal places. (5)

(January 14)

9. (a) Show that $f(x) = \ln(3x^2 - 2x - 1) - 4x^2$ has a stationary value where x satisfies

$$12x^3 - 8x^2 - 7x + 1 = 0.$$ (4)

(b) **You may assume** that the equation $12x^3 - 8x^2 - 7x + 1 = 0$ has a root α between -1 and 0.

The recurrence relation $$x_{n+1} = \left(\frac{8x_n^2 + 7x_n - 1}{12}\right)^{\frac{1}{3}}$$

with x_0 = - 0.6, can be used to find α. Find and record the values of x_1, x_2, x_3, x_4.
Write down the value of x_4 correct to four decimal places and show this is the value of α correct to four decimal places. (5)

(Summer 14)

10. (a) On the same diagram, sketch the graphs of $y = \cos^{-1} x$ and $y = 5x - 1$. (2)

(b) **You may assume** that the equation

$$\cos^{-1} x - 5x + 1 = 0$$

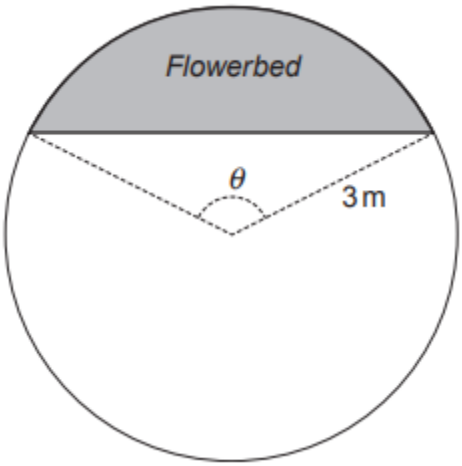
has a root α between 0.4 and 0.5.

The recurrence relation $$x_{n+1} = \frac{1}{5}(1 + \cos^{-1} x_n)$$

with x_0 = 0.4, can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to four decimal places and prove that this is the value of α correct to four decimal places. (5)

(Summer 15)

11. The diagram shows a circular garden plot of radius 3m. Alun wants to use a minor segment of the plot as a flowerbed and has a 13.5m length of edging, all of which he intends to use to form the perimeter of the shaded area below. The angle subtended at the centre of the circular plot is denoted by θ radians.



(a) Show that θ satisfies the equation

$$\theta + 2\sin\left(\frac{\theta}{2}\right) = 4.5. \qquad (3)$$

(b) Alun believes that the value of θ will turn out to be approximately 2.5.

Starting with $\theta_0 = 2.5$ use the recurrence relation

$$\theta_{n+1} = 4.5 - 2\sin\left(\frac{\theta_n}{2}\right)$$

to find the values of $\theta_1, \theta_2, \theta_3$. Write down the value of θ_3 correct to two decimal places and prove that this is the value of θ correct to two decimal places. (5)

(Summer 16)

12. A large tank in the form of a cuboid is used to store water. The width of the tank is denoted by xm. The length of the tank is 4m **greater** than its width, whilst the height of the tank is 2m **less** than its width. The volume of the tank is 150m^3.

(a) (i) Show that $x^3 + 2x^2 - 8x - 150 = 0$.
(ii) Show that $5 < x < 6$. (4)

(b) The recurrence relation $x_{n+1} = (150 + 8x_n + 1 - 2x_n^2)^{\frac{1}{3}}$,

with $x_0 = 6$, can be used to find the value of x. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to two decimal places and prove that this is the value of x correct to two decimal places. (5)

(Summer 17)